OPTIMAL PARAMETER ESTIMATION IN HETEROGENEOUS CLUTTER FOR HIGH RESOLUTION POLARIMETRIC SAR DATA

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ABSTRACT

This paper presents a new estimation scheme for optimally deriving clutter parameters with high resolution POLSAR data. The heterogeneous clutter in POLSAR data was described by the Spherically Invariant Random Vectors model. Three parameters were introduced for the high resolution POLSAR data clutter: the span, the normalized texture and the speckle normalized covariance matrix. The asymptotic distribution of the novel span estimator is also investigated. The proposed method is tested with airborne POLSAR images provided by the ONERA RAMSES system.

Index Terms— SAR, polarimetry, estimation

1. INTRODUCTION

The recently launched polarimetric SAR (POLSAR) systems are now capable of producing high quality images of the Earth's surface with meter resolution. The goal of the estimation process is to derive the scene signature from the observed data set. In the case of spatially changing surfaces ("heterogeneous" or "textured" scenes) the first step is to define an appropriate model describing the dependency between the polarimetric signature and the observable as a function of the speckle. In general, the multiplicative model has been employed for POLSAR data processing as a product between the square root of a scalar positive quantity (texture) and the description of an equivalent homogeneous surface (speckle) [1]. The objective of this paper is to present a new parameter estimation technique based on the Spherically Invariant Random Vectors (SIRV) model.

2. SIRV CLUTTER MODEL WITH NORMALIZED TEXTURE

The SIRV is a class of non-homogeneous Gaussian processes with random variance [2]. The complex m-dimensional measurement $\mathbf{k}$ is defined as the product between the independent complex circular Gaussian vector $\zeta \sim \mathcal{N}(0, [T])$ (speckle) with zero mean and covariance matrix $[T] = E\{\zeta\zeta^\dagger\}$ and the square root of the positive random variable ξ (representing the texture):

$$\mathbf{k} = \sqrt{\xi} \cdot \zeta. \tag{1}$$

It is important to notice that in the SIRV definition, the probability density function (PDF) of the texture random variable is not explicitly specified. As a consequence, SIRVs describe a whole class of stochastic processes.

For POLSAR clutter, the SIRV product model is the product of two separate random processes operating across two different statistical axes [3]. The polarimetric diversity is modeled by the multidimensional Gaussian kernel. The randomness of spatial variations in the radar backscattering from cell to cell is characterized by ξ. Relatively to the polarimetric axis, the texture random variable ξ can be viewed as a unknown deterministic parameter from cell to cell.

The texture and the covariance matrix unknown parameters can be estimated from the ML theory. For N i.i.d. secondary data, let $L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N | [T], \xi_1, ..., \xi_N)$ be the likelihood function to maximize with respect to $[T]$ and ξ_i.

$$L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N; [T], \xi_1, ..., \xi_N) = \frac{1}{\pi^{mN} \det\{[T]\}^N} \times$$
$$\times \prod_{i=1}^{N} \frac{1}{\xi_i^m} \exp\left(-\frac{\mathbf{k}_i^\dagger [T]^{-1} \mathbf{k}_i}{\xi_i} \right). \tag{2}$$

The corresponding ML estimators are given by [4]:

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N | [T], \xi_1, ..., \xi_N)}{\partial \xi_i} = 0 \Leftrightarrow \widehat{\xi}_i = \frac{\mathbf{k}_i^\dagger [T]^{-1} \mathbf{k}_i}{m}, \tag{3}$$

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N | [T], \xi_1, ..., \xi_N)}{\partial [T]} = 0 \Leftrightarrow [\widehat{T}] = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \mathbf{k}_i^\dagger}{\widehat{\xi}_i}. \tag{4}$$

As the variables ξ_i are unknown, the following normalization constraint on the texture parameters assures that the ML estimator of the speckle covariance matrix is the Sample Covariance Matrix (SCM):

$$[\widehat{T}] = \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \mathbf{k}_i^\dagger = [\widehat{T}]_{SCM} \Leftrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \mathbf{k}_i^\dagger \left(1 - \frac{1}{\widehat{\xi}_i} \right) = [0_m]. \tag{5}$$

The generalized ML estimator for ξ_i are obtained by introducing Eq. 5 in Eq. 3:

$$\widehat{\xi_i} = \frac{\mathbf{k}_i^\dagger [\widehat{T}]_{SCM}^{-1} \mathbf{k}_i}{m}. \tag{6}$$

Note the $\mathbf{k}_i$ primary data is the cell under study.

3. SIRV CLUTTER MODEL WITH NORMALIZED COVARIANCE MATRIX

Let now the covariance matrix be of the form $[T] = \sigma_0[M]$, such that $\text{Tr}\{[M]\} = 1$. The product model form Eq. 1 can be also written as:

$$\mathbf{k} = \sqrt{\tau} \cdot \mathbf{z}, \tag{7}$$

where $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, [M])$. σ_0 and ξ are two scalar positive random variables such that $\tau = \sigma_0 \cdot \xi$.

The likelihood function is:

$$L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N; [M], \tau_1, ..., \tau_N) = \frac{1}{\pi^{mN} \text{det}\{[M]\}^N} \times$$
$$\times \prod_{i=1}^{N} \frac{1}{\tau_i^m} \text{exp}\left(-\frac{\mathbf{k}_i^\dagger [M]^{-1} \mathbf{k}_i}{\tau_i}\right). \tag{8}$$

Using the same procedure as in Sect. 2, the corresponding texture and normalized covariance ML estimators are given by:

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N; [M], \tau_1, ..., \tau_N)}{\partial \tau_i} = 0 \Leftrightarrow \widehat{\tau}_i = \frac{\mathbf{k}_i^\dagger [M]^{-1} \mathbf{k}_i}{m}, \tag{9}$$

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_1, ..., \mathbf{k}_N; [M], \tau_1, ..., \tau_N)}{\partial [M]} = 0 \Leftrightarrow [\widehat{M}] = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \mathbf{k}_i^\dagger}{\widehat{\tau}_i}. \tag{10}$$

Given the fact that the covariance matrix is normalized, it is possible to compute the generalized ML estimator of $[M]$ as the solution of the following recursive equation:

$$[\widehat{M}]_{FP} = f([\widehat{M}]_{FP}) = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \mathbf{k}_i^\dagger}{\mathbf{k}_i^\dagger [\widehat{M}]_{FP}^{-1} \mathbf{k}_i}. \tag{11}$$

This approach has been used in [5] by Conte et al. to derive a recursive algorithm for estimating the matrix $[M]$. This algorithm consists in computing the Fixed Point of f using the sequence $([M]_i)_{i \geq 0}$ defined by:

$$[M]_{i+1} = f([M]_i). \tag{12}$$

This study has been completed by the work of Pascal et al. [6], [7], which recently established the existence and the uniqueness, up to a scalar factor, of the Fixed Point estimator of the normalized covariance matrix, as well as the convergence of the recursive algorithm whatever the initialization. The algorithm can therefore be initialized with the identity matrix $[\widehat{M}]_0 = [I_m]$.

The generalized ML estimator for the τ_i texture for the primary data $\mathbf{k}_i$ is given by:

$$\widehat{\tau}_i = \frac{\mathbf{k}_i^\dagger [\widehat{M}]_{FP}^{-1} \mathbf{k}_i}{m}. \tag{13}$$

4. MAIN RESULT

The span (total power) σ_0 can be derived using the covariance matrix estimators presented in Sect. 2 and Sect. 3 as:

$$\widehat{\sigma_0} = \frac{\mathbf{k}^\dagger [\widehat{M}]_{FP}^{-1} \mathbf{k}}{\mathbf{k}^\dagger [\widehat{T}]_{SCM}^{-1} \mathbf{k}}. \tag{14}$$

Note that Eq. 14 is valid when considering N identically distributed linearly independent secondary data and one primary data. It can be seen as a double polarimetric whitening filter issued from two equivalent SIRV clutter models: with normalized texture variables and with normalized covariance matrix parameter.

The main advantage of the proposed estimation scheme is that it can be directly applied with standard boxcar neighborhoods. Fig. 1 illustrates the span σ_0 estimation with high resolution POLSAR X-band data acquired by the ONERA RAMSES system with a spatial resolution of approximately 1.5 m. The 5×5 boxcar neighborhood has been selected for illustration, hence 24 secondary samples and 1 primary data. The proposed estimator from Fig. 1-(c) exhibits better performances in terms of spatial resolution preservation than the standard span estimator illustrated in Fig. 1-(b).

Finally, Fig. 2 presents the three SIRV parameters which completely describe the POLSAR data set: the total power, the normalized texture and the normalized covariance matrix.

5. ASYMPTOTIC STATISTICS OF $\widehat{\sigma_0}$

This section is dedicated to the study of large sample properties and approximations of the span estimator $\widehat{\sigma_0}$ form Eq. 14.

On one hand, the asymptotic distribution of the FP estimator from Eq. 11 has been derived in [7]. The FP estimator computed with N secondary data converges in distribution to the normalized SCM computed with $N[m/(m+1)]$ secondary data. Since the normalized SCM is the SCM up to a scale factor, we may conclude that, in problems invariant with respect to a scale factor on the covariance matrix, the FP estimate is asymptotically equivalent to the SCM computed with $N[m/(m+1)]$ secondary data. Hence one can set the degrees of freedom of FP normalized covariance matrix estimators as:

$$q_1 = N \frac{m}{m+1}. \tag{15}$$

On the other hand, the bivariate Gamma PDF has been established by Chatelain et al. in [8]:

$$P_{b\Gamma}(y_1, y_2; p_1, p_2, p_{12}, q_1, q_2).$$

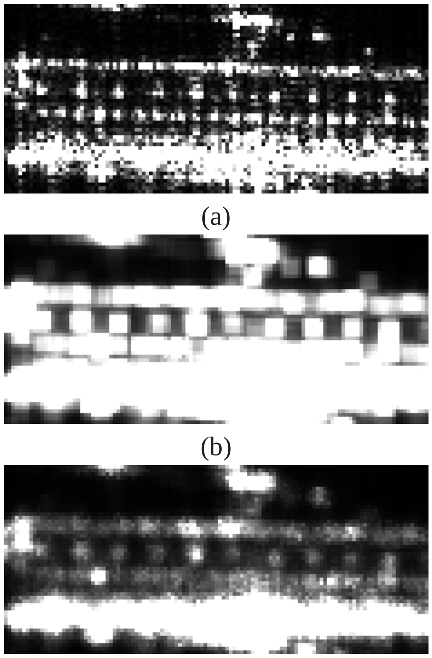


Fig. 1. Brétigny, RAMSES POLSAR data, X-band. (a) initial 1-look span estimated as $\sigma_{SLC} = \mathbf{k}^{\dagger}\mathbf{k}$, (b) 25-look span estimated as $\sigma_{SCM} = \text{Tr}\left\{[\widehat{T}]_{SCM}\right\}$, and (c) span estimated using $\widehat{\sigma_0}$ from Eq. 14

The scale parameters p_2 and p_1, the shape parameters $q_2 > q_2$ and p_{12} are linked to the mean parameters μ_1, μ_2, to the number of degrees of freedom n_1, n_2, and to the normalized correlation coefficient ρ such as:

$$q_1 = n_1,\ q_2 = n_2,\ p_1 = \frac{\mu_1}{q_1},\ p_2 = \frac{\mu_2}{q_2},\ p_{12} = \frac{\mu_1\mu_2}{q_1 q_2}(1-\rho).$$

Using these results, we derived the PDF of the ratio $R = y_1/y_2$ of two correlated Gamma random variables:

$$P_{R\Gamma}(R, p_1, p_2, p_{12}, q_1, q_2) = R^{q_1-1}\left(\frac{p_2}{p_{12}}\right)^{q_1}\left(\frac{1}{p_2}\right)^{q_2}\times$$

$$\times\left(\frac{p_{12}}{p_1 + Rp_2}\right)^{q_2+q_1}\frac{\Gamma(q_1+q_2)}{\Gamma(q_1)\Gamma(q_2)}\times \qquad (16)$$

$$\times\mathbf{H}_3\left[q_1+q_2, q_2-q_1, q_2; R\frac{p_1p_2-p_{12}}{(p_1+Rp_2)^2}, \frac{p_1p_2-p_{12}}{p_2(p_1+Rp_2)}\right],$$

where $\mathbf{H}_3(\alpha,\beta,\gamma;x,y) = \sum_{m,n=0}^{\infty}\frac{(\alpha)_{2m+n}(\beta)_n}{(\gamma)_{m+n}m!n!}x^m y^n$ is one of the twenty convergent confluent hypergeometric series of order two (Horn function), and $(\alpha)_n$ is the Pochhammer symbol such that $(a)_0 = 1$ and $(a)_{k+1} = (a+k)(a)_k$ for any positive integer k [9].

By taking into consideration both Eqs. 15, 16 and the Cochran's theorem, the PDF of the span estimator from Eq. 14 converges asymptotically to the the ratio of two correlated Gamma random variables PDF (the ratio of two quadratics). Moreover, the degrees of freedom n_1 and n_2 are set to $N[m/(m+1)]$ and N (the number of secondary data), respectively.

Fig. 3 illustrates the behavior of the σ_0 PDF with respect to the normalized correlation coefficient ρ. The PDF parameters are set according to the processing illustrated in Sect. 4, namely $N = 24$, $m = 3$, $\mu_1 = 10$, $\mu_2 = 1$. Notice that when the normalized correlation coefficient approaches to 1, the PDF tends to a Dirac.

A Monte Carlo simulation has been represented in Fig. 3, also. 5000 samples of σ_0 were obtained by computing 5000 times 24 samples drown from a zero-mean multivariate circular complex Gaussian distribution with a covariance matrix selected from the real POLSAR data. The span of the selected covariance matrix equal 10. One can observe the good correspondence between the empirical PDF of simulated σ_0 and the PDF derived in Eq. 16 for $\rho = 0.95$.

Using the same parameters as in the previous Monte Carlo simulation, Fig. 4 illustrates the behavior of the empirical mean and variance of the proposed σ_0 in Gaussian clutter (e.g. in homogeneous regions). By using 24 up to 48 secondary data, the estimation bias is negligible and the empirical variance is close to zero.

6. CONCLUSIONS

This paper presented a new estimation scheme for optimally deriving clutter parameters with high resolution POLSAR images. The heterogeneous clutter in POLSAR data was described by the SIRV model. Three estimators were introduced for describing the high resolution POLSAR data set: the span, the normalized texture and the speckle normalized covariance matrix. The asymptotic distribution of the new span estimator has been established. The estimation bias on homogeneous regions have been assessed also by Monte Carlo simulations.

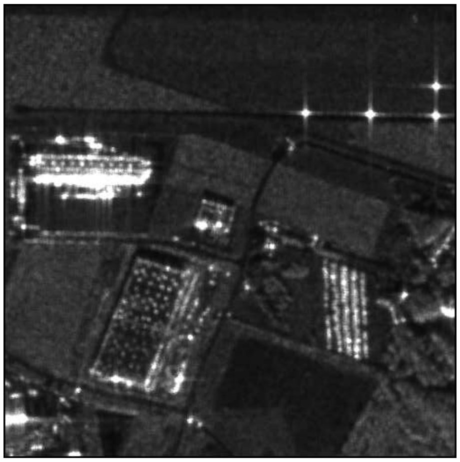
7. REFERENCES

[1] L. M. Novak and M. C. Burl, "Optimal speckle reduction in polarimetric SAR imagery," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 26, no. 2, pp. 293–305, 1990.

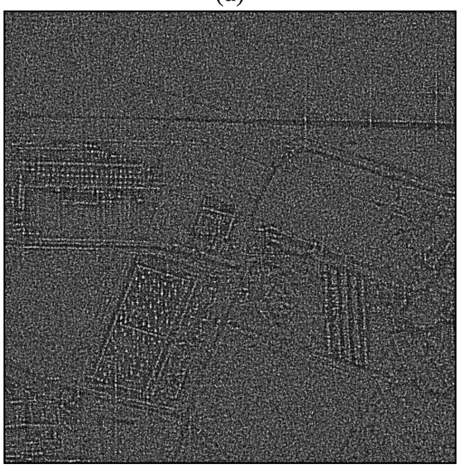
[2] B. Picinbono, "Spherically invariant and compound Gaussian stochastic processes," *IEEE Transactions on Information Theory*, vol. 16, no. 1, pp. 77–79, 1970.

[3] G. Vasile, J. P. Ovarlez, F. Pascal, and C. Tison, "Coherency matrix estimation of heterogeneous clutter in high resolution polarimetric SAR images," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 4, pp. 1809–1826, 2010.

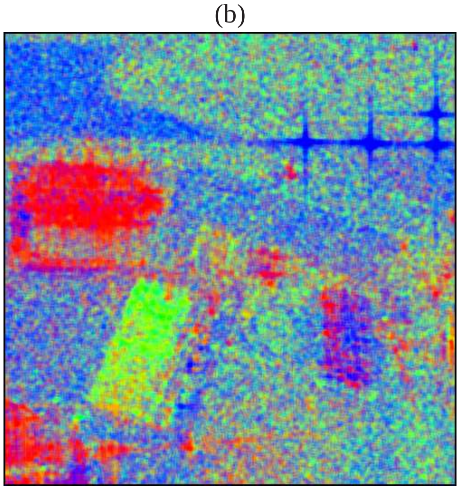
[4] F. Gini and M. V. Greco, "Covariance matrix estimation for CFAR detection in correlated heavy tailed clutter," *Signal Processing*, vol. 82, no. 12, pp. 1847–1859, 2002.



(a)



(b)



(c)

Fig. 2. Brétigny, RAMSES POLSAR data, X-band. (a) span estimated using $\widehat{\sigma_0}$ from Eq. 14, (b) normalized texture ξ, and (c) color composition of the normalized coherency diagonal elements $[M]_{11}$-$[M]_{33}$-$[M]_{22}$.

[5] E. Conte, A. DeMaio, and G. Ricci, “Recursive estimation of the covariance matrix of a compound-Gaussian process and its application to adaptive CFAR detection,” *IEEE Transactions on Image Processing*, vol. 50, no. 8, pp. 1908–1915, 2002.

[6] F. Pascal, Y. Chitour, J. P. Ovarlez, P. Forster, and P. Larzabal, “Covariance structure maximum-likelihood estimates in compound Gaussian noise: existence and algorithm analysis,” *IEEE Transactions on Signal Processing*, vol. 56, no. 1, pp. 34–48, 2008.

[7] F. Pascal, P. Forster, J. P. Ovarlez, and P. Larzabal, “Performance analysis of covariance matrix estimates in impulsive noise,” *IEEE Transactions on Signal Processing*, vol. 56, no. 6, pp. 2206–2216, 2008.

[8] F. Chatelain, J. Y. Tourneret, and J. Inglada, “Change detection in multisensor SAR images using bivariate gamma distributions,” *IEEE Transactions on Image Processing*, vol. 17, no. 3, pp. 249–258, 2008.

[9] F. O. A. Erdlyi, W. Magnus, and F. Tricomi, *Higher Transcendental Functions*, vol. 1, New York: Krieger, 1981.

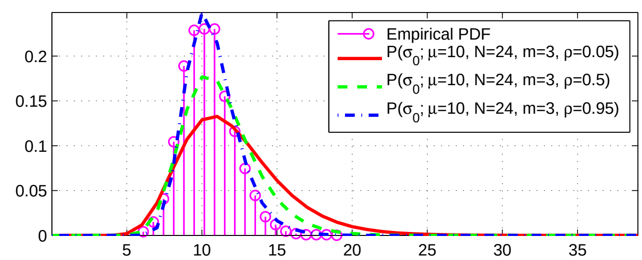


Fig. 3. Ratio PDF of two correlated Gamma random variables (Eq. 16) for different ρ and the empirical PDF of simulated σ_0 in Gaussian clutter

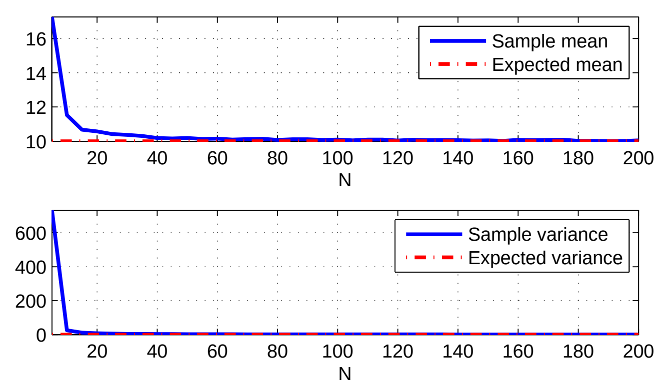


Fig. 4. Empirical mean and variance of the σ_0 estimator from Eq. 14 and the their expected values for simulated Gaussian clutter.