Bayesian Optimum Radar Detector Performances Against Ground Clutter Data

Jean-Philippe Ovarlez, Emmanuelle Jay and Frédéric Pascal

ONERA, DEMR/TSI, BP 72, 92322 Chatillon, France
Email : ovarlez@onera.fr, ejay@noos.fr

Abstract : In this paper detection performances of the Bayesian Optimum Radar Detector (BORD) against non-Gaussian ground clutter data are showed in the case of an unknown complex amplitude target. Recalling first how BORD was derived from the non-Gaussian SIRP model (Spherically Invariant Random Process) clutter we derive theoretical performances of the asymptotical expression of BORD which is CFAR with respect to the texture PDF of the SIRV. Then we compare BORD performances obtained in non-Gaussian ground clutter data with those of well-known optimum radar detectors such as Optimum K-Detector (OKD) and Optimum Gaussian Detector (OGD).

Keywords : Radar Detection, Non-Gaussian Clutter, SIRP

1. Introduction

Coherent radar detection against non-Gaussian clutter has gained many interests in the radar community since experimental clutter measurements made by organizations like MIT [1] showed to fit non-Gaussian statistical models. One of the most tractable and elegant non-Gaussian model results in the so-called *Spherically Invariant Random Process* (SIRP) theory which states that some non-Gaussian random processes are the product of a Gaussian random process - called *speckle* - with a non-negative random variable (r.v.) - called *texture*. This model is the base of many results [2, 3, 4, 5, 6, 7] where, for example in Gini and al.'s works [7], is derived the optimum detector in the presence of composite disturbance of known statistics modeled as SIRP.

In previous authors'works [8, 9], a bayesian approach was proposed to determine the PDF of the *texture* (the characteristic PDF of the SIRP) from N reference clutter cells. For this task, the Bayes'rule and a Monte Carlo integration given a *non informative* prior on the variance PDF were used. This approach exploits the SIRP model particularity to describe non-Gaussian processes as compound processes and allows to derive the expression of the optimum detector called Bayesian Optimum Radar Detector (BORD). Detection performances of BORD showed that this detector gives optimum performances whatever is the nature of the simulated data. Moreover BORD was showed to be an adaptive detector : it is so no more necessary to have any knowledge about the clutter statistics but BORD deals directly with the received data.

In this paper, we propose to evaluate BORD performances against non-Gaussian ground clutter data and to compare the results to other optimum detectors such as Optimum K-detector and Optimum Gaussian detector. These comparisons are possible and significant since parts of the ground clutter data statistics are showed to be closed to a K-distribution or a Gaussian PDF.

In section 2 and 3 we briefly recall the formulation of a detection problem and describe the bayesian approach used to determine a bayesian estimator to the *texture* PDF and give the expression of the resulting Bayesian Optimum Radar Detector (BORD). Section 4. is devoted to the theoretical performances of the asymptotical expression of the BORD (called Asymptotical BORD) in the case where the correlation matrix is non-singular. In section 5. we first briefly describe the ground clutter data and then we show BORD performances obtained thanks to Monte Carlo computation. Conclusion and outlook are given in section 6.

2. General relations of detection theory

We consider here the basic problem of detecting the presence (H_1) or absence (H_0) of a complex signal **s** in a set of N measurements of m-complex vectors $\mathbf{y} = \mathbf{y}_\mathrm{I} + \mathrm{j}\,\mathbf{y}_\mathrm{Q}$ corrupted by a sum **c** of independent additive complex noises (noises + clutter). The problem can be described in terms of a statistical hypothesis test :

$$H_0 : \mathbf{y} \quad = \quad \mathbf{c} \tag{1}$$

$$H_1 : \mathbf{y} \quad = \quad \mathbf{s} + \mathbf{c} \tag{2}$$

When present, the target signal **s** corresponds to a modified version of the perfectly known transmitted signal **t** and can be rewritten as $\mathbf{s} = A\,T(\underline{\theta})\,\mathbf{t}$. A is the target amplitude and we suppose determined all the others parameters ($\underline{\theta}$) which characterize the target (Doppler frequency, time delay, ...). In the following, $\mathbf{p} = T(\underline{\theta})\,\mathbf{t}$. The observed vector **y** is used to form the Likelihood Ratio Test (LRT) $\Lambda(\mathbf{y})$ which is compared with a threshold η set to a desired false alarm probability (P_{fa}) value :

$$\Lambda(\mathbf{y}) = \frac{p_\mathbf{y}(\mathbf{y}/H_1)}{p_\mathbf{y}(\mathbf{y}/H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{3}$$

LRT performances follow from the statistics of the data. P_{fa} is the probability of choosing H_1 when the target is absent, and the detection probability (P_d) is the probability of choosing H_1 when the target is present, that is :

$$P_{fa} = \mathbb{P}(\Lambda(\mathbf{y}) \underset{H_0}{>} \eta) \quad \text{and} \quad P_d = \mathbb{P}(\Lambda(\mathbf{y}) \overset{H_1}{>} \eta) \tag{4}$$

3. Non-Gaussian clutter case : SIRV and Bayesian Optimum Radar Detector

In the case of non-Gaussian clutter, detection strategies can be derived if an *a priori* hypothesis is made on the clutter statistics. To model non-Gaussian clutter and derive general detector expressions, we use the SIRP representation [2, 10, 11].

3.1 Description and general expressions

SIRV (Vector) model interprets each element of the clutter vector **c** as the product of a m-complex Gaussian vector **x** ($\mathcal{CN}(\mathbf{0}, 2\,\mathbf{M})$) with a positive r.v. τ, the so-called *texture*, that is $\mathbf{c} = \mathbf{x}\,\sqrt{\tau}$.

The PDF of the variable τ is the so-called *characteristic PDF* of the SIRV and the so formed vector **c** is, conditionally to τ, a complex Gaussian random vector ($\mathcal{CN}(\mathbf{0}, 2\,\tau\,\mathbf{M})$) with joint PDF $p(\mathbf{c}/\tau)$. The marginal PDF of the clutter is then :

$$p(\mathbf{c}) = \int_0^{+\infty} \frac{1}{(2\pi\tau)^m|\mathbf{M}|} \exp\left(-\frac{\mathbf{c}^\dagger\mathbf{M}^{-1}\mathbf{c}}{2\tau}\right) p(\tau)d\tau. \tag{5}$$

where † is the transpose conjugate operator, and $|\mathbf{M}|$ is the determinant of the matrix **M**.

This general expression allows to determine, for a known $p(\tau)$, joint PDFs of non-Gaussian random vectors.

3.2 SIRP Optimum Detector

Applied to the detection problem, expression (5) gives $p_{\mathbf{c}}(\mathbf{y}/H_0)$ and $p_{\mathbf{c}}(\mathbf{y}/H_1) = p_{\mathbf{c}}(\mathbf{y}-\mathbf{s}/H_0)$ when the target signal **s** is known. The LRT becomes (with the same notations as in [7]) :

$$\int_0^{+\infty} \left[\exp\left(-\frac{q_1(\mathbf{y})}{2\tau}\right) - \exp\left(\lambda - \frac{q_0(\mathbf{y})}{2\tau}\right)\right] \frac{p(\tau)}{\tau^m}\, d\tau \underset{H_0}{\overset{H_1}{\gtrless}} 0 \tag{6}$$

where $q_0(\mathbf{y}) = \mathbf{y}^\dagger\mathbf{M}^{-1}\mathbf{y}$, $q_1(\mathbf{y}) = q_0(\mathbf{y}-\mathbf{s})$ for a known signal **s** and $\lambda = \ln(\eta)$.

When the target signal **s** is unknown, ML estimation of A is performed and the detection strategy is given by (6) where now :

$$q_1(\mathbf{y}) = \mathbf{y}^\dagger\mathbf{M}^{-1}\mathbf{y} - \frac{|\mathbf{p}^\dagger\mathbf{M}^{-1}\mathbf{y}|^2}{\mathbf{p}^\dagger\mathbf{M}^{-1}\mathbf{p}}. \tag{7}$$

3.3 Bayesian Optimum Radar Detector (BORD)

For a known *texture* PDF $p(\tau)$ it is possible to derive the associated detector expression. The idea of a bayesian approach is to determine, from N clutter reference cells of size m, $\mathbf{R} = [\mathbf{r}_1, \ldots, \mathbf{r}_N]^T$ where $\mathbf{r}_i = [r_i(1), \ldots, r_i(m)]^T$, a bayesian estimator $\hat{p}_N(\tau)$ of the *texture* PDF $p(\tau)$. We write $p(\tau)$ as follows :

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r})p(\mathbf{r})\, d\mathbf{r}, \tag{8}$$

Given $\mathbf{r}_{i=1}^N$ a Monte Carlo estimation of (8) is [12] :

$$\hat{p}_N(\tau) = \frac{1}{N}\sum_{i=1}^N p(\tau/\mathbf{r}_i), \tag{9}$$

and the Bayes'rule provides :

$$p(\tau/\mathbf{r}_i) = \frac{p(\mathbf{r}_i/\tau)\, g(\tau)}{p(\mathbf{r}_i)}, \tag{10}$$

where $g(\tau)$ is the prior distribution of τ for the reference cells and the normalization constant $p(\mathbf{r}_i)$ is obtained by integrating the numerator in (10) over $g(\tau)$. As the clutter statistics is unknown, a non-informative prior distribution $g(\tau) = 1/\tau$ is chosen to retrieve the *a posteriori* PDF of τ given the N reference cells. So (10) becomes :

$$p(\tau/\mathbf{r}_i) = \frac{1}{(2\pi)^m\,|\mathbf{M}|\,p(\mathbf{r}_i)}\tau^{-m-1}\,\exp\left(-\frac{\mathbf{r}_i^\dagger\mathbf{M}^{-1}\mathbf{r}_i}{2\tau}\right), \tag{11}$$

and $p(\mathbf{r}_i)$ can be computed easily :

$$p(\mathbf{r}_i) = \frac{\Gamma(m)}{\pi^m\,|\mathbf{M}|\,(\mathbf{r}_i^\dagger\mathbf{M}^{-1}\mathbf{r}_i)^m}. \tag{12}$$

Replacing (11) and (12) in (10) and then (10) in (9) gives :

$$\hat{p}_N(\tau) = \frac{1}{N}\sum_{i=1}^N h_i(\tau). \tag{13}$$

where $h_i(\tau)$ is exactly an Inverse Gamma PDF with parameters m and $2/\mathbf{r}_i^\dagger\mathbf{M}^{-1}\mathbf{r}_i$.

The so-called BORD expression which is given for each observation cell $\mathbf{y}_{obs}$ (size m) and given the N reference clutter vectors $\mathbf{r}_{i=1}^N$ becomes after integration of (6) over $\hat{p}_N(\tau)$ (in lieu of $p(\tau)$) :

$$\Lambda_N(\mathbf{y}_{obs}) = \frac{\displaystyle\sum_{i=1}^N \left[\frac{\mathbf{r}_i^\dagger\,\mathbf{M}^{-1}\,\mathbf{r}_i}{(q_1(\mathbf{y}_{obs}) + \mathbf{r}_i^\dagger\,\mathbf{M}^{-1}\,\mathbf{r}_i)^2}\right]^m}{\displaystyle\sum_{i=1}^N \left[\frac{\mathbf{r}_i^\dagger\,\mathbf{M}^{-1}\,\mathbf{r}_i}{(q_0(\mathbf{y}_{obs}) + \mathbf{r}_i^\dagger\,\mathbf{M}^{-1}\,\mathbf{r}_i)^2}\right]^m} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{14}$$

where q_0 and q_1 are the same than for (6).

BORD expression depends only on the reference clutter cells which provide all the necessary information about the clutter statistics. Its adaptive version is obtained when the correlation matrix is estimated from the reference clutter cells. This estimation can be made as described in [5], which is independent from the *texture* PDF :

$$\widehat{M} = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{r}_i \, \mathbf{r}_i^\dagger}{\mathbf{r}_i^\dagger \, \mathbf{r}_i}. \tag{15}$$

Replacing **M** by (15) in (14) gives the adaptive BORD.

4. Theoretical performances of the Asymptotical BORD

In [9] authors gave the asymptotical result of BORD (convergence in law, $\mathcal{L}$) when $N \to +\infty$ which is :

$$\lim_{\substack{N \to +\infty \\ \mathcal{L}}} \Lambda_N(\mathbf{y}_{obs}) = \left(\frac{q_0(\mathbf{y}_{obs})}{q_1(\mathbf{y}_{obs})} \right)^m \tag{16}$$

$$= \left(1 - \frac{|\mathbf{p}^\dagger \mathbf{M}^{-1} \mathbf{y}_{obs}|^2}{\mathbf{p}^\dagger \mathbf{M}^{-1} \mathbf{p} \, \mathbf{y}_{obs}^\dagger \mathbf{M}^{-1} \mathbf{y}_{obs}} \right)^{-m} \tag{17}$$

Asymptotical BORD expression coincides with other detector expressions derived under different hypothesis [3, 4, 6]. For example, K.J.Sangston and al. in [6] consider a deterministic *texture* variable and replace its value by its ML estimate under each of the hypothesis test (1), (2). Expression (17) is also commonly called GLRT-LQ (GLRT-Linear Quadratic) and can be expressed like the classical matched filter [4].

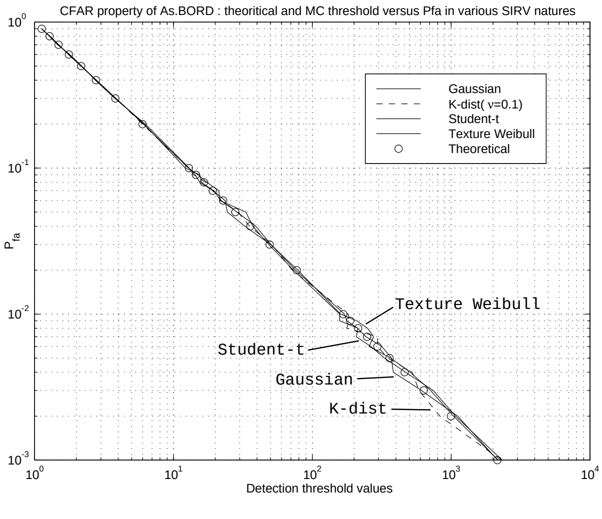

Figure 1. CFAR property of the Asymptotical BORD on synthetic signals. Comparison between MC threshold values and theoretical values given by (19). Threshold computation is independent on the *texture* PDF of the SIRV clutter.

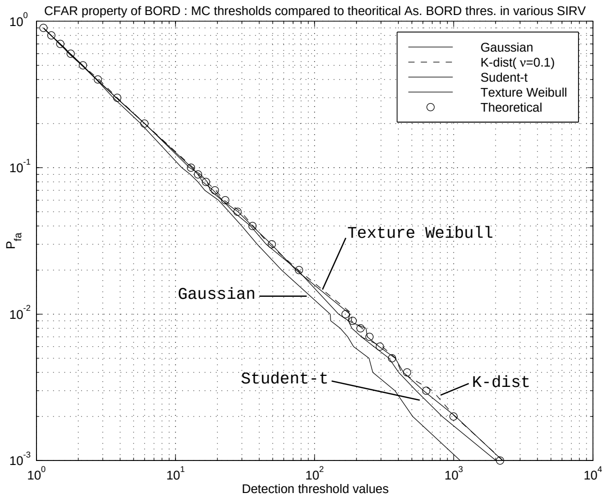

Figure 2. CFAR property of the BORD on synthetic signals. Comparison between MC threshold values and theoretical values given by (19). Threshold computation is independent on the *texture* PDF of the SIRV clutter.

Expressions (16) and (17) are statistically independent of the *texture* PDF. This means that the Asymptotical BORD is CFAR with respect to the *texture* PDF as showed in [8] and figure 1. Following the same process, it is possible to show ([8] in figure 2 that BORD is also CFAR with respect to the *texture* PDF.

Expression (17) is so only made up of Gaussian vectors. Under non-singularity condition on the correlation matrix it is possible to prove that numerator and denominator in (17) are statistically independent and then to derive the PDF of Asymptotical BORD [8].

Considering (16) as the following detection test :

$$\left(\frac{q_0(\mathbf{y}_{obs})}{q_1(\mathbf{y}_{obs})} \right)^m \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{18}$$

then theoretical performances of (18) are :

$$\eta = P_{fa}^{\frac{m}{1-m}}. \tag{19}$$

Figure 2 shows that this result applies also for the BORD.

5. BORD performances against ground clutter data

The ground clutter data presented in this paper were collected by an operational radar at THALES Air Defence, placed at 13 meters height and illuminating the ground at low grazing angle. Ground clutter complex echoes were collected in $N = 868$ range bins for 70 different azimuth angles and for $m = 8$ recurrences of the pulse repetition frequency (PRF). Near the radar, echoes represent non-Gaussian ground clutter whereas beyond the radioelectric horizon of the radar (around 15 km) only Gaussian thermal noise is present (figure 3).

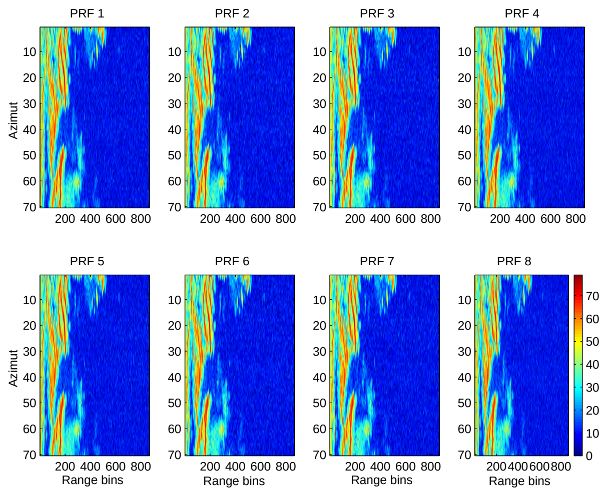

Figure 3. Ground clutter data level (in dB) for the $m = 8$ recurrences of PRF. Y-coordinates represent 70 azimuth angles and X-coordinates represent $N = 868$ range bins.

BORD performances are evaluated in some particular zones of the data representing either non-Gaussian complex clutter ($m = 8$, $N = 2660$ from range bin 22 to 212 and from azimuth 18 to 32) or complex Gaussian thermal noise ($m = 8$, $N = 3296$ from range bin 568 to 774 and from azimuth 22 to 38).

The detection threshold value is derived by Monte Carlo computation for $P_{fa} = 10^{-2}$. Figure 4 represents BORD performances in a non-Gaussian ground clutter zone whereas figure 5 deals with a Gaussian thermal noise zone. An artificial target with complex and unknown

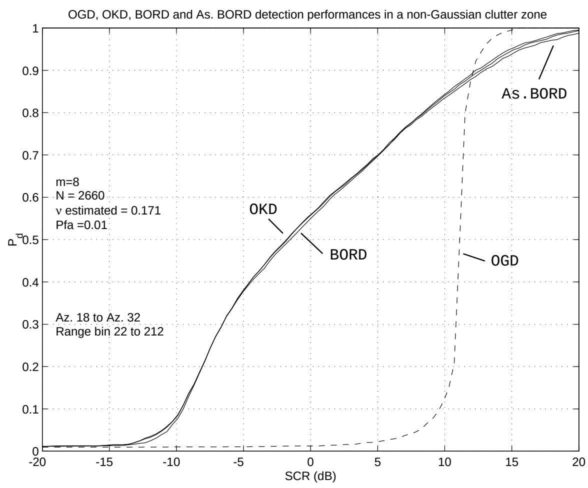

Figure 4. Performance comparison between the OGD, OKD, BORD and As.BORD in a non-Gaussian ground clutter zone for an unknown complex target amplitude. $P_{fa} = 10^{-2}$, $\widehat{\nu} = 0.171$ (spiky K-clutter).

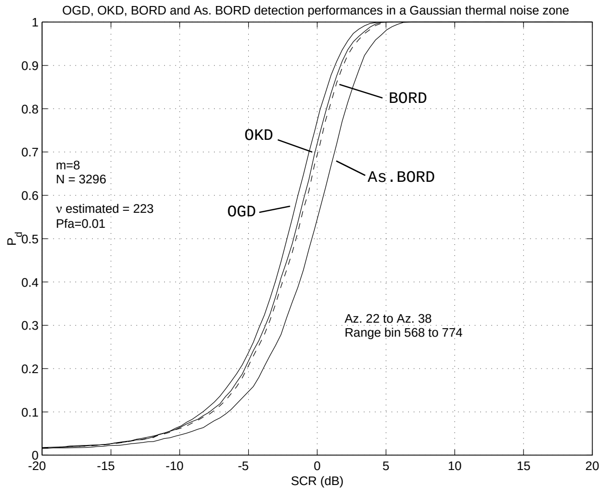

Figure 5. Performance comparison between the OGD, OKD, BORD and As.BORD in a Gaussian thermal noise zone for an unknown complex target amplitude. $P_{fa} = 10^{-2}$, $\widehat{\nu} = 223$ (closed to Gaussian noise).

amplitude is embedded in each of these zones in order to evaluate the probability to detect such signal in such clutter. All the curves represent the detection probability P_d versus the Signal-to-Clutter-Ratio (SCR) given for one pulse.

In order to compare BORD performances with those of optimum detectors we fit experimental data envelope to a K-distribution. More the value of K-distribution form parameter ν is low and spikier is the K clutter. When ν is high, the clutter tends to be Rayleigh distributed. With the results we obtain in [8], we conclude that these data can be connected with a K-distribution.

6. Conclusion and outlooks

The present paper has addressed detection performances on experimental data of the new detector BORD, based on a bayesian estimation of the clutter statistics. This detector is adaptive and applies with significant results to experimental clutter data. Moreover, in the case where the correlation matrix of the data is non-singular it is possible to apply the theoretical threshold value of the Asymptotical BORD to the BORD. In this case, it is not necessary to have many reference clutter cells to make BORD efficient. Only ten data can be enough, except of course for the correlation matrix estimation.

From the experimental point of view, it would be interesting to evaluate BORD performances in various clutter cases such as coastal clutter where ground/sea transitions can be observed for a deterministic or fluctuating target amplitude.

Special thanks to THALES Air Defence for allowing us to make use of the experimental data.

References

[1] J.B. Billingsley, Ground Clutter Measurements for Surface-Sited Radar, *Technical Report 780, MIT*, February 1993.

[2] T.J. Barnard and D.D. Weiner, Non-Gaussian Clutter Modeling with Generalized Spherically Invariant Random Vectors, *IEEE Trans.-SP*, **44**(10) (October 1996), 2384-2390.

[3] E. Conte, M. Lops and G. Ricci, Asymptotically Optimum Radar Detection in Compound-Gaussian Clutter, *IEEE Trans.-AES*, **31**(2) (April 1995), 617-625.

[4] F. Gini, Sub-Optimum Coherent Radar Detection in a Mixture of K-Distributed and Gaussian Clutter, *IEE Proc.Radar, Sonar Navig*, **144**(1) (February 1997), 39-48.

[5] F. Gini and M.V. Greco, Sub-Optimum Approach to Adaptive Coherent Radar Detection in Compound-Gaussian Clutter, *IEEE Trans.-AES*, **35**–3 (July 1999), 1095-1103.

[6] K.J. Sangston, F. Gini, M.V. Greco and A. Farina, Structures for Dadar Detection in Compound Gaussian Clutter, *IEEE Trans.-AES* **35**(2) (April 1999), 445-458

[7] F. Gini, M.V. Greco, A.Farina and P. Lombardo, Optimum and Mismatched Detection Against K-distributed Clutter Plus Gaussian Clutter, *IEEE Trans.-AES*, **34**(3) (July 1998), 860-876.

[8] E. Jay, Détection en Environnement non-Gaussien, *Ph.D. Thesis*, University of Cergy-Pontoise / ONERA, France, June 2002.

[9] E. Jay, J.P. Ovarlez, D. Declercq and P. Duvaut, BORD : Bayesian Optimum Radar Detector, *Signal Processing*, **83**(6) (June 2003), 1151-1162

[10] M. Rangaswamy, D. Weiner and A. Öztürk, Non-Gaussian Random Vector Identification Using Spherically Invariant Random Processes, *IEEE Trans.-AES* **29**(1) (January 1993), 111-123

[11] K. Yao, A Representation Theorem and its Application to Spherically Invariant Random Processes, *IEEE Trans.-IT* **19**(2) (1973), 600-608.

[12] C. Robert, Méthodes de Monte Carlo par Chaînes de Markov, *Statistique mathématique et probabilité, Economica, Paris* (1996)