ONERA
DEMR

École Doctorale d'ORSAY

Apport de Techniques d'Analyse, de Détection et d'Estimation en Radar et Imagerie SAR

Habilitation à Diriger des Recherches

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par

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Acronymes

Organismes

CELAR	Centre d'Électronique de l'Armement (DGA Maîtrise de l'Information)
CNES	Centre National d'Études Spatiales
CNRS	Centre National de La Recherche Scientifique
CSB	Comité Scientifique de Branche (ONERA)
CSD	Conseil Scientifique de Département (ONERA)
DEMR	Département Électromagnétisme et Radar (ONERA)
DGA	Délagation Générale de l'Armement
DSO	Defence Science Organization (Singapore)
DOTA	Département d'Optique Théorique et Appliquée (ONERA)
DMPH	Département Mesures Physiques (ONERA)
DESP	Département Environnement Spatial (ONERA)
DSTA	Defence Science and Technology Agency (Singapore)
EDA	European Defence Agency
ENS	École Normale Supérieure
ENST	École Nationale Supérieure des Télécommunications
IETR	Institut d'Électronique et de Télécommunications de Rennes
ISAE	Institut Supérieur de l'Aéronautique et de l'Espace
MIT	Massachusets Institute of Technology
LSS	Laboratoire des Signaux et Systèmes
NTU	Nanyang Technology University (Singapore)
NUS	National University od Singapore
ONERA	The French Aerospace Lab
SATIE	Systèmes et Applications des Technologies de l'Information et de l'Énergie
SONDRA	SUPELEC-ONERA-NUS-DSTA Research Alliance
TAD	THALES Air Defence

Acronymes généraux

AMF	Adaptive Matched Filter
AML	Approximate Maximum Likelihood
ANMF	Adaptive Normalized Matched Filter
BORD	Bayesian Optimum Radar Detector
CFAR	Constant False Alarm Rate
FP	Fixed Point
GEA	Groupe d'Électronique Appliquée
GLRT	Generalized Likelihood Ratio Test
GLRT - LQ	GLRT - Linear Quadratic
GMTI	Ground Moving Target Indicator
HFSW	High Frequency Surface Wave
ISAR	Inverse Synthetic Aperture Radar
LIDAR	Laser Detection and Ranging
LRT	Likelihood Ratio Test
MIMO	Multiple Input Multiple Output
MTI	Moving Target Indicator
MV / ML	Maximum de Vraisemblance / Maximum Likelihood
NSCM	Normalized Sample Covariance Matrix
OGD	Optimum Gaussian Detector
P-AMF	Persymmetric Adaptive Matched Filter
P-ANMF	Persymmetric Adaptive Normalized Matched Filter
PDF	Probability Density Function
POLINSAR	Polarimetric and Interferometric Synthetic Aperture Radar
POLSAR	Polarimetric Synthetic Aperture Radar
PSCM	Persymmetric Sample Covariance Matrix
RV	Rapport de Vraisemblance
SAR / RSO	Synthetic Aperture Radar / Radar à Synthèse d'Ouverture
SAS	Synthetic Aperture Sonar
SCM	Sample Covariance Matrix
SER / RCS	Surface Équivalente Radar / Radar Cross Section
SIRP	Spherically Invariant Random Process
SIRV	Spherically Invariant Random Vector
SNR	Signal to Noise Ratio
STAP	Space Time Adaptive Processing
TFAC	Taux de Fausse Alarme Constant
THR	Très Haute Résolution
TSVM	Target Scattering Vector Model

Références

Par souci de commodité et de lecture, j'ai choisi de référencer mes publications et mes travaux dans un ordre chronologique comme suit :

- [**Oxx**] : Ouvrages
- [**Jxx**] : Journaux à comité de lecture
- [**Cxx**] : Articles de Conférence avec actes
- [**Bxx**] : Brevets logiciels
- [**Sxx**] : Participations à des Séminaires avec ou sans actes
- [**Txx**] : Thèses encadrées
- [**PDxx**] : Post-Doc encadrés

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Préambule

Le goût indéfectible de la recherche, le goût de l'encadrement, de l'enseignement et du transfert de connaissances m'ont été enseignés en grande partie par Pierre et Jacqueline Bertrand, à qui je voudrais ici rendre hommage, lors de ma thèse de doctorat qui a débuté à l'ONERA en septembre 1988 et dont le thème portait sur l'*Analyse des Signaux à Large Bande*. Durant cette thèse, l'accent était mis sur l'étude, la recherche de nouveaux concepts permettant de combler les lacunes de techniques classiques, en l'occurrence, l'extension des techniques dites *bande étroite* au signaux *large bande*. L'exemple de l'effet Doppler, souvent assimilé à tort à un simple décalage Doppler, en est un exemple. Dès lors que le signal possède une largeur de bande relative importante, cet effet Doppler agit sur le signal comme un facteur d'échelle et le fait de l'assimiler comme un simple décalage peut alors dégrader les performances des traitements conventionnels. Durant les années passées, je me suis ainsi toujours intéressé aux lacunes potentielles des techniques de traitement de signal couramment utilisées et souvent considérées à tort comme valides. Malgré la charge journalière de travail contractuel, j'ai eu la chance à l'ONERA, que je tiens vraiment encore ici à remercier, d'avoir pu constamment garder le contact avec le monde universitaire, de lire l'abondante littérature, de publier, d'enseigner. Cette liberté m'a permis d'élaborer et d'étoffer de nouvelles pistes de recherche, de proposer des sujets amont, toujours en lien avec les problématiques de l'ONERA et en m'appuyant sur les stagiaires d'école d'ingénieur ou de Master mais surtout sur les doctorants et leurs directeurs de thèse. Cette curiosité et ces collaborations toujours fructueuses et riches d'enseignements m'ont permis d'intégrer en 2003 la filière recherche de l'ONERA, comme Maître de Recherche, puis, plus récemment, de participer au développement du laboratoire SONDRA de Supélec.

Introduction

Ce mémoire présente une synthèse des travaux de recherche ou d'études que j'ai menés depuis une vingtaine d'années au sein de l'unité Traitement du Signal du Département Électromagnétisme et Radar de l'ONERA, The French Aerospace Lab à Palaiseau. Ces travaux ont, pour la plupart, été essentiellement consacrés à l'apport des techniques de Traitement du Signal pour le radar et l'imagerie radar.

L'évolution technologique grandissante des radars (veille, poursuite, ou encore imageur) et des capteurs leur permet aujourd'hui d'obtenir une résolution des cellules (distance, vitesse, angulaire, ...) à analyser de plus en plus fine. Cette résolution de plus en plus petite est un atout essentiel pour la détection, le pistage, la caractérisation et/ou la reconnaissance de cibles. Cependant, la haute résolution induit une plus grande hétérogénéité et impulsivité des échos environnant la cible ou se superposant à la cible à détecter. Cette hétérogénéité et ce caractère non gaussien peuvent s'expliquer respectivement par le fait que le nombre de réflecteurs des cases de résolution est de plus en plus faible et que celui-ci varie aléatoirement d'une case à une autre. Dès lors, l'hypothèse d'une statistique gaussienne pour ce bruit n'est plus valide. En conséquence, la détection d'objets et la régulation de fausses alarmes deviennent de plus en plus difficiles. Il est donc impératif de mettre en place de nouveaux outils robustes, performants et adaptés à cet environnement non-gaussien permettant d'améliorer les performances des détecteurs, des estimateurs et des classifieurs.

De manière duale, cette fine résolution d'analyse peut-être caractérisée par l'émission d'une large bande spectrale (résolution distance), un long temps d'intégration (résolution vitesse), une large ouverture angulaire (résolution azimutale), un large réseau d'antennes (résolution angulaire), ... De ce fait, les cibles, qui sont souvent supposées répondre de la même manière (notion de blancheur spectrale, d'isotropie, de stationnarité temporelle) sur tous les domaines sur lesquels elles sont analysées, ont des caractéristiques qui varient. Il est donc nécessaire de définir des outils d'analyse de ces non-stationnarités. C'est là qu'interviennent les notions de distributions temps-fréquence (Classe de Cohen [2], [3], Classe Affine [Bert85], [Bert88], [Bert92]) que j'ai longuement étudiées pendant ma thèse et qui peuvent rendre de multiples services notamment pour l'imagerie SAR (Synthetic Aperture Radar).

De manière plus générale, toutes les techniques de traitement de signal utilisant l'hypothèse gaussienne et l'hypothèse de stationnarité peuvent être étendues au cas non-gaussien et au cas non-stationnaire. Cette perspective ouvre un champ d'investigation assez étourdissant. Mes axes de recherches ont ainsi largement été guidés et inspirés par cette problématique qui restera, je l'espère, assez longtemps, source d'inspiration.

De nombreuses applications potentielles ayant trait au radar sont concernées par cette thématique :

- Le radar de veille en général : radar maritime pour la détection de petites cibles en mer agitée, radar de surveillance pour la détection basse altitude,

- Le radar MIMO (Multiple Input, Multiple Output), généralisation à plusieurs émetteurs et récepteurs de la fonction radar,

- la détection GMTI (Ground Moving Target Indicator) de cibles mobiles à partir de capteurs aéroportés et les techniques de type STAP (Space Time Adaptive Processing),

- la détection de cibles sous couvert (FOPEN), détection de mines,
- la détection et la reconnaissance des cibles et des fonds dans les images SAR (Synthetic Aperture Radar) ou SAS (Synthetic Aperture Sonar),
- la détection de changement et détection de cibles mobiles dans les images SAR,
- l'analyse de la stationnarité spectrale, angulaire, polarimétrique des images SAR.

Toutes ces thématiques figurent dans les trois principaux axes de recherche que je vais décrire dans ce mémoire :

- **L'analyse Temps-Fréquence pour l'imagerie SAR**

Le premier axe sur lequel je me suis penché provient des perspectives logiques issues de mes travaux de thèse sur la transformation de Mellin et consiste à étendre à l'imagerie SAR ou l'imagerie ISAR (Inverse Synthetic Aperture Radar) les techniques temps-fréquence utilisées pour l'analyse des signaux non-stationnaires. Depuis longtemps et même encore maintenant, l'imagerie radar fait implicitement l'hypothèse simplificatrice que les points brillants de la scène analysée possèdent la même réponse quelle que soit la fréquence émise et quel que soit l'angle sous lesquels ils sont vus. Tous les algorithmes de synthèse d'images SAR exploitent cette hypothèse. L'Analyse Temps-Fréquence permet de pouvoir étudier la dispersivité et l'anisotropie des réflecteurs dans une image SAR ou ISAR. Principalement basé sur des analyses en sous bandes et sous ouvertures, ce type d'analyse a également permis d'analyser la non-stationnarité angulaire et spectrale des mécanismes polarimétriques de rétrodiffusion des réflecteurs d'une image SAR polarimétrique mais également d'améliorer l'estimateur de la cohérence interférométrique voire la cohérence polarimétrique et interférométrique qui permet l'estimation de la hauteur des points du sol. Cette analyse Temps-Fréquence couplée aux méthodes polarimétriques de décomposition cohérentes (Pauli, Krogager, Cameron, ...) était initialement dédiée à l'étude du comportements des cibles manufacturées (véhicules, bâtiments, ...) principalement pour la reconnaissance et l'identification des cibles. Deux thèses conduites respectivement par Mohamed Tria **[T2]** en octobre 2001 et Mickaël Duquenoy **[T4]** en décembre 2004 ont été proposées et encadrées sur cette thématique.

- **Détection et Estimation en Environnement de Fouillis Non Gaussien**

Le deuxième axe de recherche que je poursuis encore actuellement a débuté à la suite d'une étude interne ONERA sur l'Analyse de la Détectabilité de Cibles à Basse Altitude (MODEM). Le problème consistait à analyser la détection radar d'une cible manœuvrant à basse altitude au dessus du sol, la cible étant caractérisée par son diagramme site-gisement de rétrodiffusion (SER), par sa trajectoire réelle au dessus d'un terrain réel. L'étude de la détectabilité radar nécessitait d'effectuer un bilan radar complet comprenant tous les éléments vitaux : facteur de propagation au dessus d'un sol réaliste, niveaux de réflectivité du fouillis de sol ou de mer en fonction de la bande de fréquence, de l'angle d'attaque du sol, du type de fouillis intercepté, de la polarisation. Le bilan radar ainsi calculé permettait ainsi, sous hypothèses simplificatrices (à l'époque) d'une statistique gaussienne du fouillis, de calculer les probabilités de détection pour une probabilité de fausse alarme fixée. En explorant la bibliographie existante sur les modèles de fouillis de sol ainsi que leur niveau de rétrodiffusion, il s'est avéré que cette hypothèse gaussienne pouvait être mise en défaut, spécialement dans des situations d'angles de vue rasants. Cela a été le déclencheur de mes recherches encore menées aujourd'hui : la détection et l'estimation en environnement non gaussien. Les modèles gaussiens composés SIRV (Spherically Invariant Random Vectors - vecteurs multivariés gaussiens dont la variance est aléatoire) sur lesquels je me suis penché permettent d'étendre de manière très élégante et efficace toutes les méthodes classiques basées sur l'hypothèse restrictive gaussienne. C'est ainsi que nous avons développé et étudié des détecteurs optimaux pour ces environnements non gaussiens. Couplées à de nouveaux estimateurs puissants et robustes des paramètres de l'environnement (matrice de covariance par exemple), ces techniques peuvent s'appliquer à la détection radar mono-voie, la détection

radar multi-voies (antenne réseau pour la localisation de sources, techniques de type STAP (Space Time Adaptive Processing), MIMO, interférométrie, polarimétrie, multi-spectral, ...).

Trois thèses proposées et encadrées, celles d'Emmanuelle Jay **[T1]** en septembre 1998, de Frédéric Pascal **[T3]** en septembre 2003 et de Guilhem Pailloux **[T5]** en décembre 2008 ont véritablement contribué à l'avancée de ces recherches. La nouvelle thèse de Mélanie Mahot **[T8]** financée par la DGA a donc logiquement débuté en septembre 2009 pour poursuivre leurs avancées et analyser la robustesse de ces estimateurs face à des données aberrantes (cibles, brouilleurs dans les données secondaires) mais également réfléchir à comment s'affranchir ou comment faire face aux non-stationnarités spectrales du fouillis que l'on rencontre par exemple en STAP de pointe avant ou encore dans des types de fouillis dynamique comme la mer.

- **Classification des Images SAR**

Nous nous sommes vite rendu compte qu'il fallait également analyser le fouillis présent dans les images SAR avec les méthodes polarimétriques dites non-cohérentes (H/alpha, Freeman-Durden, ...). Le fouillis étant considéré comme un processus aléatoire, ces techniques dites non-cohérentes exploitent la statistique de ces processus, généralement à travers leurs caractéristiques du second ordre comme la matrice de covariance en encore la matrice de cohérence. Récemment, des modèles ont également été étendus à la classification polarimétrique des fonds dans les images SAR hautement texturées (c'est à dire hétérogènes et non-gaussiennes) lors du post-doc de Gabriel Vasile **[PD1]** en septembre 2007 qui a su établir un lien très fort entre les deux précédentes thématiques citées plus haut. Ce lien peut également s'appliquer à la détection multi-bandes ou multi-ouvertures des images SAR mono-voies ainsi qu'à la détection de changement. La nouvelle thèse DGA/ONERA de Pierre Formont **[T9]** qui a débutée en septembre 2009 a ainsi été proposée sur cette thématique à la suite du travail de Gabriel Vasile. Le travail de post-doc de Lionel Bombrun **[PD2]** du Gipsa-Lab, détaché pour 6 mois au laboratoire SONDRA en octobre 2009 ouvre actuellement un certain nombre de perspectives très prometteuses sur la détection polarimétrique des cibles dans les images SAR.

Les deux premiers axes de recherches qui ont été construits et exploités de manière indépendante, peuvent ainsi se combiner pour résoudre de nouveaux problèmes que je développerai dans mes perspectives de recherche. Les applications dérivées de ces deux thèmes de recherche sont assez nombreuses : l'analyse des images SAR (segmentation, classification), détection de cibles mobiles dans les images SAR, l'imagerie hyperspectrale, la détection de changement.

Enfin, de manière surprenante, et cela *bouclera la boucle*, ces modèles SIRV ont également une connexion très forte avec la transformation de Mellin étudiée pendant ma thèse. Cette transformation qui possède la propriété d'invariance au changement d'échelle d'un signal, d'une statistique, d'une image permet la "factorisation" de la densité de probabilité des processus SIRV, produit d'un processus gaussien par un paramètre d'échelle (variance) aléatoire indépendant, de la même manière que la transformation de Fourier "factorise" la densité de probabilité de la somme de deux variables aléatoires indépendantes. Cette thématique a été largement couverte par Jean-Marie Nicolas de l'ENST qui a utilisé cette connexion pour établir des modèles statistiques de deuxième espèce par les log-moments et les log-cumulants des images SAR texturées [Nico06].

Le document proposé est divisé en quatre chapitres :

- Le premier chapitre **Parcours Personnel** présente différentes informations générales mais détaillées retraçant mon parcours, mes activités et mes diverses productions depuis la fin de ma thèse en 1992. On y trouvera la liste de mes activités contractuelles, de mes encadrements de thèse, de mes cours, de mes publications, etc.

- Le deuxième chapitre **Synthèse des Activités de Recherche** dresse la ligne directrice thématique de mes activités de recherche. J'y expose le raisonnement méthodologique qu'il m'a été

nécessaire de mener pour effectuer ces différentes études ainsi que l'évolution de mes recherche au cours des vingt dernières années : trois axes seront privilégiés, un axe dédié à l'imagerie SAR, un axe dédié à l'estimation et la détection radar en environnement non gaussien et le dernier, plus axé sur la classification en imagerie SAR qui établira le lien entre les deux axes précédents. Des publications choisies données en Annexes B à M étayeront ce chapitre

- Le troisième chapitre **Perspectives** présente enfin quelques perspectives personnelles sur mon travail et les orientations futures que je souhaite lui donner à court terme.

- Le dernier chapitre **Conclusions** conclura ce travail.

Chapitre 1

Parcours Personnel

1.1 État Civil

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1.2 Biographie Succincte

Doctorant de septembre 1988 à décembre 1991 dans la division *Systèmes Radar* au sein de la Direction des Études de Synthèse (DES) de l'ONERA (The French Aerospace Lab), je suis embauché comme ingénieur de Recherche le 1er janvier 1992 et soutiens ma thèse en avril 1992. Lors de la réorganisation de l'ONERA, je rejoins en septembre 1997 l'unité *Traitement du Signal* du Département Électromagnétisme et Radar (DEMR) de la branche Physique. Le 1er septembre 2002, je deviens responsable scientifique du projet ALBATROS visant à développer, contrôler et valoriser les nouvelles techniques ou recherches développées dans le cadre du capteur SAR aéroporté RAMSES du DEMR (algorithmie de formation d'images, interférométrie, polarimétrie, suivi de thèses, développement logiciel associé). Je suis nommé Maître de Recherches le 1er août 2003 et entre au Comité Scientifique de la Branche Physique de l'ONERA en mars 2004, reconduit en 2009. Je suis élu président du Conseil Scientifique du Département DEMR en janvier 2006 et janvier 2007. En 2007, je deviens l'animateur scientifique du Club STAP ayant pour but de fédérer les recherches étatiques (DGA, CELAR, ONERA), industrielles (THALES) et universitaires ayant trait à la thématique STAP. À partir de janvier 2008, je suis détaché deux jours par semaine au laboratoire SONDRA de Supélec en qualité de responsable des études et recherches relatives au traitement du signal. En janvier 2010, je suis chercheur attaché au DSO (Defence Science Organization) National Laboratories à Singapour (Séjour de recherche ERE d'un an sous financement DGA).

1.3 Domaines de Compétences

- Analyse Spectrale, Localisation de sources
- Analyse Temps-Fréquence, Temps-Échelle, Ondelettes
- Radar, Sonar
- Détection, Estimation de paramètres, Traitement d'antenne
- Imagerie radar (ISAR, SAR), Imagerie en laboratoire
- Statistiques (estimation de densités, méthodes bayésiennes, statistiques d'ordre supérieur, élaboration de tests statistiques), méthodes de reconnaissance
- Algorithmie et développement de logiciels scientifiques (C, Fortran, Matlab, Maple, LaTeX)
- Suivi de projets transversaux, contacts avec industriels et universitaires
- Langues : russe, anglais (parlé, lu, écrit).

1.4 Expériences Professionnelles

Jan 2010 Séjour de recherche de longue durée d'un an (stage ERE financé par la DGA) au DSO National Laboratories, Singapour sur la thématique *Méthodes d'Estimation Robustes dans les Processus Gaussiens Composés - Application au Radar*,

Jan 2008 Chercheur associé dans le laboratoire franco-singapourien SONDRA de Supélec, détaché de l'ONERA deux jours par semaine, responsable des activités de recherche en Traitement du Signal,

1992-2010 Actuellement Maître de Recherche (2003) à l'ONERA - Position repère 3B - Ingénieur Spécialiste niveau 1 dans l'Unité *Traitement du Signal* du DEMR *Département Électromagnétisme et Radar* : Études et recherches amont en Traitement du Signal,

1988-1992 Thèse de doctorat soutenue le 13 avril 1992 et effectuée dans la division *Systèmes Radar* de l'ONERA : "*La Transformation de Mellin : un Outil pour l'Analyse des Signaux à Large Bande*",

1987-1988 Appelé en qualité de Scientifique du Contingent en août 1987 au SAMAN à la base Aéronautique Navale de Toussus-Le-Noble (78),

Avril-Juin 1987 Stage de DEA à la CSEE (Compagnie des Signaux et d'Entreprises Électriques) et LSS : *Reconnaissance de Parole en Milieu Fortement Bruité*,

Mars-Déc. 1986 Projet ESIEA à l'INRIA (Institut National de Recherches en Informatique et Automatique) : *Filtrage Numérique et Filtrage Optimal de Kalman.*

1.5 Formation

Mars 2011 Préparation à l'Habilitation à Diriger des Recherches (HDR) à l'Université de Paris XI, Orsay,

1988-1992 Doctorat de l'Université de Paris 6, Spécialité Traitement du Signal (Mention Très Honorable),

1986-1987 DEA Automatique et Traitement du Signal au L.S.S. (Mention Bien), Laboratoire des Signaux et Systèmes - Supélec - 91 Gif sur Yvette

1984-1987 Diplôme d'ingénieur ESIEA. (Mention TB), École Supérieure d'Informatique Électronique Automatique - 9 Rue Vésale, Paris 5

1981-1984 Mathématiques Supérieures et Spéciales - Option M

1981 Baccalauréat série C (Lycée Henri Wallon - 59300 Valenciennes)

1.6 Activités de Recherche

1.6.1 Travaux de Thèse

1.6.1.1 Manuscrit de Thèse

[T0] J.P. Ovarlez, "*La Transformation de Mellin : un Outil Pour l'Analyse des Signaux à Large Bande*", Thèse de Doctorat de l'Université de Paris 6, avril 1992

Un grand nombre de méthodes actuelles d'analyse de signaux n'a de justification théorique acceptable que pour la catégorie bien particulière des signaux à bande étroite, associée au groupe des translations en temps et en fréquence. C'est le cas des Représentations Temps-Fréquence de la classe de Cohen, mais aussi des Fonctions d'Ambiguïté de Woodward, qui trouvent ainsi une limitation dans leurs conditions d'utilisation. Ces deux grandes familles possèdent toutefois leur extension dans le domaine large bande : les Représentations Temps-Fréquence Affines de J. et P. Bertrand et les Fonctions d'Ambiguïté Généralisées. Ces formes sont étroitement liées au mode de transformation du groupe affine qui agit sur les signaux par translation et compression de temps. Elles sont, de ce fait, beaucoup plus difficiles à utiliser et à calculer numériquement. La Transformation de Mellin s'est alors, par sa puissance et sa simplicité d'emploi, révélée être un outil particulièrement adapté pour résoudre des problèmes théoriques et algorithmiques associés à ces effets de compression. On propose donc dans cette thèse de découvrir cette nouvelle transformation qui possède de nombreuses analogies avec celle de Fourier (interprétation physique de la variable de Mellin, propriétés, forme discrétisée, théorème d'échantillonnage). Toutes les nouvelles applications, résultats théoriques ou algorithmes rapides qui en découlent, sont présentés : le calcul des Distributions Temps-Fréquence Affines et leurs régularisées (Transformation en Ondelettes) ainsi que les Fonctions d'Ambiguïté Généralisées, l'élaboration des bornes de Cramèr-Rao pour les estimateurs retard/vitesse en large bande, et finalement une application opérationnelle, l'Imagerie Radar Multidimensionnelle par Transformées en Ondelettes Dimensionnées.

1.6.1.2 Publications Relatives aux Travaux de Thèse

Les publications relatives aux travaux de thèse sont mentionnés dans la liste de publications données au paragraphe 1.6.7. On trouve principalement les références suivantes : **[O1]**, **[O2]**, **[J1]**, **[J2]**, **[C1]** à **[C10]**.

1.6.2 Participation à des Jurys de Thèse

1.6.2.1 Jurys de thèse encadrée (voir détails plus bas au paragraphe 1.6.4)

- Guilhem Pailloux, *Estimation Structurée de la Covariance du Bruit en Détection Adaptative*, Université de Nanterre, 10 juin 2010,
- Mickaël Duquenoy, *Analyse Temps Fréquence Appliquée à l'Imagerie SAR Polarimétrique*, Université de Rennes 1, 20 octobre 2009,
- Frédéric Pascal, *Détection en Environnement Non Gaussien*, Université de Nanterre, 14 novembre 2006,
- Mohamed Tria, *Imagerie Radar à Synthèse d'Ouverture par Analyse en Ondelettes Continues Multidimensionnelles*, Université d'Orsay, 18 novembre 2005,
- Emmanuelle Jay, *Détection en Environnement Non Gaussien*, Université de Cergy-Pontoise, 14 juin 2002.

1.6.2.2 Autres participations

- Fabien Millioz, *Deux Approches de Segmentation Temps-Fréquence : Détection par Modèle Statistique et Extraction de Contours par le Champ de Vecteurs de Réallocation*, Université de Grenoble,

16 septembre 2009,

Président :	Jocelyn CHANUSSOT	Professeur des Universités,
Rapporteurs :	Patrick FLANDRIN	Directeur de Recherche CNRS,
	Peter DJURIĆ	Professeur Université de New York,
Examinateurs :	Jean Philippe OVARLEZ	Maître de Recherche,
	Pascal LARZABAL	Professeur des universités,
	Nadine MARTIN	Directeur de Recherche CNRS, Directeur de Thèse.

- Qi Cheng, *Contribution à l'Estimation d'État dans les Modèles Non Linéaires*, Université d'Orsay, 9 juillet 2009,

Président :	Messaoud BENIDIR	Professeur des Universités,
Rapporteurs :	Pierre-Olivier AMBLARD	Directeur de Recherche CNRS,
	François DUFOUR	Professeur des Universités,
Examinateurs :	Jean Philippe OVARLEZ	Maître de Recherche,
	Alexandre RENAUX	Maître de Conférence,
	Pascal BONDON	Directeur de Recherche CNRS, Directeur de Thèse.

1.6.3 Encadrements de Post-Doc

[PD2] Participation à l'encadrement du post-doc de Lionel Bombrun (octobre 2009-avril 2010) du Gipsa Lab en visite au laboratoire SONDRA pour 6 mois sur le thème "*Processus Stochastiques pour l'Analyse de la Rétrodiffusion SAR*".

The new generation of recently launched Synthetic Aperture Radar (SAR) sensors are able to produce high quality images of the Earth's surface with meter resolution. The number of scatterers present in each resolution cell decreases considerably and homogeneous hypothesis of the Polarimetric SAR (Pol- SAR) clutter can be reconsidered. Heterogeneous clutter models have therefore recently been studied. In 1973, Kung Yao has first introduced the use of Spherically Invariant Random Vectors (SIRV) and their applications to estimation and detection in communication. From a PolSAR point of view, the observed target vector $\mathbf{k}$ *is defined as the product of a square root of a positive random variable* τ *(representing the texture) with an independent complex Gaussian vector* $\mathbf{z}$ *with zero mean. SIRVs describe a whole class of stochastic processes. This class includes the conventional clutter models having Gaussian, K,* $\mathcal{G}'$ *and KummerU PDFs which correspond respectively to dirac, Gamma, Inverse Gamma and Fisher distributed texture. Once the SIRV parameters are estimated (covariance matrix and texture parameters), optimal SAR detectors can be applied to detect particular targets. Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) detectors have been successfully applied to detect trihedral scattering. In 2007, Ridha Touzi has proposed a new target scattering vector model (TSVM) to extract physical parameters. Based on the Kennaugh-Huynen decomposition, the TSVM allows to extract four roll-invariant parameters (in- dependent of the incidence angle). Those parameters are necessary for an unambiguous characterization of target scattering. The first step of the proposed work consists in the implementation of the TSVM in the GLRT-LQ for the detection of any targets (trihedral, dihedral, oriented-dihedral, helix, . . .). Based on recent works done on texture modeling of PolSAR data and maximum likelihood parameter estimation, the second part of this work will deal with the development of new similarity measures for segmentation and displacement estimation. The developed methods will be applied and validated on high resolution data (RADARSAT-2, RAMSES, TerraSAR-X, . . .) over different test-sites (dams, glaciers, urban scenes, . . .). This work will take part of the EFIDIR and EDF projects.*

[PD1] Encadrement (100%) du post-doc de Gabriel Vasile (2007-2008), financé par le CNES et effectué à l'ONERA, sur le thème "*Analyse Statistique de la Texture Polarimétrique des images SAR*". Gabriel Vasile est actuellement chercheur au CNRS au Gipsa-Lab de Grenobble.

En s'appuyant sur les avancées récentes dans le modélisation SIRV (Spherically Invariant Random Vectors), ce travail présente une nouvelle approche pour l'estimation des matrices de cohérence du fouillis hétérogène. Ainsi, les données RSO polarimétriques (POLSAR) sont complètement caractérisées par l'estimation jointe des deux paramètres : la matrice de cohérence normalisée (information polarimétrique) et le SPAN (information de

puissance). Un nouvel estimateur de la cohérence polarimétrique normalisée est introduit, l'estimateur du point fixe (FP). Mis à part son degré de généralité, l'estimation FP est indépendante de la densité de probabilité "a priori" du SPAN. La méthode proposée utilise l'estimateur du Point Fixe, couplé avec des approches multi-résolution basées sur le principe du voisinage adaptatif (SDAN). Une nouvelle mesure de distance, optimale au sens du rapport de vraisemblance généralisé, est également introduite pour la classification non-supervisée POLSAR. Ces méthodes sont appliquées à la fois sur des images POLSAR simulées et sur deux jeux de données aéroportées haute et très haute résolution fournis par le système RAMSES.

1.6.4 (Co)-Encadrements de Thèses

1.6.4.1 Encadrements de Thèses en Cours

[T9] Encadrement (20%) de la thèse DGA/ONERA/SONDRA de Pierre Formont (2009-2012), effectuée à SONDRA, sur le thème "*Classification Polarimétrique dans les Images SAR Hautement Texturées*" en collaboration avec Laurent Ferro-Famil (Directeur de thèse) de l'IETR de Rennes, Gabriel Vasile (CNRS) du Gipsa-Lab à Grenoble et Frédéric Pascal de SONDRA.

La texture d'une image peut être définie comme la variation locale spatiale du niveau d'intensité d'une image. L'analyse de cette texture permet, à une certaine échelle, de caractériser l'image en zones homogènes nécessaires, par exemple, à la mise en œuvre d'un processus de détection et/ou de classification. Contrairement à une image en niveau de gris ou à une image en couleur, chaque pixel d'une image provenant d'un capteur SAR (Synthetic Aperture Radar) est caractérisé par une amplitude et une phase. Ces images sont généralement multi-voies (utilisation de canaux de polarisation, interférométrie), multi-vues (même scène pour différents angles de vue) ou multi-spectrales (même scène analysée pour plusieurs bandes de fréquence). Par conséquent, les processus d'analyse de la texture de l'image deviennent de plus en plus complexes et font, de ce fait, généralement appel à des outils de classification basés sur des hypothèses de type gaussiennes. Ainsi, dans la littérature, la plupart des travaux conduisent aux classifieurs classiques construits sur des matrices de covariances de type Wishart (estimation au sens du Maximum de Vraisemblance de la matrice de covariance à partir de données supposées gaussiennes). Une analyse approfondie de ces méthodes montre que leurs performances se dégradent de manière significative sur des zones hétérogènes, non-gaussiennes et/ou texturées. C'est le cas pour des images SAR Très Haute Résolution provenant des dernières générations de capteurs ou pour des images hautement texturées (zones urbaines par exemple). Nous nous proposons dans cette thèse de mettre en place des outils de classification robustes à ces différents scénarii. Ces outils sont basés sur une généralisation de la statistique gaussienne pour la modélisation des données : les processus aléatoires gaussiens composés (processus SIRP sphériquement invariants, processus SIRP généralisés). Ces processus (loi K, Weibull, ...) sont basés sur une modélisation multidimensionnelle gaussienne (définie par une matrice de covariance) dont la puissance est elle-même aléatoire, caractérisant de ce fait les fluctuations de la texture.

[T8] Encadrement (co-direction 20%) de la thèse DGA/ONERA/SONDRA de Mélanie Mahot (2009-2012), effectuée à SONDRA, sur le thème "*Techniques robustes en STAP dans des environnements de fouillis hétérogène et non-stationnaire*" en collaboration avec Philippe Forster (Directeur de thèse) du GEA (Groupe d'Électromagnétisme Appliqué) de l'Université Paris X et Frédéric Pascal de SONDRA.

La détection des cibles mobiles par un capteur aéroporté de surveillance du sol (GMTI) ou par une antenne de pointe avant de chasseur est un sujet de recherche qui suscite encore un grand intérêt opérationnel. A l'aide d'un réseau d'antennes, le principe du STAP est alors d'éliminer conjointement par traitement temporel et spatial les échos fixes du sol (fouillis) pouvant considérablement détériorer les performances de détection d'une cible d'intérêt. Cette technique repose principalement sur l'estimation d'une matrice de covariance des signaux temporels collectés sur le réseau de capteurs dans une zone spatiale de fouillis située autour de la cible d'intérêt (principe de blanchiment du fouillis). La qualité d'estimation de cette matrice conditionne fortement les performances attendues de l'algorithme de réjection de fouillis. Une analyse bibliographique a montré que tous les algorithmes proposés sont majoritairement basés sur l'hypothèse d'un fouillis gaussien, homogène spatialement. Cette hypothèse simplificatrice n'est plus respectée lorsque la zone de fouillis n'est plus homogène et lorsque la cellule d'analyse tend à devenir de plus en plus petite. Ceci entraîne une dégradation des performances de réjection de fouillis et implicitement

de détection. Or, actuellement, les traitements STAP tendent à devenir de plus en plus Haute Résolution (HR) distance pour, d'une part, améliorer la qualité de localisation de la cible et, d'autre part, permettre l'utilisation conjointe de formes d'onde dédiées à l'imagerie SAR. Trois thèses menées à l'ONERA depuis 1998 ont permis de définir de nouveaux types de détecteurs en environnement non gaussien et non homogène. Ces nouveaux détecteurs sont basés sur l'hypothèse très générale d'un fouillis considéré comme processus gaussien composé, c'est-à-dire un processus gaussien dont la puissance est elle-même aléatoire. Ces processus permettent de représenter une zone de fouillis hétérogène spatialement et de prendre en compte des éventuelles hétérogénéités de fluctuations spectrales dues au vent par exemple. Ces nouveaux détecteurs adaptatifs, basés sur une nouvelle estimation de la matrice de covariance du fouillis (appelée matrice du point fixe), sont alors invariants à la texture (variation spatiale de la puissance de fouillis) mais également invariants aux éventuelles fluctuations spectrales du fouillis. Ces nouveaux détecteurs et processus d'estimation des matrices de covariance ont été étendus dans le cadre du STAP et apportent une amélioration très nette des performances de détection par rapport à celles d'un traitement classique. Cette thèse, continuité des travaux de thèses menés précédemment, s'attaque à deux problèmes majeurs. Le premier concerne la présence éventuelle d'une ou plusieurs cibles ou de brouilleurs dans les données secondaires permettant d'estimer la matrice de covariance. L'estimateur classique de la matrice de covariance s'avère alors complètement biaisé par la présence de ces cibles ou brouilleurs et dégrade fortement les performances attendues du détecteur. L'objectif de ces travaux est d'étudier et de caractériser la propriété de robustesse de la matrice de covariance du point fixe. Cette robustesse caractérise le pouvoir d'invariance à un écart sur les hypothèses faites pour le modèle : ici, les données secondaires sont polluées par la présence de cibles et s'écartent de ce fait de l'hypothèse d'homogénéité sur la densité de probabilité les régissant. Le deuxième problème concerne plus spécifiquement le STAP de pointe avant (air-air ou air-sol) voire le STAP bistatique où la matrice de covariance n'est plus stationnaire sur les cases distance.

[T7] Participation à l'encadrement de la thèse ONERA de Julien Totems (2007-2010), effectuée à l'ONERA, sur le thème "*Émission Laser Impulsionnelle et Traitements Temps-Fréquence en Vibrométrie par Lidar à Détection Cohérente*" en collaboration avec Nadine Martin (Directeur de thèse) du Gipsa Lab du CNRS et de Véronique Jolivet (ONERA/DOTA/SLS). Soutenance prévue en février 2011.

Les lidars à détection cohérente hétérodyne permettent la mesure par effet Doppler des vibrations micrométriques de surfaces situées à plusieurs dizaines de kilomètres de distance. Le principe est de mesurer la phase modulée par la vibration en transférant, par hétérodynage, le spectre optique en bande de base. Une des applications de cette technique est l'identification de cible à très longue distance. L'utilisation d'instruments impulsionnels augmente de façon notable les performances mais dépend fortement de l'adaptation des paramètres de la modulation laser choisie aux caractéristiques de la vibration que l'on souhaite mesurer. Les solutions actuellement proposées dans la littérature (doublets ou encore pulses «agiles» de durée variable) supposent une connaissance a priori sur la vibration ou impose la répétition de la mesure à chaque changement de durée du pulse pour l'adapter à la vibration inconnue. Les travaux proposés dans ce travail de thèse portent sur l'étude de formats de modulation innovants, inspirés des formes d'onde radar et télécoms, qui permettront une auto-adaptation de l'instrument quelles que soient les caractéristiques de la vibration à mesurer. Le développement de ces nouveaux formats de modulation est rendu possible grâce aux avancées récentes de la technologie des lasers à fibre de puissance et des modulateurs fibrés rapides. Une partie des travaux de recherche portera sur les méthodes et algorithmes de démodulation adaptés, en particulier dans les cas de bas rapports signal à bruit, où le signal est fortement perturbé par sa propagation le long du canal de mesure (speckle, turbulence, Doppler résiduel, ...). Les résultats obtenus par l'ONERA dans le domaine de la modélisation de la physique des lidars cohérents, en particulier de la modélisation mathématique du champ optique spatio-temporel au cours de sa propagation, ainsi que dans le domaine de la réalisation de sources fibrées impulsionnelles à fine largeur de raie (laboratoire SOFIA), et dans le domaine des radars pour l'étude des formats de modulation et de démodulation adaptés seront pris en compte. La réalisation d'un algorithme de traitement adapté s'appuiera sur les travaux déjà réalisés par le GIPSA (ex LIS) dans le domaine de la modélisation des signaux modulés et dans le traitement temps fréquence. Ces travaux conduiront à la réalisation à terme d'un système de vibrométrie impulsionnelle à 1.55 microns à longue distance, pour des applications militaires (identification de cibles) et civiles (diagnostic sismique ou surveillance volcanique).

[T6] Participation à l'encadrement de la thèse Supelec/SONDRA de Chin Yuan Chong (2008-2011) sur le

thème "*Signal Processing for MIMO Radars : Detection Under Gaussian and non-Gaussian Environments and Application to STAP*", en collaboration avec Marc Lesturgie (Directeur de Thèse) et Frédéric Pascal de SONDRA. Soutenance prévue en novembre 2011.

A Multiple-Input Multiple Output (MIMO) radar can be broadly defined as a radar system employing multiple transmit waveforms and having the ability to jointly process signals received at multiple receive antennas. In terms of configurations, the antennas can be widely separated or co-located. The first part of the thesis is on detection under Gaussian and non- Gaussian environments using a MIMO radar which contains several widely separated subarrays with one or more elements each. Two different situations are considered. Firstly, we consider that the interference is Gaussian but correlation between subarrays can arise due to insufficient spacing and the imperfect orthogonality of waveforms. Secondly, we consider that the interference is non-Gaussian, a situation which arises under sea and ground clutter and when the resolution is very high.
The second part is on the application of MIMO techniques to Space-Time Adaptive Processing (STAP). The coherent MIMO configuration is studied in terms of antenna element distribution and inter-element spacing to improve detection and estimation performance. A preliminary study is also done on the use of spatial diversity to improve detection stability w.r.t. target Radar Cross Section (RCS) fluctuations and velocity direction changes.

1.6.4.2 Encadrements de Thèses Soutenues

[T5] Encadrement (50%) de la thèse ONERA de Guilhem Pailloux (2007-2010), effectuée à l'ONERA, sur le thème "*Estimation Structurée de la Covariance du Bruit en Détection Adaptative*" en collaboration avec Philippe Forster (Directeur de thèse) du GEA (Groupe d'Électromagnétisme Appliqué) de l'Université Paris X et Frédéric Pascal du laboratoire SONDRA de Supélec. Cette thèse a été soutenue (Mention Très Honorable) le 10 juin 2010 à l'Université de Nanterre. Actuellement Ingénieur de Recherche chez MBDA, Paris.

Composition du Jury

Président :	Sylvie MARCOS	Directeur de Recherche CNRS,
Rapporteurs :	Yannick BERTHOUMIEU	Professeur des Universités,
	François VINCENT	Professeur ISAE,
Examinateurs :	Jean Philippe OVARLEZ	Maître de Recherche,
	Philippe FORSTER	Professeur des Universités,
	Frédéric PASCAL	Chercheur SONDRA.

Dans le cadre de la détection radar en environnement gaussien comme non-gaussien, de nombreux détecteurs ont été développés en se basant sur des modèles de fouillis précis et des estimateurs de matrices adaptés à ces modèles. Les modèles gaussiens, simples d'utilisation, montrent rapidement leurs limites face à la réalité physique et laissent ainsi place aux SIRP, processus aléatoires sphériquement invariants qui rendent compte beaucoup plus fidèlement de la non-gaussianité du fouillis. Les détecteurs adaptés à ces environnements sont alors construits sur la base d'un estimateur de la matrice de covariance adapté. Or, dans de nombreuses applications, cette matrice de covariance présente une structure particulière dite persymétrique. L'objet de cette thèse est donc d'exploiter cette structure particulière de la matrice de covariance du fouillis afin d'en diminuer l'erreur d'estimation. Par cette exploitation, deux nouveaux estimateurs de la matrice ont été déterminés pour les environnement gaussiens et non-gaussiens. Ces détecteurs nommés PAMF et GLRT-PFP, ont été caractérisés statistiquement et une validation des travaux théoriques a été menée sur des données opérationnelles tant gaussiennes que non-gaussiennes. Une application de la persymétrie a également été effectuée dans le cadre des algorithmes spatio-temporels (STAP) ainsi que sur des algorithmes dits "à rang réduit". Les résultats probants en détection obtenus sur tous ces types de données confirment donc l'intérêt de la technique étudiée. Enfin, un élargissement de la structure persymétrique a été étudié par l'extension des détecteurs aux matrices dites de Toeplitz. Ces matrices obtenues dans le cas de traitements spatio-temporels présentent une structuration plus riche encore que la persymétrie et permettent d'envisager des développements futurs intéressants en vue de l'amélioration des performances des détecteurs. Les premiers résultats sont présentés pour conclure ce travail de thèse.

[T4] Encadrement (50%) de la thèse ONERA de Mickaël Duquenoy (2004-juin 2009), effectuée à l'ONERA, sur le thème "*Analyse Temps Fréquence Appliquée à l'Imagerie SAR Polarimétrique*" en collaboration

avec Éric Pottier (Directeur de thèse) et Laurent Ferro-Famil de l'IETR Rennes. Cette thèse a été soutenue (Mention Très Honorable) le 20 octobre 2009 à l'IETR de Rennes. Actuellement activement en recherche de postes universitaires (post-doc, maître de conférence et CNRS).

Composition du Jury

Président :	Joseph SAILLARD	Professeur des Universités,
Rapporteurs :	Nadine MARTIN	Directeur de Recherche CNRS,
	Jean-Marie NICOLAS	Professeur ENST,
Examinateurs :	Éric POTTIER	Professeur des Universités,
	Jean Philippe OVARLEZ	Maître de Recherche,
	Laurent FERRO-FAMIL	Maître de Conférence,
Invités :	Jean-Marc BOUTRY	Directeur DEMR-ONERA.

L'imagerie SAR considère les cibles comme un ensemble de diffuseurs ponctuels, isotropes et non-dispersifs. Ce modèle devient obsolète pour les nouvelles applications radar qui utilisent une large bande et une forte excursion angulaire. Il s'agit donc d'étudier les non-stationnarités par l'outil usuel : l'analyse temps-fréquence. Les transformées temps-fréquence usuelles permettent de construire des hyperimages (images à 4 dimensions) décrivant l'énergie en fonction de la fréquence émise et de l'angle de visée. Cette source d'information est reliée à la section efficace radar (SER). Une autre source d'information en imagerie radar est la polarimétrie. Il s'agit d'utiliser conjointement ces deux sources d'information afin de caractériser les diffuseurs. Par application des ondelettes continues et des décompositions cohérentes sur la matrice de Sinclair, des hyperimages polarimétriques sont générées. Ces hyperimages permettent de décrire le comportement polarimétrique en fonction de l'angle de visée et de la fréquence émise. Dans certains cas, elles décrivent globalement le mécanisme de rétrodiffusion, par sa nature, son orientation relative dans le plan horizontal et son orientation absolue dans le plan vertical. De plus, elles mettent en évidence un nouveau phénomène : la non-stationnarité polarimétrique. Celle-ci serait causée par le fait que le radar ne voit pas la même géométrie au cours de l'acquisition. De ces hyperimages polarimétriques, l'information est synthétisée et une nouvelle classification phénoménologique est proposée.

[T3] Encadrement (50%) de la thèse ONERA de Frédéric Pascal (2003-2006), effectuée à l'ONERA, sur le thème "*Détection en Environnement Non Gaussien*" en collaboration avec Philippe Forster (Directeur de thèse) du GEA (Groupe d'Électromagnétisme Appliqué) de l'Université Paris X et Pascal Larzabal de l'École Normale Supérieure de Cachan (LESIR/SATIE). Cette thèse a été soutenue (Mention Très Honorable avec les félicitations du jury) le 14 novembre 2006 à l'Université de Nanterre. Frédéric Pascal est actuellement Chercheur dans le laboratoire SONDRA de Supélec.

Composition du Jury

Président	Jean-Jacques FUCHS	Professeur des Universités,
Rapporteurs :	Olivier BESSON	Professeur des Universités,
	Sylvie MARCOS	Directeur de Recherche CNRS,
Examinateurs :	Pascal LARZABAL	Professeur des Universités,
	Philippe FORSTER	Professeur des Universités,
	Fulvio GINI	Professeur Université de Pise,
	Jean Philippe OVARLEZ	Maître de recherche.

Dans le contexte très général de la détection radar, les détecteurs classiques, basés sur l'hypothèse d'un bruit Gaussien, sont souvent mis en défaut dès lors que l'environnement (fouillis de sol, de mer) devient hétérogène, voire impulsionnel, s'écartant très vite du modèle Gaussien. Des modèles physiques de fouillis basés sur les modèles de bruit composé (SIRP, Compound Gaussian Processes) permettent de mieux représenter la réalité (variations spatiales de puissance et nature de fouillis, transitions, ...). Ces modèles dépendent cependant de paramètres (matrice de covariance, loi de texture, paramètres de "nuisance") qu'il devient nécessaire d'estimer. Une fois ces paramètres estimés, il est possible de construire des détecteurs radar optimaux (Generalized Likelihood Ratio Test - Linear Quadratic) pour ces environnements. Cette thèse, qui s'appuie sur ces modèles, propose une analyse

complète de diverses procédures d'estimation de matrices de covariance, associées à ce problème de détection. Une étude statistique des principaux estimateurs de matrice de covariance, utilisés actuellement, est réalisée. De plus, un nouvel estimateur est proposé : l'estimateur du point fixe, très attractif grâce à ses bonnes propriétés statistiques et "radaristiques". Elle décrit également les performances et les propriétés théoriques (SIRV-CFAR) du détecteur GLRT-LQ construits avec ces nouveaux estimateurs. En particulier, on montre l'invariance du détecteur à la loi de la texture mais également à la matrice de covariance régissant les propriétés spectrales du fouillis. Ces nouveaux détecteurs sont ensuite analysés sur des données simulées mais également testés sur des données réelles de fouillis de sol.

[T2] Encadrement (70%) de la thèse ONERA de Mohamed Tria (2001-2004), effectuée à l'ONERA, sur le thème "*Imagerie Radar à Synthèse d'Ouverture par Analyse en Ondelettes Continues Multidimensionnelles*" en collaboration avec Messaoud Bénidir (Directeur de thèse) du Laboratoire des Signaux et Systèmes (LSS) de Supélec. Cette thèse a été soutenue (Mention Très Honorable) le 18 novembre 2005 à l'Université d'Orsay. Mohamed Tria est actuellement Ingénieur de Recherche dans la société Noveltis, Toulouse.

Composition du Jury

Président	Jean Christophe PESQUET	Professeur des Universités,
Rapporteurs :	Jean Pierre ANTOINE	Professeur des Universités,
	Éric POTTIER	Professeur des Universités,
Examinateurs :	Messaoud BÉNIDIR	Professeur des Universités,
	Jean François GIOVANELLI	Maître de Conférence,
	Jean Philippe OVARLEZ	Maître de Recherche.

Les méthodes classiques d'imagerie SAR (Synthetic Aperture Radar) font l'hypothèse d'isotropie (les points imagés se comportent de la même manière selon l'angle sous lequel ils sont vus) et de blancheur (les points imagés ont le même comportement dans la bande spectrale émise) des réflecteurs imagés. Néanmoins, ces hypothèses ne sont plus valables avec l'émergence de nouvelles capacités d'imagerie (large bande spectrale, grandes excursions angulaires, ...). L'objectif du travail de thèse a consisté à développer une méthode d'analyse temps-fréquence basée sur l'utilisation d'ondelettes continues permettant de mettre en lumière les phénomènes d'anisotropie et de coloration des points imagés. Une grande partie du travail a consisté à exploiter le caractère coloré et anisotrope des réflecteurs pour discriminer une cible dans une image SAR. Enfin l'analyse par ondelettes continues a été étendue à la polarimétrie pour améliorer la classification des cibles aléatoires et cibles déterministes puis à l'interférométrie afin obtenir une meilleure estimation de la hauteur des cibles.

[T1] Encadrement (50%) de la thèse ONERA/MNRT d'Emmanuelle JAY (1998-2001), effectuée à l'ONERA, sur le thème "*Détection en Environnement Non Gaussien*" en collaboration avec Patrick Duvaut (Directeur de thèse) et David Declercq du laboratoire ETIS de l'ENSEA. Cette thèse a été soutenue (Mention Très Honorable avec les félicitations du jury) le 14 juin 2002 à l'Université de Cergy-Pontoise. Emmanuelle Jay est actuellement Ingénieur de Recherche en Mathématiques Financières à la Société Générale Asset Managment, Paris.

Composition du Jury

Président :	Jean-Louis LACOUME	Professeur des Universités,
Rapporteurs :	Francis CASTANIÉ	Professeur des Universités,
	Alfonso FARINA	Professeur Université de Rome,
Examinateurs :	David DECLERCQ	Maître de Conférence,
	Dominique PICARD	Professeur des Universités,
	Jean Philippe OVARLEZ	Ingénieur de Recherche,
	Patrick DUVAUT	Professeur des Universités.
Invités :	Christian DELHOTE	

Les échos radar provenant des diverses réflexions du signal émis sur les éléments de l'environnement (le fouillis) ont longtemps été modélisés par des vecteurs gaussiens. La procédure optimale de détection se résumait alors en la mise en œuvre du filtre adapté classique. Avec l'évolution technologique des systèmes radar, la nature réelle du fouillis s'est révélée ne plus être gaussienne. Bien que l'optimalité du filtre adapté soit mise en défaut dans pareils cas, des techniques TFAC (Taux de Fausses Alarmes Constant) ont été proposées pour ce détecteur, dans le but d'adapter la valeur du seuil de détection aux multiples variations locales du fouillis. Malgré leur diversité, ces techniques se sont avérées n'être ni robustes ni optimales dans ces situations. A partir de la modélisation du fouillis par des processus complexes non-gaussiens, tels les SIRP (Spherically Invariant Random Process), des structures optimales de détection cohérente ont pu être déterminées. Ces modèles englobent de nombreuses lois non-gaussiennes, comme la K-distribution ou la loi de Weibull, et sont reconnus dans la littérature pour modéliser de manière pertinente de nombreuses situations expérimentales. Dans le but d'identifier la loi de leur composante caractéristique qu'est la texture, sans a priori statistique sur le modèle, nous proposons, dans cette thèse, d'aborder le problème par une approche bayésienne. Deux nouvelles méthodes d'estimation de la loi de la texture en découlent : la première est une méthode paramétrique, basée sur une approximation de Padé de la fonction génératrice de moments, et la seconde résulte d'une estimation Monte Carlo. Ces estimations sont réalisées sur des données de fouillis de référence et donnent lieu à deux nouvelles stratégies de détection optimales, respectivement nommées PEOD (Padé Estimated Optimum Detector) et BORD (Bayesian Optimum Radar Detector). L'expression asymptotique du BORD (convergence en loi), appelée le "BORD Asymptotique", est établie ainsi que sa loi. Ce dernier résultat permet d'accéder aux performances théoriques optimales du BORD Asymptotique qui s'appliquent également au BORD dans le cas où la matrice de corrélation des données est non singulière. Les performances de détection du BORD et du BORD Asymptotique sont évaluées sur des données expérimentales de fouillis de sol. Les résultats obtenus valident aussi bien la pertinence du modèle SIRP pour le fouillis que l'optimalité et la capacité d'adaptation du BORD à tout type d'environnement.

1.6.5 Encadrements de Stages

- Encadrement de Mélanie Mahot du Master Recherche Systèmes de Télécommunications Numériques (STN), de l'université Paris 6 (UPMC) en partenariat avec Télécom ParisTech de mars à juin 2009 sur le thème "*Techniques Robustes Pour le STAP en Environnement de Fouillis Hétérogène et Non-Stationnaire*"

- Encadrement de Pierre Formont du Master Recherche Signal Images Parole Télécom du GipsaLab, Grenoble de février à juillet 2009 sur le thème "*Classification Polarimétrique Dans les Images SAR Hautement Texturées*"

- Encadrement de Frédéric Pascal du DEA de l'Université de Paris VII d'avril à juillet 2003 sur le thème "*Détection en Environnement Non Gaussien*"

- Encadrement de Mohamed Tria du DEA du LSS d'avril à juillet 2001 sur le thème *"Analyse SAR par Ondelettes*"

- Encadrement d'Éloi Thoumie du DEA de l'ENSEA d'avril à juillet 2000 sur le thème *"Analyse SAR par Ondelettes*"

- Encadrement d'Emmanuelle Jay du DEA de l'ENSEA d'avril à juillet 1998 sur le thème "*Détection en Environnement Non Gaussien*"

- Encadrement de Véronique Hirsinger de l'École Polytechnique Féminine de janvier à juin 1994 (stage de fin d'études) sur le thème "*Analyse et Modélisation du Fouillis de Sol*"

- Encadrement de Christophe Corroyer du DEA de l'ENSEA d'avril à juillet 1992 sur le thème "*Analyse de Signaux Large Bande par Représentations Temps-Fréquence Affines*"

1.6.6 Enseignement

- 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007 : Cours et Travaux Dirigés en Traitement du Signal à l'Ecole Centrale de Paris (30 heures),
- 2003, 2005, 2007, 2009 : Cours SAE (stage de formation ELA 030) sur l'Analyse et la Reconnaissance des Cibles Radar (3h),
- 2006, 2009 : Cours SAE (stage de formation ELS 043) sur l'Analyse par Ondelettes (6h),
- 2004, 2005, 2006, 2007, 2008, 2009 : Cours sur l'Analyse Temps-Fréquence appliquée à l'Imagerie Radar, 3ème année, ENSEA (4h),
- 2008, 2009 : Encadrement de projets d'élèves à l'ENSAE,
- 1997, 1998, 1999, 2000, 2001 : Cours *"Ondelettes et Temps-Fréquence"* de troisième année à l'École Nationale Supérieure des Télécommunications (10 heures). Rédaction d'un polycopié de cours. Encadrement de projets,
- 1993, 1994, 1995 : Cours de Traitement du Signal à l'ESIEA, première année, (36 heures).

1.6.7 Publications

Ma liste de publications se compose de :

- 5 publications dans des ouvrages ou collections référencés [**Oxx**] dans tout le document,
- 20 publications dans des journaux à comité de lecture référencées [**Jxx**] dans tout le document, dont deux en révision et trois soumises,
- 63 publications dans les conférences à comité de lecture référencées [**Cxx**] dans tout le document,,
- 14 participations dans différents colloques ou séminaires sans acte référencées [**Sxx**] dans tout le document.

Tous ces articles sont en grande partie accessibles sur ma page web :

http://web.mac.com/jeanphilippe.ovarlez.

1.6.7.1 Ouvrages et Collections

[**O5**] P. Goncalves, **J.P. Ovarlez** and R. Baraniuk, "*Quadratic Time-Frequency Analysis III : the Affine Class and Other Covariant Classes*", Chapitre 7 de "*Time-Frequency Analysis : Concepts and Methods*" dans la collection Digital Signal and Image Processing Series, Eds. F. Hlawatch and F. Auger, ISTE Ltd and John Wiley & Sons, Inc., 2008.

[**O4**] **J.P. Ovarlez**, P. Goncalves and R. Baraniuk, "*Analyse Temps-Fréquence Quadratique III : La Classe Affine et Autres Classes Covariantes*", Chapitre 7 de "*Temps-Fréquence : Concepts et Outils*" dans la collection *Information, Commande et Contrôle*, Eds. F. Hlawatch and F. Auger, Edition Hermes, 2005.

[**O3**] **J.P. Ovarlez** and J. Dulost, "*New Methods of Designing Optimum Broad-Band Radar Signals*", dans "Ultra-Wideband, Short Pulse Electromagnetics 3", Eds A. Stone, C. Baum and L. Carin, Editions Plenum Press, 1997.

[O2] J. Bertrand, P. Bertrand and **J.P. Ovarlez**, "*The Mellin Transform*", Chapitre 12 de "The Transforms and Applications Handbook", Ed. A.D. Poularikas, Volume de la collection "The Electrical Engineering Handbook", CRC Press Inc, 1995.

[O1] J. Bertrand, P. Bertrand and **J.P. Ovarlez**, "*The Wavelet Approach in Radar Imaging*" dans "*Progress in Wavelet Analysis and Applications*", Ed. Y. Meyer, Editions Frontières, 1993.

1.6.7.2 Revues à Comité de Lecture

[J20] G. Ginolhac, P. Forster, F. Pascal and **J.P. Ovarlez**, "*Robust STAP Detectors In Low-Rank Homogeneous or Non-Homogeneous Clutter*", IEEE Trans. on AES, soumis, septembre 2010.

[J19] J. Totems, V. Jolivet, **J.P. Ovarlez** and N. Martin, "*Maximum Likelihood Instantaneous Frequency Estimation and Time-Frequency Processing for Pulsed Laser Vibrometry*", IEEE Trans. on Signal Processing, juillet 2010, en révision.

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[C62] G. Vasile, F. Pascal and **J.P. Ovarlez**, "*Heterogenous Clutter Model for High Resolution Polarimetric SAR Data Parameter Estimation*", Proc. POLINSAR, Jan. 2011.

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[C58] M. Mahot, P. Forster, **J.P. Ovarlez** and F. Pascal, "*Robustness Analysis of Covariance Matrix Estimates*", Proc. EUSIPCO 2010, Aalborg, Denmark, Aug. 2010.

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[C45] L. Bombrun, J.M. Beaulieu, G. Vasile, **J.P. Ovarlez**, F. Pascal and M. Gay, "*Segmentation of Polarimetric SAR Images using Heterogeneous Clutter Models*", IEEE-IGARSS09, Cape Town, South Africa, Jul. 2009.

[C44] M. Duquenoy, **J.P. Ovarlez**, E. Pottier, L. Ferro-Famil, "*Supervised Classication by Neural Networks Using Polarimetric Time-Frequency Signatures*", IEEE-IGARSS09, Cape Town, South Africa, Jul. 2009

[C43] M. Duquenoy, **J.P. Ovarlez**, L. Ferro-Famil and E. Pottier, "*HyperImage Concept : Multidimensional Time-Frequency Analysis Applied to SAR Imaging*", IEEE-IGARSS09, Cape Town, South Africa, Jul. 2009.

[**C42**] J. Totems, V. Jolivet, **J.P. Ovarlez**, N. Martin, "*Comparison of Signal Processing Methods and Poly-Pulse Waveforms for Laser Vibrometry in Pulsed Mode*", 15th Coherent Laser Radar Conference, Toulouse, France, 22-26 June 2009.

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[C18] C. Musso, E. Jay and **J.P. Ovarlez**, *"Saddlepoint Approximation Applied to Fusion in Multiple Sensor and to Detection in Clutter*", Proc. IEEE-Information Fusion, Cairns, Queensland, Australia, Jul. 2003.

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[C11] **J.P. Ovarlez**, "*Optimum Signal Synthesis For Time-Scale Estimation*", Proc. IEEE-ICASSP98, Seattle NY, USA, May 1998.

[C10] **J.P. Ovarlez**, "*La Transformation de Mellin et l'Analyse des Signaux Large Bande*", Coll. Temps-Fréquence, Ondelettes et Multirésolution : Théorie, Modèles et Applications, INSA, Lyon, France, 9-11 Mar. 1994.

[C9] **J.P. Ovarlez**, *"L'application de la Transformation de Mellin en Théorie du Radar Large Bande"*, Proc. SEE, Journée Méthodes Modernes et Tendances en Radar et Sonar, ENST, France, 1993.

[C8] **J.P. Ovarlez**, "*Cramer-Rao Bounds Computation for Velocity Estimate in the Broad-Band Case Using the Mellin Transform*", Proc. IEEE-ICASSP93, Minneapolis MT, USA, April 1993.

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[C2] J. Bertrand, P. Bertrand and **J.P. Ovarlez**, "*Compression d'Impulsions en Large Bande*", Proc. XIIème Coll. GRETSI, Juan-les-Pins, Juin, France, 1989.

[C1] J. Bertrand, P. Bertrand and **J.P. Ovarlez**, *"Discrete Mellin Transform for Signal Analysis"*, Coll. Ondelettes, Université de Luminy, Marseille, France, May 1989.

1.6.7.4 Conférences Nationale/Internationales Sans Acte

[S15] Workshop on Statistical Analysis, Learning and Signal Processing (SALSIP-2010), DSO National Laboratories, 1 Novembre 2010, *SIRV Modelling for Estimation and Detection Problems*

[S14] Séminaire IEEE Communication Society Singapore Chapter, National University of Singapore, Singapore, 2 septembre 2010, *The SIRV Modelling for Estimation and Detection Problems*

[S13] Séminaire IEEE Oceanic Engineering Society Singapore Chapter, DSO National Laboratories, Singapore, 13 juillet 2010, *Time-Frequency Analysis and Its Use in Radar Imaging*

[S12] Colloque SEE/IET, Paris, 9 june 2009, Radar Clutter - Observation, Modelling And Related Processing, *Optimal Detectors in Non Gaussian and Non Homogeneous Clutter*

[S11] Séminaire GDR ISIS Modèle Stochastique Multivarié en Traitement des Images, Paris, 4 mars 2009, *Méthodes d'Estimation Robuste dans les Processus Gaussiens Composés*

[S10] Séminaire invité au GIPSA Lab, Grenoble, 26 février 2009, *Robust Estimation in Non-Gaussian and/or Heterogeneous Environment*

[S9] Séminaire invité au Laboratoire IMS Groupe Signal, 3 février 2009, *Détection robuste à un environnement non Gaussien, Application au Traitement Adaptatif Spatio-Temporel (STAP) - Classification Polarimétrique dans les Images SAR Hautement Texturées*

[S8] Séminaire invité au 8th DLR-CNES Workshop Information Extraction & Scene Understanding for Meter Resolution Images, *Coherency Matrix Estimation for Heterogeneous Clutter Through the SIRV Model. Application to High Resolution POLSAR Images, Toulouse, October 15-17, 2008*

[S7] First French Singaporean Workshop on Signal Processing, Singapore, 3-4 décembre 2007, *CFAR Radar Detection in Non-Gaussian Heterogeneous Clutter*

[S6] Séminaire GDR ISIS, Traitement d'Antennes pour le Radar, Paris, 2 juillet 2007, *Détection Robuste à un Environnement Non-Gaussien, Application au Traitement STAP*

[S5] Séminaire Détection Electromagnétique organisé par la DGA, Paris, 22 novembre 2006, *STAP HR dans un Fouillis Hétérogène et/ou Non Gaussien pour des Applications Air-Sol*,

[S4] Séminaire invité CNES, Toulouse, 22 juin 2004, "Non-Gaussianité, Non-Linéarité et Non-Stationnarité", *Les Distributions Temps-Fréquence : Outils d'Analyse pour l'Etude des Signaux Non-Stationnaires*

[S3] Séminaire SEE - Ruptures en Radar, 19-20 novembre 2003, *Détection Radar en Environnement de fouillis non Gaussien,*

[S2] Séminaire Société de Calcul Mathématique - 18 novembre 1999, *Détection Radar et Évaluation des Performances*

[S1] Colloque SEE "Methodes Modernes de Traitement de Signal en Radar et Sonar", ENST, 8 Avril 1993, Paris, *Application de la Transformation de Mellin en Théorie du Radar Large Bande*

1.6.8 Liste des Brevets/Logiciels

[B1] Référencement du logiciel MODEM (Modélisation et Observation de la Détection Electromagnétique des Missiles) développé conjointement par C. Aumasson, P. Fely, J.P. Ovarlez et F. Tardivel, 2002

1.6.9 Vie Scientifique

- Relecteur pour les revues et congrès nationaux et internationaux de Traitement du Signal : Traitement du Signal, Signal Processing, IEEE Transactions on Signal Processing, IEEE Transactions on Image Processing, IEEE Transactions on Geoscience and Remote Sensing, Processing IEEE-ICASSP, GRETSI, EUSIPCO, ...

- Membre du Comité Scientifique de la conférence IEEE-Radar 2004, octobre 2004, Toulouse,

- Membre du Comité Scientifique de la conférence IEEE-Radar 2009, octobre 2009, Bordeaux,

- Membre du Comité Scientifique de la conférence CIP2010, Second IAPR International Workshop on Cognitive Information Processing, 14-16 Juin 2010, Elba, Italy,

- Chairman à la conférence IEEE-RadarCon 2008, Rome,

- Chaiman à la conférence IEEE-Radar 2009, octobre 2009, Bordeaux,

- Organisateur et pilote depuis 2007 du «Club STAP» ayant pour but de fédérer les activités de recherche industrielles (THALES, ...), universitaires (laboratoires CNRS et universitaires) et étatiques (ONERA, DGA, ...) ayant trait aux techniques de type STAP (Space Time Adaptive Processing). Le club STAP créé en janvier 2007 met à disposition des données synthétiques ou expérimentales provenant des différents partenaires et sur lesquelles peuvent être mis en œuvre les différents algorithmes développés par la communauté scientifique. Environ quatre réunions annuelles sont organisées. Le club STAP a été récemment étendu (juin 2010) à la communauté scientifique européenne dans le cadre d'un appel à proposition de l'agence de défense européenne (EDA) initié par l'ONERA et THALES. Ces travaux font actuellement l'objet d'un numéro spécial de la revue Traitement du Signal présentant les activités du groupe mais également la synthèse des travaux engagés depuis trois ans. Développement d'un site web coopératif : http ://clubstap.free.fr

- Membre du Comité Scientifique de la Branche (CSB) Physique de l'ONERA depuis 2003 : évaluation des doctorants, des propositions de thèse ONERA, organisations internes d'événements scientifiques au sein des quatre départements (DOTA, DEMR, DESP, DMPH) de la branche Physique de l'ONERA. Reconduit pour cinq ans par la Direction Scientifique Générale de l'ONERA en 2009,

- Président du Conseil Scientifique du Département (CSD) DEMR de l'ONERA en janvier 2006 et janvier 2007. Le CSD conseille la direction du DEMR dans le domaine scientifique et instruit les dossiers préparatoires aux actions qui relèvent de ce domaine. En particulier, le CSD porte un avis sur :
  — les demandes de congrès et les propositions de publications, qu'il stimule ou suscite le cas échéant.
  — les propositions de thèse et de stage cours d'étude, qu'il stimule ou suscite le cas échéant.
  — les projets de partenariat avec les laboratoires extérieurs,
  — les sujets présentés en Comité d'Expertise et d'Orientations (CEO), le suivi des recommandations faites par le CEO,
  — les sujets présentés au compte-rendu d'activité annuel du département.

  Le CSD contribue à la préparation des dossiers à instruire par le CSB et assure un travail de veille scientifique en liaison avec les structures internes et externes à l'ONERA dans ce domaine. Il élabore enfin un document annuel de synthèse de son activité.

1.7 Activités Contractuelles - Rapports de Recherche ou d'Études Internes ONERA

Environ 40 rapports internes numérotés chronologiquement et provenant d'études contractuelles, majoritairement pour la DGA, sont ici très succinctement présentés en différentes thématiques. Ces rapports sont en grande majorité non accessibles pour des raisons de confidentialité.

1.7.1 Aspects Détection

[I42] Rapport RTS 2/13019 DEMR, *"Expertise sur la Contre-Contre Mesure (CCME) face à une modulation de cible*, décembre 2009,

[I40] Rapport RTS 1/13019 DEMR, *"Expertise pour la Lutte Contre le Leurrage"*, mai 2008.
La modulation de la signature d'une cible perturbe les performances de détection d'un radar. Si le radar diagnostique une telle modulation, il peut alors mettre en place des techniques de Contre Contre-Mesure. L'objectif de ces études était de décrire des méthodes de diagnostic qui permettent de caractériser ces types de modulation et de les appliquer pour contrer la contre-mesure.

[I39] Rapport RT 2/11569 DEMR, *"Détection et Pistage par Filtrage Particulaire"*, juillet 2008,
[I38] Rapport RT 1/11569 DEMR, *"Détection et Pistage par Filtrage Particulaire"*, octobre 2006.
Ces deux rapports présentent essentiellement la modélisation du signal et une architecture de traitement à partir desquels sont développés les algorithmes de détection et de pistage par filtrage particulaire. Il fait enfin une analyse non-exhaustive mais critique de la mise en place d'un filtre particulaire pour l'intégration longue. Les avantages et inconvénients de cette technique sont analysés.

[I24] Rapport RT 5/03424 DPRS, *"Modèle Comportemental d'un Radar Sol"*, nov. 2000.
Élaboration d'un module de détection radar et d'environnement réaliste de fouillis pour la simulation comportementale COSMOS.

[I23] Rapport RT 53/7259 DEMR/Y, *"Modélisation des Performances de Détection de Cibles Scintillantes",* octobre 1999.
Cette étude présente une modélisation générique des performances de détection radar en présence de cibles scintillantes. Cette étude se décompose en deux parties : l'une s'appuie sur une méthode statistique (Monte-carlo), l'autre, sur une méthode purement mathématique permettant de valider les résultats issus de la première partie à partir d'une analyse statistique des phénomènes mis en jeu.

[I30] Rapport RT DEMR/TRA/RTFCA/05073, *"Traitements Radar Avancés et Exploitation - Contrôle de la fausse alarme"*, 23 juillet 2002.
Mise en œuvre, pour THALES, de détecteurs radar optimaux dans un environnement de fouillis non-gaussien. Exploitation des performances sur signaux réels de fouillis.

[I10] Rapport RT 6/5275 SN, *"Détection en Environnement Non gaussien*", octobre 1996.
Des méthodes originales de calcul de probabilité de détection sont présentées dans le cadre de la détection à site-bas d'une cible fluctuante noyée dans un bruit gaussien et dans un fouillis de sol modélisable en un processus SIRP (processus composé non gaussien).

[I37] Rapport RT 1/11569 DEMR, *"Mesure de la Longueur Radioélectrique*", décembre 2006.
Ce document décrit quelques algorithmes Haute-Résolution qui permettent de déterminer la longueur radioélectrique d'un corps composé de plusieurs points brillants. Il propose une description de l'algorithme des Moindres Carrés non Linéaires, des algorithmes Music et Esprit couplés avec les moindres carrés et enfin de l'algorithme *CLEAN*.

[I15] Rapport RT 173/4348 DEMR/Y, *"Discrimination Après Haute Résolution"*, novembre 1998.
Analyse de la potentialité de discrimination polarimétrique de deux corps différents dans la même cellule de résolution radar.

[I6] Rapport RT 26/5272 SY, "*Étude de Formes d'Onde Pour Radar de Trajectographie*", juin 1995.
Cette étude présente une description d'outils d'analyse et d'évaluation des performances de codes radar ainsi qu'une synthèse des formes d'onde les plus couramment utilisées en radar.

[I2] Rapport RT 2/5169 SY, "*Généralisation Large Bande de la Fonction d'Ambiguïté et des Représentations Temps-Fréquence. Calcul par Transformée de Mellin Discrète*", avril 1991,

[I3] Rapport RT 5/5169 SY, "*Les Fonctions d'Ambiguïté en Large Bande : Analyse de Scènes, Synthèse de Codes Optimaux*", 1993.
Cette étude décrit le comportement large-bande de l'analyse d'une scène constituée d'un ensemble de points brillants se déplaçant avec des vitesses différentes. Les problèmes d'interaction entre les points brillants à travers la représentation du diagramme d'ambiguïté sont soulevés grâce à l'étude des résolutions vitesse-retard en large bande. On montre, grâce à la transformation de Mellin qui joue un rôle primordial en analyse de signaux large-bande, que les résolutions vitesse ne sont plus inversement proportionnelles à la durée d'analyse (comme dans le cas bande étroite), mais à l'étendue du signal dans l'espace de Mellin. On montre de plus que les différentes notions usuelles d'ambiguïté distance-vitesse ne sont plus conservées. Pour pouvoir contrôler à la fois la forme des fonctions d'autocorrélation distance et vitesse (qui fixe le pouvoir séparateur du radar), une nouvelle méthode de synthèse de code optimaux, pour l'analyse radar large-bande, basée sur la méthode de la phase stationnaire, est alors proposée.

1.7.2 Modélisation du Fouillis

[I33] Rapport RT 1/07104 DEMR, *"PEA Fouillis pour Radar Haute Résolution et Autodirecteurs - Rapport final"*, octobre 2003,

[I32] Rapport RT 2/07102 DEMR, *"Fouillis pour Radars et Autodirecteurs à Haute Résolution : Caractéristiques de fouillis"*, 1 octobre 2002,

[I31] Rapport RT 1/07102 DEMR, *"Fouillis pour Radars et Autodirecteurs à Haute Résolution : Expression du Besoin"*, 1 octobre 2002.
Description d'une proposition à la DGA d'un Plan d'Étude Amont concernant le montage d'une expérimentation de mesure de fouillis ainsi que son analyse.

[I13] Rapport RT 12/4401 DPRS/Y/DEMR, *"Modélisation Affinée du Fouillis de Sol"*, mars 1998.
Modélisation réaliste des caractéristiques spectrales et statistiques du fouillis de sol. Analyse de l'impact des performances de détecteurs radar conventionnels face à ce fouillis. Modélisation de la statistique du fouillis par les Approximants de Padé.

[I8] Rapport RT 30/5272 SY, "*Analyse des Données de Fouillis de Sol de l'Expérimentation Site-Bas 1995*", août 1996.
Modélisation statistique et spectrale de données de fouillis de sol.

[I4] Rapport RT 25/5272 SY, "*Évaluation de la Détectabilité des Cibles Volant à Basse Altitude par des Radars Sols*", mai 1995.
Analyse des performances de détection radar face à une cible évoluant à très basse altitude.

1.7.3 Nouveaux Concepts

[I36] Rapport RT 1/09244 DEMR, *"Synthèse Technique du PRF ROMULUS 2004"*, mars 2005.
Cette étude concerne l'exploration de nouveaux concepts de détection, de localisation et de reconnaissance basés sur l'utilisation de radars spatiaux distribués sur des plates-formes volant en formation (PRF ROMULUS (Radars Orbitaux MULtisatellites à Usage de Surveillance). Analyse de la faisabilité de techniques STAP pour analyse de trafic auto-routier par réseau lacunaires de satellites.

1.7.4 Imagerie SAR/ISAR

[I29] Rapport RT 1/03506 DEMR, *"Imagerie SAR par Analyse en Ondelettes"*, mars 2001,

[I11] Rapport RT 22/5169 SY, "*Imagerie ISAR de Satellites : Traitement des Données FGAN et ARMOR*", février 1997.
Cette étude s'inscrit dans la caractérisation et le développement de nouvelles méthodes temps-fréquence et ondelettes continues pour l'analyse des images SAR et plus particulièrement pour

l'analyse de réflecteurs dits dispersifs (ex : non isotropes, colorés, ...) présents dans une zone de terrain. Mise en œuvre de techniques d'imagerie ISAR sur cibles réalistes.

[I34] Rapport RT 5/06116 DEMR, *"Rapport d'Analyse sur la Simulation d'Images SAR THR pour Drones MALE"*, janvier 2004,

[I35] Rapport RT 1/08306 DEMR, *"Rapport d'Expertise ESPRITSAR"*, février 2004.
Rapports d'expertise (évaluation de performances, avantages, limitations) sur toutes les méthodes de synthèse d'image SAR et leur exploitation (qualité image).

[I1] Rapport RT 4/3708 SY, "*Groupe des Similitudes et Ondelettes en Imagerie Radar*", juin 1990.
Rapport sur l'analyse par ondelettes des images radar de laboratoire.

1.7.5 Simulateurs - Simulations

[I28] Rapport RT 9835833 Radar/PVE/1.1, *"Plan de Vérification du module de propagation radar sur terrain réel"*, mars 2001,

[I27] Rapport RT 9835833 Radar/PVA/1.1, *"Plan de Validation du module de propagation radar sur terrain réel"*, mars 2001,

[I26] Rapport RT 9835833 Radar/DCD/1.1, *"Dossier de Conception Détaillée du module de propagation radar sur terrain réel"*, janvier 2001,

[I25] Rapport RT 9835833 Radar/DSL/1.1, *"Dossier de Spécification Logiciel d'un module de propagation radar sur terrain réel"*, novembre 2000.
Élaboration d'un module de propagation permettant de calculer, dans un plan vertical radar-cible et au dessus d'un terrain réel, les pertes de propagation.

[I22] Rapport RT 183/4348 DEMR/Y, *"Justification du Modèle Comportemental SAR/MTI de SICAST"*, septembre 1999,

[I21] Rapport RT 182/4348 DEMR/Y, *"Plan de Validation de l'Élément Logiciel du Modèle Comportemental SAR/MTI de SICAST"*, septembre 1999,

[I20] Rapport RT 181/4348 DEMR/Y, *"Cahier d'Essai du Modèle Comportemental SAR/MTI de SICAST"*, septembre 1999,

[I19] Rapport RT 180/4348 DEMR/Y, *"Spécifications Techniques d'Interface de l'Élément Logiciel du Modèle Comportemental SAR/MTI de SICAST"*, septembre 1999,

[I18] Rapport RT 179/4348 DEMR/Y, *"Spécifications Techniques de Besoin Logiciel de l'Élément Logiciel du Modèle Comportemental SAR/MTI de SICAST"*, septembre 1999,

[I17] Rapport RT 177/4348 DEMR/Y, *"Rapport Technique d'Avancement sur la Modélisation du Modèle Comportemental SAR/MTI de SICAST"*, avril 1999.
Cette étude avait pour but de construire un simulateur comportemental de photo-interprétation d'une scène SAR. La modélisation SAR consistait à simuler l'image SAR d'une scène réaliste constituées de fonds et de cibles (imagettes réelles) et d'évaluer la probabilité de détection du photo-interprète de cette scène.

[I14] Rapport RT 11/4401 DPRS/Y, *"ATHOS : Logiciel de Pénétration du Missile APACHE. Rapport Final de Synthèse"*, mars 1998.
Analyse technico-opérationnelle par le logiciel MODEM des performances de détection d'un raid de missiles par un ensemble de radars disposés sur un champ de bataille.

[I7] Rapport RT 2/4401 SY, "*Logiciel d'Analyse de la Détectabilité Électromagnétique d'un Missile Volant à Basse Altitude. Logiciel Modem*", octobre 1995.
L'objectif de cette étude était de développer une modélisation fiable mais pas trop complexe permettant de répondre à la question de la détectabilité d'une cible volant à basse altitude au dessus du sol, face à un radar.

1.7.6 Analyse de Détectabilité Radar

[I41] Rapport RT DEMR-BFR-1.0, *"Analyse Théorique du Signal BFR et Emulation"*, avril 2010.
Analyse de différentes techniques de détection pour la détection de navires dans des images SAR haute résolution de fouillis de mer,

[I16] Rapport RT 14/4406 DPRS/Y/DEMR, *"Détectabilité du Missile SCALP/EG Face Aux Systèmes de Défense Sol-Air CROTALE NG et SAMP/T"*, décembre 1998,

[I12] Rapport RT 7/4401 SY, "*Capacité de Pénétration du Missile APACHE*", août 1997.
Analyse par le logiciel MODEM des performances de détection radar d'une cible réaliste caractérisée par son vrai diagramme de rétrodiffusion,

[I9] Rapport RT 5/4401 SY*, "Détectabilité de Différentes Configurations du Missile APACHE/EG*", août 1996.
Analyse par le logiciel MODEM des performances de détection radar d'une cible réaliste caractérisée par son vrai diagramme de rétrodiffusion,

[I5] Rapport RT 1/4401 SY, "*Influence de la SER sur la Détectabilité du Missile Apache*", avril 1995.
Analyse par le logiciel MODEM des performances de détection radar d'une cible réaliste caractérisée par son vrai diagramme de rétrodiffusion.

Chapitre 2

Synthèse des Activités de Recherche

En qualité d'Ingénieur de Recherche à l'ONERA depuis 1992, mes activités de recherche se sont réparties entre les activités de recherche à court et moyen terme sous contrat, principalement issues de la DGA, des contrats d'expertise également issus de la DGA, des activités contractuelles pour les industriels (THALES, MBDA, Alcatel, ...) et des activités de recherche plus amont (études propres, élaboration de sujets de thèses, travail avec les doctorants) financées par les ressources internes de l'ONERA et contrôlées par le Direction Scientifique de la Branche Physique de l'ONERA dont une des missions principales est la formation par la recherche. Celle-ci se concrétise par de l'enseignement et de l'encadrement de thèses et des publications par les différentes équipes de l'ONERA. Près de 150 doctorants sont donc encadrés chaque année.

La trame générale de mon travail de recherche a toujours consisté à mettre en œuvre les outils de Traitement de Signal ou de Traitement de l'Information les plus adaptés à une thématique donnée, principalement axée dans le domaine du radar : détection et séparation de sources, codes radar optimaux, traitement radar adaptés aux cibles, détection des cibles, estimation de paramètres, analyse de la qualité de ces paramètres (bornes), imagerie radar (SAR, ISAR, laboratoire), modélisation du fouillis, élaboration de simulateurs radar, SAR et STAP pour répondre à des demandes d'expertise bien précises.

Ces différentes activités contractuelles, même celles non étiquetées "recherche", m'ont sans aucun doute beaucoup apporté et énormément appris. L'expérience que l'on acquiert, par exemple, après avoir analysé des signaux réels provenant d'expérimentations auxquelles quelquefois on a participé devient une source d'inspiration et d'informations très riche. Les signaux réels nous causent des problèmes qu'il faut impérativement résoudre. Ces problèmes peuvent provenir soit des défauts des chaînes d'acquisition ou d'émission des radars, soit du non-respect des conditions souvent bien établies des algorithmes de traitement de signal. L'unité *Traitement du Signal* dans laquelle j'ai évolué, a toujours été sensible à cette problématique et a toujours pointé du doigt l'importance, la nécessité et l'omniprésence des méthodes de traitement du signal pour un département radar comme le DEMR.

Les trois axes principaux de mes recherches passées ou actuelles sont présentés. Le premier, **Poursuite des Travaux de Thèse**, est directement issu de mes perspectives de thèse, notamment sur l'Analyse Temps Fréquence appliquée à l'Imagerie Radar. Le deuxième, **la Détection et Estimation pour le Radar**, présente la méthodologie et les principaux résultats obtenus pour la détection radar en général. Le troisième, plus récent, **Classification des Images SAR**, synthétise la méthodologie et les résultats acquis dans les deux premiers axes pour l'application de classification, détection de changement dans les images SAR hautement texturées.

2.1 Poursuite des Travaux de Thèse

Parallèlement aux activités contractuelles qui sont une part non négligeable du travail fourni à l'ONERA, je me suis toujours senti attiré par la recherche, attirance que mes trois ans de thèse passées à l'ONERA ont certainement dû contribuer à renforcer. C'est ainsi que, jeune ingénieur dans la division Systèmes Radar de l'ONERA et face à mes nouvelles obligations contractuelles, j'ai continué par tous les moyens à faire évoluer mes travaux et perspectives de thèse. Ces perspectives m'ont conduit, de 1992 à 2008 à publier quelques papiers et chapitres d'ouvrage dans la ligne droite de ma thèse ([**J2**], [**O1**], [**O2**], [**O3**], [**O4**], [**O5**], [**C1**] à [**C11**]).

2.1.1 Qualité d'Estimation du Facteur de Compression Doppler

Le résultat de thèse [**C8**], [**C9**], [**C10**] le plus original (que je n'ai jamais pris le temps, et je le regrette vraiment, de publier dans une quelconque revue) est celui de la qualité d'estimation par les bornes de Cramèr Rao du facteur d'échelle Doppler affectant un signal radar large bande émis sur une cible mobile. Je ne peux vraiment pas résister, dans ce document d'HDR, au plaisir de vous le faire ou refaire découvrir en Annexe A. Une synthèse de code optimale, similaire à la construction de code radar à grand produit bande-durée (grand BT) a pu alors être mise en œuvre ([**O3**], [**C11**], [**I3**]). Malheureusement, ces idées n'ont jamais pu être mises en application dans le domaine du radar, la notion de fréquence Doppler (au lieu de compression Doppler) étant tellement ancrée dans les mœurs ! Il faut aussi admettre que les signaux radar n'ont jamais été jusqu'ici assez "large-bande" pour véritablement être affectés par la compression Doppler dans l'impulsion radar. Mais ces techniques peuvent néanmoins être appliquées au domaine du sonar actif ou aux radars *ultra-wideband*, qui peuvent être amenés à rencontrer ce genre de problème.

2.1.2 Analyse Temps-Fréquence pour l'Imagerie Radar

Le seul axe que j'ai véritablement continué de développer est l'analyse Temps-Fréquence appliquée à l'imagerie radar [**I1**], [**J2**], l'imagerie SAR [**J5**], [**J6**], [**J7**], [**I29**] et l'imagerie ISAR [**I11**], notamment avec l'encadrement du stage de DEA d'Éloi Thoumie en 2000, la thèse ONERA de Mohamed Tria ([**T2**], 2001 - soutenue en 2005) et celle de Mickaël Duquenoy [**T4**], [**J11**], [**J15**] en 2004. En 2004, Mohamed Tria a également travaillé avec Élise Colin-Koeniguer, doctorante ONERA sur l'adaptation des méthodes Temps-Fréquence pour améliorer la qualité de l'estimation de la cohérence interférométrique et polarimétrique [Col05].

La thèse de Mickaël Duquenoy (2004 - soutenue en 2009) que j'ai proposée et encadrée a permis d'initier un certain nombre de résultats pour l'analyse des images SAR polarimétriques et notamment sur l'analyse polarimétrique dite *cohérente* des cibles manufacturées. Ce travail est d'ailleurs en connexion certaine avec le travail de post-doc [**PD1**] de Gabriel Vasile sur la classification polarimétrique dite *non cohérente* des images SAR [**J12**]. Enfin, les méthodes générales de l'Analyse Temps-Fréquence ont pu être mises en applications dans la thèse de Julien Totems [**T7**] pour l'analyse de la vibration des cibles ou objets par laser [**J16**], [**C42**], [**C50**], [**C52**].

2.1.2.1 Imagerie Radar

Les méthodes d'imagerie radar fournissent un moyen d'analyse qui permet, entre autres, de pouvoir caractériser, à l'aide d'une image, les propriétés rétrodiffusantes des cibles ou des zones observées par un radar aéroporté. De manière conventionnelle, cette image est souvent porteuse d'informations relatives à la seule répartition spatiale des points brillants qui réfléchissent une partie d'un signal radar émis sur la cible :

- En chambre anéchoïque, notamment dans le cadre de l'analyse de la furtivité des cibles, il est nécessaire d'identifier les parties de la cible éclairée qui réfléchissent le signal émis sur elle. Cette identification permet aux "furtivitiens" de masquer (à l'aide de matériaux par exemple), voire au

contraire, d'amplifier les échos de certaines parties de cette cible.

- Dans le cas de l'imagerie ISAR (Inverse Synthetic Aperture Radar), l'objectif est d'imager, à partir de l'émission d'une station radar fixe, une cible en évolution par la seule analyse du signal reçu. Cette analyse permet, à partir de l'image formée, de lancer des processus de reconnaissance et d'identification des cibles analysées.

- Dans le cas du SAR (Synthetic Aperture Radar), l'image du sol est construite à partir du signal reçu par un radar imageur aéroporté évoluant sur une trajectoire et éclairant le sol. Les applications relatives à cette imagerie portent principalement sur l'aspect reconnaissance et identification de cibles ou de zones de terrain.

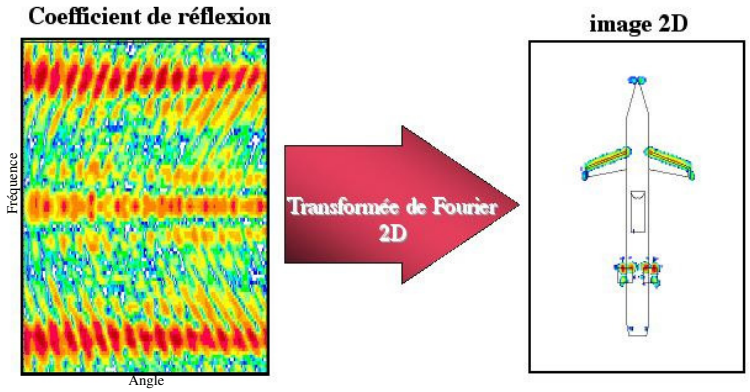


FIGURE 2.1 – Imagerie bidimensionnelle conventionnelle d'une maquette de missile.

On attend souvent du processus d'imagerie radar qu'il fournisse une information qui s'apparente à la notion d'image *optique* (imagerie infrarouge, imagerie classique par capteur CCD, ...). Les procédés de reconnaissance ou d'identification automatique radar en étant encore à leurs balbutiements, le seul moyen utilisé aujourd'hui est d'analyser "à l'oeil" les images délivrées par le moyen d'imagerie. On oublie alors que la réflexion électromagnétique est très différente de l'imagerie optique du fait de la complexité du comportement des ondes électromagnétiques dans la gamme de fréquence des radars (HF-W). Pour améliorer le procédé de reconnaissance ou d'identification, il est possible d'étendre la notion d'imagerie en fournissant d'autres éléments caractérisant la cible. Cela permet de conférer au terme *image* un sens beaucoup plus général que le sens optique auquel on se réfère souvent.

Ainsi, une première démarche permet de décrire la répartition spatiale des points de la cible dans un domaine angulaire, spectral ou polarimétrique restreint. Si l'on étend cette démarche, il est possible de décrire également, à l'aide d'une image, le comportement cinématique [Bert97],[Vig96] ou la stationnarité de ces points brillants (vitesse, accélération, réflecteurs variant dans le temps, ...). L'apport de la polarimétrie, nous le verrons, permet également d'enrichir les images d'informations sur le comportement électromagnétique des cibles ou des fonds, voire des informations de type interférométrique (hauteur) [Titin99], [Titin00a], [Titin00b]. L'*image radar* doit ainsi être prise au sens large du terme car elle peut s'étendre sur des dimensions autres que la largeur, la longueur ou la hauteur.

L'axe de recherche que nous avons initié s'inspire de ces réflexions et s'inscrit dans la description et le développement de nouvelles distributions temps-fréquence et ondelettes continues pour l'analyse des images SAR et plus particulièrement pour l'analyse de réflecteurs dits dispersifs (ex : non isotropes, colorés, ...) présents dans une zone de terrain imagée par un radar SAR.

Le modèle le plus couramment utilisé en imagerie radar est celui des points brillants ([Pouit80], [Men81], [Men92], [Aus84]). On modélise la cible par un ensemble de points brillants, c'est-à-dire des

sources ponctuelles, indépendantes entre elles, isotropes et répondant de la même manière à toutes les fréquences. En notant $I(\mathbf{x})$ l'amplitude du point brillant caractérisé par $\mathbf{x}$, le vecteur définissant sa position dans un repère d'axes lié à la cible et $\mathbf{k}$ le vecteur d'onde relié à la fréquence et à la direction d'illumination, le coefficient de rétrodiffusion $H(\mathbf{k})$ complexe pour l'ensemble des points brillants est donné par la sommation en amplitude et en phase de la contribution de chaque réflecteur :

$$H(\mathbf{k}) = \int I(\mathbf{x})\, e^{-2i\pi \mathbf{k}.\mathbf{x}}\, d\mathbf{x}. \tag{2.1}$$

La transformée de Fourier de ce coefficient permet d'obtenir la densité de répartition spatiale $I(\mathbf{x})$ des points brillants pour une fréquence et un angle moyens d'analyse (fréquence centrale et angle central d'illumination). La figure (Fig. 2.1) représente l'image de la répartition spatiale des points brillants d'une maquette de missile élaborée avec cette technique issue des mesures en chambre anéchoïque de l'ONERA.

Devant l'émergence de nouvelles capacités d'imagerie (grande largeur de bande, bande basse, fortes excursions angulaires d'analyse, ...), le système d'imagerie classique fait l'hypothèse majeure d'isotropie (les points imagés se comportent de la même manière selon tous les angles sous lesquels ils sont vus) et de blancheur (les points imagés ont le même comportement dans toute la bande spectrale émise). Cette hypothèse n'est plus valable dans ces conditions et on peut mettre en évidence, dans ce cas, une dégradation certaine de l'image. Ces constatations sont illustrées simplement sur une image SAR Très Haute Résolution (THR) construite en trois sous-bandes spectrales (figures (Fig. 2.2) et (Fig. 2.3) et codée dans les canaux de couleur rouge, verte et bleue. Ainsi, un diffuseur blanc dans les trois bandes de fréquence aura une couleur grise (même niveau dans les trois couleurs) et sera ainsi caractérisé comme non-dispersif. Au contraire, un diffuseur coloré sera considéré comme dispersif.

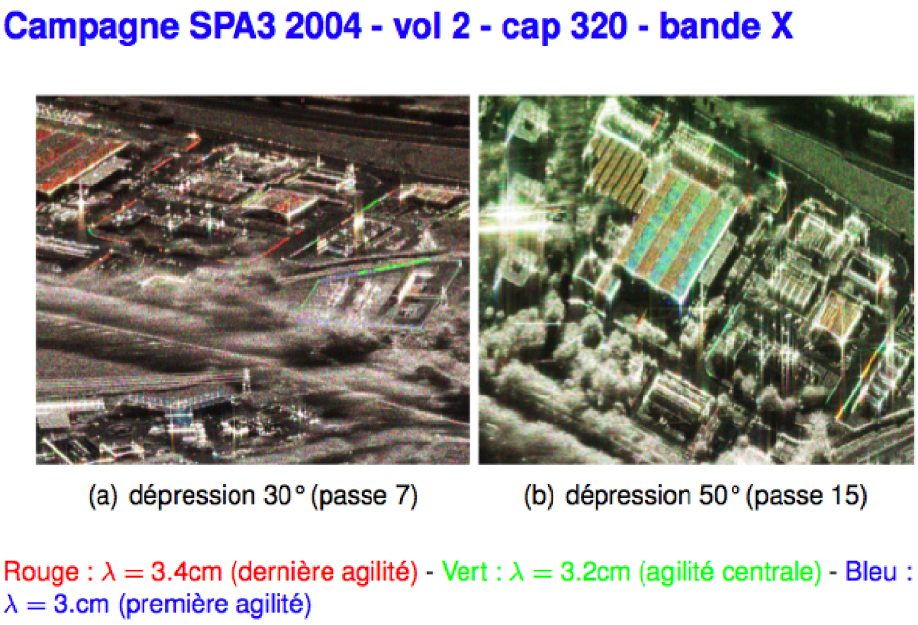


FIGURE 2.2 – Mise en évidence de la dispersion spectrale des diffuseurs sur des images THR

La théorie des groupes [Bert91] et l'analyse dimensionnelle permettent alors de construire de manière élégante des images radar, appelées Images Radar par Transformée en Ondelettes Dimensionnées ou encore Hyperimages **[J2]**, [Bert96] [Bert97]. L'information correspondante est construite au moyen d'une analyse par transformée d'ondelettes continues multi-dimensionnelle définie, à un facteur de normalisation près, comme le produit scalaire entre le coefficient de rétrodiffusion H et une ondelette $\Psi_{\mathbf{x}_0,\mathbf{k}_0}(\mathbf{k})$ représentant le déplacement par le groupe affine dans le plan spatial autour de $\mathbf{x}_0$ et spectral autour de $(|\mathbf{k}_0|, \Omega_0)$ d'une

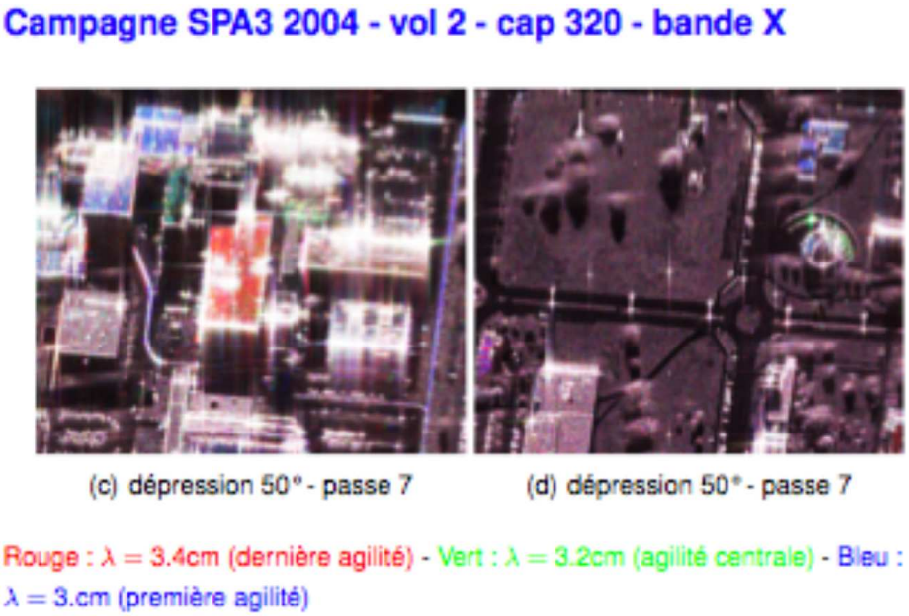


FIGURE 2.3 – Mise en évidence de la dispersion spectrale des diffuseurs sur des images THR

ondelette mère $\phi(\mathbf{k}) = \phi(k, \Omega, \Omega_0)$ caractérisant la rétrodiffusion d'une cible de référence localisée autour de l'origine $\mathbf{x} = \mathbf{0}$ et réfléchissant essentiellement dans la direction du vecteur unitaire Ω de $\mathbf{k}$ autour de $|\mathbf{k}| = 1$:

$$C(\mathbf{x}_0, \mathbf{k}_0) = \int H(\mathbf{k})\, \Psi^*_{\mathbf{x}_0,\mathbf{k}_0}(\mathbf{k})\, d\mathbf{k}. \tag{2.2}$$

où,

$$\Psi_{\mathbf{x}_0,\mathbf{k}_0}(\mathbf{k}) = \frac{1}{k_0}\, e^{-2i\pi \mathbf{k}.\mathbf{x}_0}\, \phi\left(\frac{k}{k_0}, \Omega, \Omega_0\right). \tag{2.3}$$

L'hyperimage d'énergie étant définie comme le carré du module de $C(\mathbf{x}_0, \mathbf{k}_0)$, on peut, par ce type de représentation, suivre l'évolution de la position des points brillants en fonction de la fréquence et de la direction d'illumination. La figure (Fig. 2.4) montre l'association du comportement angulaire et spectral pour les points brillants de la cible.

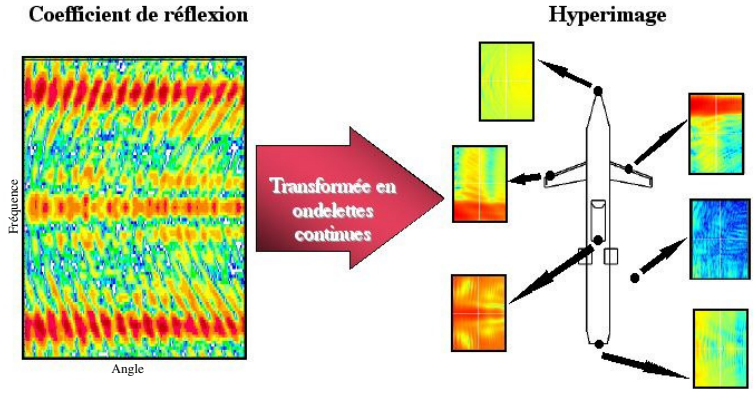


FIGURE 2.4 – Imagerie par ondelettes continues d'une maquette de missile.

La série des deux figures (Fig. 2.5) et (Fig. 2.6) représente l'analyse d'un avion d'armes et montre la sensibilité de l'image SAR pour deux angles de visée particuliers du domaine $[-6\,\text{deg}, 6\,\text{deg}]$. L'extension

spatiale correspond à une petite zone de 30 m^2 avec une résolution spatiale d'environ 30 cm. L'analyse considérée est effectuée pour privilégier à la fois les résolutions spatiales et les résolutions angulaires et spectrales.

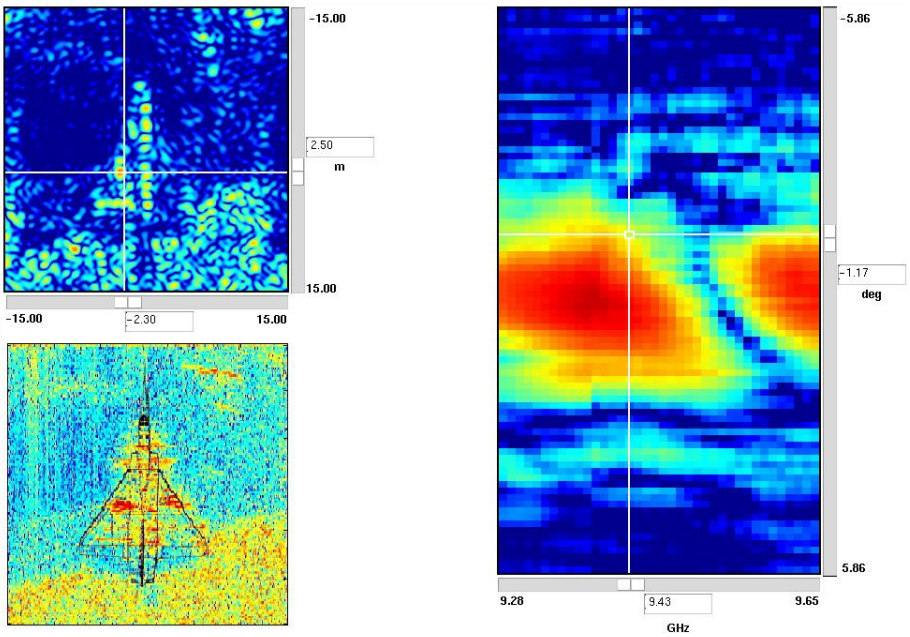


FIGURE 2.5 – Analyse SAR par Ondelettes d'une cible de type « avion d'armes ». L'image de gauche correspond aux points de la cible répondant à la fréquence (fréquence 9.43 Ghz) et l'angle -1.17 deg référencés par le curseur dans l'image de droite. L'image de droite correspond à la répartition spectrale et angulaire du point référencé par le curseur dans l'image de gauche. On peut mettre en évidence le bord de fuite de l'aile gauche.

Les méthodes d'analyse en ondelettes peuvent donc mettre en évidence des effets que l'imagerie radar classique est impuissante à caractériser. Ces effets sont d'autant plus présents que les caractéristiques relatives à l'image (fréquence centrale, bande, excursion angulaire autour de la zone d'analyse) peuvent ne pas être nécessairement extraordinaires : sur l'image des avions d'arme, la largeur de bande relative n'excède pas 6% et l'excursion angulaire de la zone ne dépasse pas 12 deg. Ces phénomènes doivent se renforcer pour des signaux ou des conditions de prises d'image dépassant ces caractéristiques (cas des images THR bande X, cas de la bande P utilisée dans le cadre de la pénétration sous couvert, ...).

Ces analyses montrent également que la résolution nominale de l'image SAR annoncée de manière globale sur l'image est loin d'être atteinte sur ce type de points brillants puisque ceux-ci ne répondent pas de manière identique dans toute la bande et dans tout le domaine angulaire. Si un point dispersif ne répond que sur le tiers de la bande émise, alors, il s'étalera alors sur trois cases de résolution optimale dans l'image finale. La figure (Fig. 2.7) montre une comparaison entre une image SAR de points isotropes et blancs et une image SAR de points anisotropes et dispersifs.

Le travail de thèse de Mohamed Tria **[T2]**, **[J5]**, **[J7]**, **[C20]**, **[C24]**, **[C30]** a permis d'exploiter la richesse du comportement des réflecteurs pour tenter d'extraire de ces hyperimages à 4 dimensions, des informations permettant de caractériser les objets. Supposons que l'on connaisse la réponse angulaire et spectrale $I_{ref}(f, \theta)$ exacte d'un réflecteur de référence (un dièdre, trièdre, sphère, par exemple). Il est alors possible, dans une hyperimage SAR, de chercher la position (x, y) des réflecteurs de l'image qui possèdent ce type de réponse. L'idée est alors de construire une nouvelle image $C(x, y)$ provenant de la

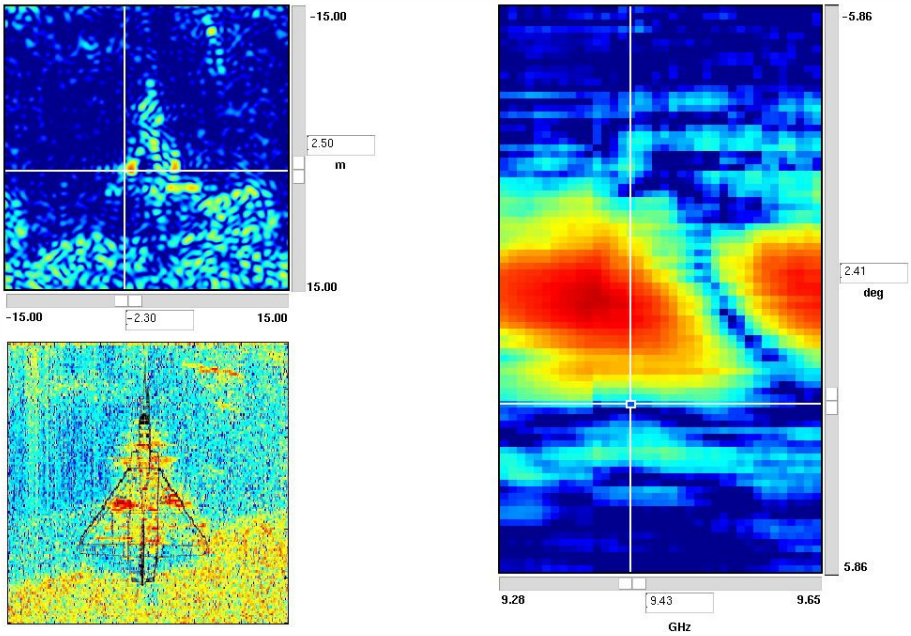


FIGURE 2.6 – Analyse SAR par Ondelettes d'une cible de type « avion d'armes ». L'image de gauche correspond aux points de la cible répondant à la fréquence (fréquence 9.43 Ghz) et l'angle 2.41 deg référencés par le curseur dans l'image de droite. L'image de droite correspond à la répartition spectrale et angulaire du point référencé par le curseur dans l'image de gauche. On peut mettre en évidence le bord de fuite de l'aile droite.

corrélation entre la réponse spectrale et angulaire $I_{ref}(f,\theta)$ du réflecteur et l'hyperimage $I(x,y,f,\theta)$:

$$C(x,y) = \int I(x,y,f,\theta)\, I^*_{ref}(f,\theta)\, df\, d\theta \tag{2.4}$$

La figure (Fig. 2.9) illustre ce genre de procédé appliqué ici pour l'extraction de toits de bâtiments. L'objectif est ici d'extraire le sommet des bâtiments latéraux présentés sur la figure (Fig. 2.8) tout en *annulant* la réponse de l'arête du bâtiment central.

Le papier proposé ci-après en Annexe B décrit en détail la construction de ces hyperimages :

[**J7**] M. Tria, **J.P. Ovarlez**, L. Vignaud, J.C. Castelli, M. Tria and M. Bénidir, "*Discriminating Real Objects in Radar Imaging by Exploiting the Squared Modulus of the Continuous Wavelet Transform*", IET Proc. Radar Sonar and Navigation, Vol.1, No.1, pp.27-37, Feb. 2007.

Les applications de cette techniques sont évidemment la classification des cibles et l'exploitation de l'information apportée par ces techniques d'hyperimage.

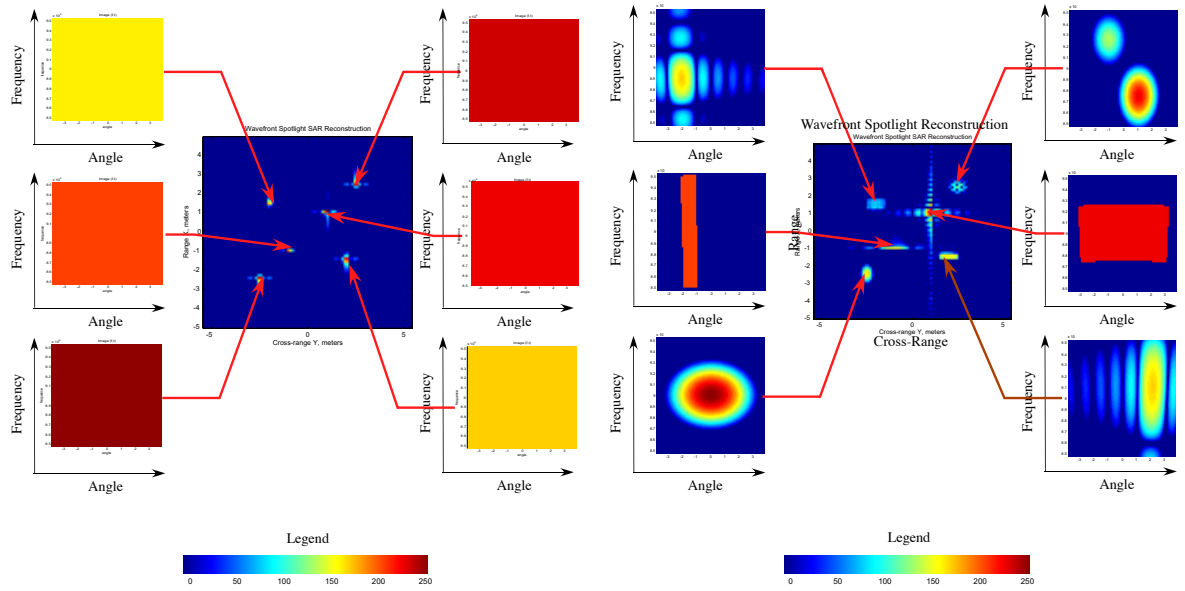
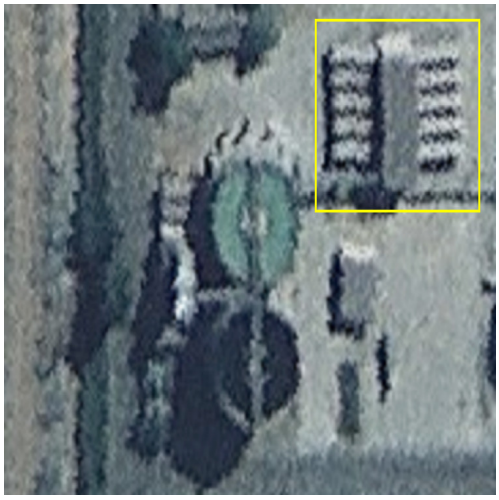
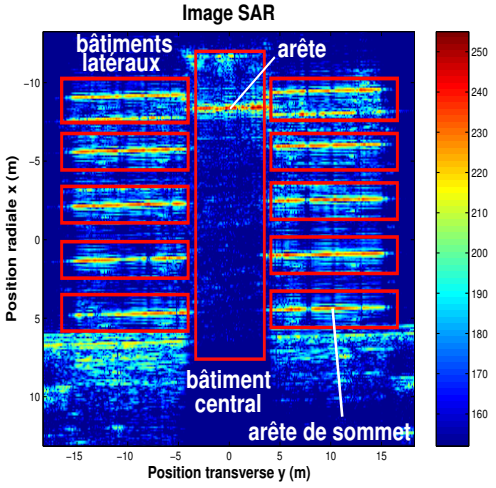


FIGURE 2.7 – Comparaison entre une image SAR de points isotropes et blancs et une image SAR de points anisotropes et dispersifs.



(a) Photographie aérienne de la région.



(b) Image SAR contenant le bloc de bâtiment.

FIGURE 2.8 – Image optique et image SAR contenant le bloc de bâtiment.

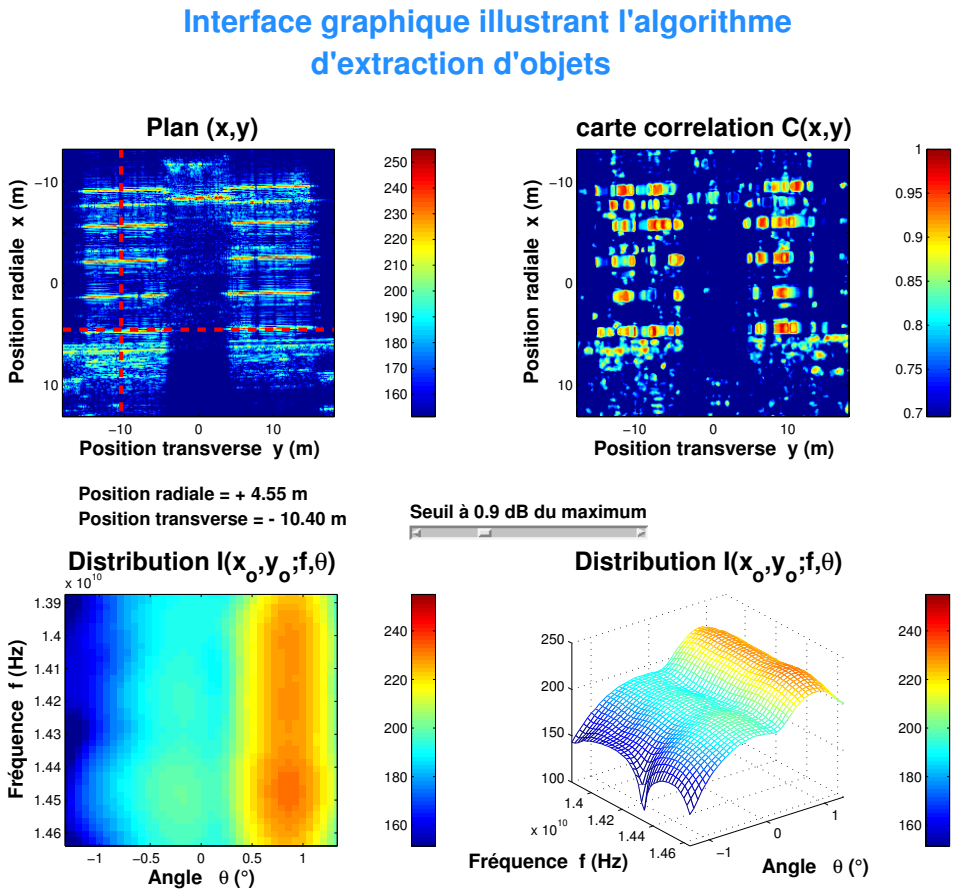


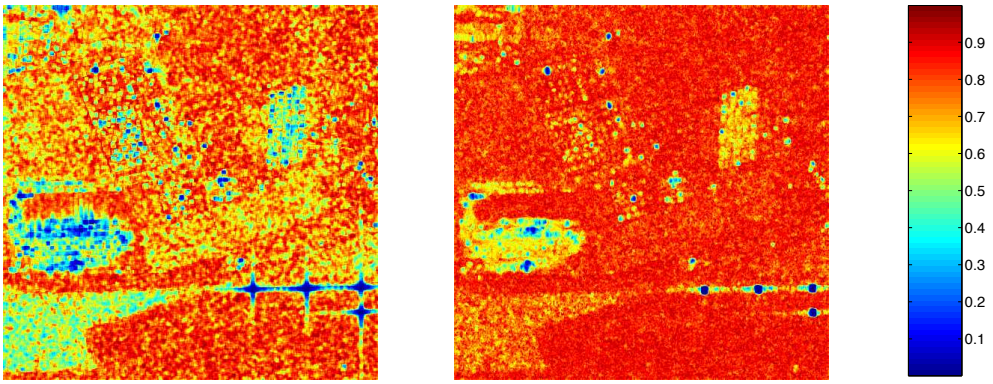
FIGURE 2.9 – Extraction des sommets des bâtiments latéraux en appliquant l'algorithme pour la distribution énergétique du réflecteur localisée dans l'image en $(x_{ref}, y_{ref}) = (4.55\text{ m}, -10.40\text{ m})$ comme distribution de référence.

2.1.2.2 Application à l'Imagerie SAR Interférométrique et Polarimétrique

Le but de l'analyse polarimétrique est de séparer et d'identifier les mécanismes électromagnétiques de chaque réflecteur de la cible analysée (réflexions par simple, double rebond, ...) en étudiant soit les relations d'amplitude et de phase des canaux polarimétriques, si la cible est considérée comme déterministe, soit, la matrice de cohérence ou de covariance polarimétrique (caractéristique statistique d'ordre deux) pour ce qui est d'une cible aléatoire. En polarimétrie, on distingue donc deux types de cibles :

- Les cibles déterministes dites "simples" ; elles sont caractérisées par un nombre réduit de réflecteurs. Les ondes renvoyées par les réflecteurs s'additionnent de manière cohérente. Chaque réflecteur élémentaire est caractérisé par sa *matrice de diffusion* [Pot92][Clou96] (théorie déterministe). En général ces cibles sont fabriquées par l'homme : véhicules, bâtiments, etc.

- Les cibles aléatoires dites "distribuées" ; par exemple : cultures, prairies, forêts, etc. Le signal renvoyé par ce type de cible, résulte de l'addition non cohérente d'un nombre indéterminé de processus élémentaires. Ces cibles sont caractérisées par une grandeur statistique du second ordre (théorie non déterministe) [Clou97] : matrice de covariance ou matrice de cohérence. La théorie non déterministe englobe, en tant que cas particulier, la théorie déterministe.



(a) Entropie initiale (b) Entropie maximale obtenue grâce à l'analyse en ondelettes.

FIGURE 2.10 – Amélioration du contraste entropique de l'image SAR par analyse en ondelettes.

L'interférométrie est, quant à elle, un outil permettant de reconstruire la topographie d'une zone donnée à l'aide de deux voies de mesure. Cette technique est basée sur la mesure des différences de phase entre deux signaux cohérents en phase obtenus par le biais de deux antennes faiblement délocalisées spatialement.

Dans le cadre du calcul de la matrice de cohérence interférométrique ou polarimétrique, il est nécessaire d'estimer de manière locale (masque autour du pixel d'analyse de l'image SAR) et spatialement l'information interférométrique ou polarimétrique. Supposons que le vecteur d'information (polarimétrique, interférométrique, interférométrique et polarimétrique) de chaque pixel (x, y) de l'image SAR soit caractérisé par $\mathbf{c}(x, y)$. La caractéristique du second ordre, la matrice de covariance, de ce processus est généralement donnée par :

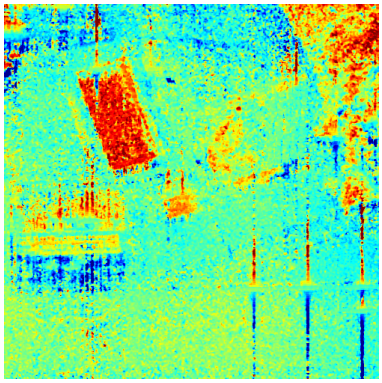
$$[\mathbf{T}] = \frac{1}{K} \sum_{i=1}^{K} \mathbf{c}_i(x, y)\, \mathbf{c}_i^H(x, y) \tag{2.5}$$

où K est le nombre de données du masque spatial et $\mathbf{c}_i(x, y)$ est le vecteur d'information relative au pixel i appartenant au masque spatial autour du pixel (x, y).

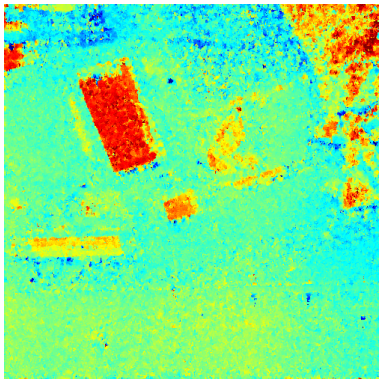
Ce moyennage spatial entraîne des imperfections dues à l'hétérogénéité de la zone spatiale du masque. Pour améliorer cette estimation, l'idée de Mohamed Tria [T2] et Élise Colin [J6], [C19] était ici la suivante : plutôt que de moyenner l'information spatiale des canaux interférométriques et polarimétriques de l'image SAR, la cohérence interférométrique ou polarimétrique s'estime à partir d'un moyennage de l'information sur le domaine spectral et angulaire calculée grâce aux décompositions en ondelettes des différents canaux interférométriques ou polarimétriques :

$$[\mathbf{T}] = \frac{1}{K}\sum_{i=1}^{K} \mathbf{c}(x, y, f_i, \theta_i)\, \mathbf{c}_i^H(x, y, f_i, \theta_i) \tag{2.6}$$

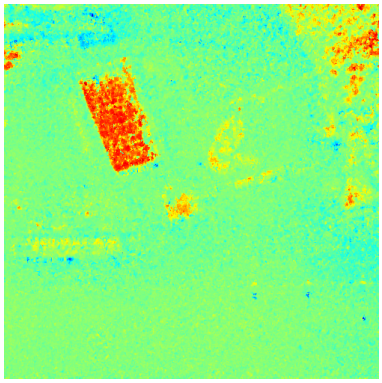
où K est le nombre de données spectrales et angulaires d'un pixel $\mathbf{c}(x, y, f, \theta)$ de l'hyperimage.



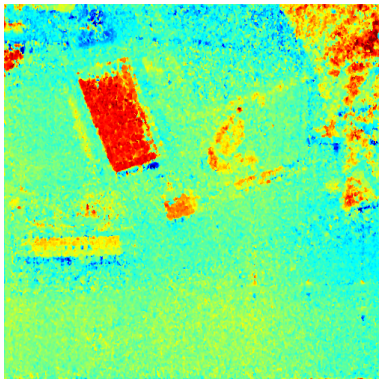
(a) Interférogramme initial en polarisation Hh.



(b) Interférogramme après optimisation par ondelettes.



(c) Interférogramme après optimisation par ondelettes puis polarimétrique.

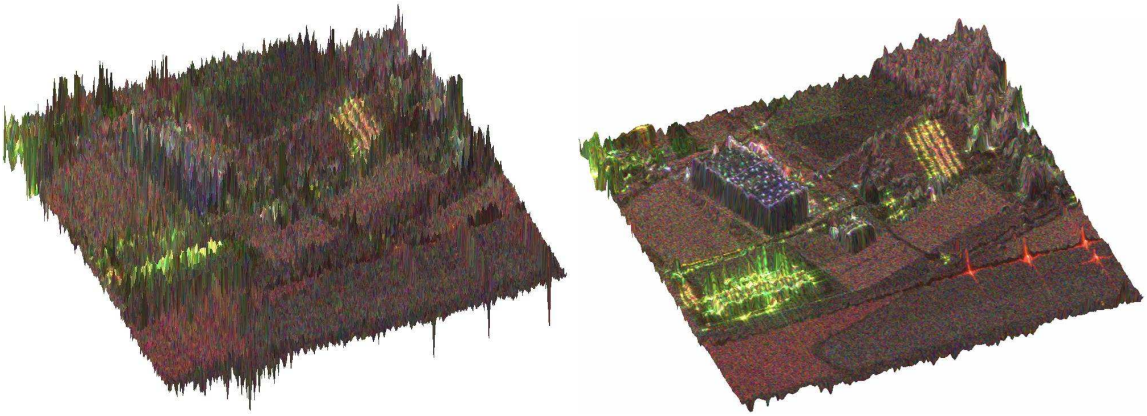


(d) Interférogramme obtenu par la méthode "moyennage des coefficients d'ondelettes puis optimisation polarimétrique"

FIGURE 2.11 – Comparaison des différentes méthodes étudiées pour l'estimation de la hauteur des cibles : Les méthode "optimisation par ondelettes" et "moyennage coefficients ondelettes + optimisation polarimétrique" sont les méthodes les plus performantes.

On peut également choisir, plutôt que la moyenne, l'information rendant maximale la cohérence. Ceci a donné lieu à des résultats tout à fait intéressants. Une image d'entropie polarimétrique maximale est donnée en figure (Fig. 2.10-b) et comparée à la carte entropique initiale illustrée en (Fig. 2.10-a) : on remarque, sur la carte d'entropie initiale, que certains champs (cibles aléatoires), comme celui sur le bas gauche de l'image, ont un niveau d'entropie anormalement faible. Dans la carte d'entropie, après analyse par ondelettes, ces champs ont un niveau d'entropie élevé conformément à ce que prévoit la théorie : ceci a pour conséquence de mieux discerner les cibles déterministes d'entropie plus faible.

Ces méthodes d'optimisation interférométrique et polarimétrique par ondelettes ont été utilisées pour construire une carte d'élévation donnant la hauteur estimée des différents réflecteurs de l'image SAR. Pour cela, pour chaque pixel de l'image, on calcule la cohérence puis on effectue une optimisation polarimétrique qui consiste à déterminer les canaux polarimétriques les mieux adaptés (voir Fig. 2.11). La hauteur de chaque pixel de l'image est ainsi estimée et on établit la carte d'élévation. À la carte d'élévation ainsi construite, on peut superposer la composition polarimétrique RVB (Rouge : HH+VV, Vert HH-VV, Bleu : 2HV) de chaque réflecteur de l'image. Le résultat final est donné en figure (Fig. 2.12-b) et comparé à la carte d'élévation initiale en (Fig. 2.12-a) sans traitement par ondelettes.



(a) (b)

FIGURE 2.12 – Carte de hauteur, colorée grâce aux canaux polarimétriques. (a) Carte d'élévation initiale. (b) après traitement par ondelettes

2.1.2.3 Classification SAR Polarimétrique

L'objectif de cette thématique est de combiner l'Analyse Temps-Fréquence et l'analyse polarimétrique cohérente ou non-cohérente afin d'identifier le comportement polarimétrique des réflecteurs et des fonds dans les images SAR en fonction de l'angle sous lequel ils sont vus et de la fréquence à laquelle ils répondent.

La thèse **[T4]** de Mickaël Duquenoy **[J11]**, **[J15]**, **[C27]**, **[C28]**,**[C32]**, **[C33]**, **[C34]**, **[C40]**, **C[43]**, **[C44]**, **[C51]**, proposée en décembre 2004 (et soutenue en octobre 2009) dans la suite logique du travail de thèse de Mohamed Tria **[T2]**, a été consacrée à l'élaboration d'outils d'analyse polarimétrique basés sur les hyperimages SAR par ondelettes. Ce travail, qui a obtenu le *Best Paper Award* (voir (Fig. 2.13)) à l'EUSIPCO 2009 **[C48]** et une invitation à publier dans la revue Signal Processing **[J17]**, a permis de définir des méthodes d'extraction de paramètres physiques pertinents (coloration, anisotropie, stationnarité polarimétrique) précurseurs aux développement des méthodes de reconnaissance de cibles, de fonds, de structures qu'il a d'ailleurs initiées tout à la fin de sa thèse.

Dans les images radar, chaque pixel est peut être représenté par une matrice 2×2 ou de manière équivalente par un vecteur $\mathbf{k}$ de 4 composantes donnant la réponse des points de l'image dans les quatre polarisations (horizontales et verticales) d'émission et de réception, HH, HV, VH et VV (base de Sinclair [Sinc48]). De la même manière, grâce aux techniques temps-fréquence, il est possible de définir une hyperimage polarimétriques constituées de 4 hyperimages mono canal. Le but de la classification polarimétrique est d'exploiter cette source d'information pour caractériser les propriétés de rétrodiffusion des cibles via des méthodes de décomposition.

Les techniques de décomposition les plus connues sont soit les décompositions polarimétriques dites cohérentes (Pauli, Kogager, Cameron, Hyunen) soit les décompositions non cohérentes (H-α-Anisotropie,

EUSIPCO 2009

17th European Signal Processing Conference
August 24-28, 2009 Glasgow - Scotland

Texas Instruments Student Paper Award

The Organising Committee of the 17th European Signal Processing Conference (EUSIPCO) has the pleasure of awarding this prize to

Michaël Duquenoy

for his paper "Supervised Classification of Scatterers on SAR Imaging Based on Incoherent Polarimetric Time-Frequency Signatures", co-authored with Jean-Philippe Ovarlez, Laurent Ferro-Famil and Eric Pottier.

Many congratulations on behalf of the EUSIPCO 2009 Organising Committee:

Robert W. Stewart, General Chair

Stephan Weiss, Technical Co-Chair

FIGURE 2.13 – Best Paper Award Eusipco 2009

Freeman Durden, ...). Elles seront ici appliquées aux hyperimages SAR par ondelettes.

2.1.2.4 Décompositions Polarimétriques Cohérentes

Le but des décompositions polarimétriques cohérentes est de décomposer le vecteur $\mathbf{k}$ d'information polarimétrique sur une base (impropre et incomplète) de N vecteurs ou matrices caractérisant la réponse polarimétrique de réflecteurs de référence (sphère, dièdre, trièdre, hélice, ...) :

$$\mathbf{k} = \sum_{i=1}^{N} c_i \, [\mathbf{S}]_i \tag{2.7}$$

où les $[\mathbf{S}]_i$ sont les réponses des objets canoniques et où les c_i indiquent le poids des $[\mathbf{S}]_i$ dans la combinaison menant à $\mathbf{k}$ mesurée par le radar. Ainsi, les propriétés physiques de la cible sont extraites et interprétées à travers l'analyse de réponses simples $[\mathbf{S}]_i$ et de leurs coefficients correspondants c_i. Il est souhaitable que les $[\mathbf{S}]_i$ soient indépendantes entre elles, de telle manière qu'un phénomène de rétrodiffusion ne soit contenu que dans une cible canonique. D'ailleurs, la condition d'orthogonalité entre les $[\mathbf{S}]_i$ est souvent requise. Il existe une infinité de familles d'objets canoniques sur lesquelles nous pouvons décomposer la matrice de Sinclair [Sinc48].

Le papier suivant présenté en Annexe C, qui doit paraître en juin 2010, explique cette méthodologie :

[J11] M. Duquenoy, **J.P. Ovarlez**, L. Ferro-Famil, L. Vignaud and E. Pottier, "*Scatterers Characterisation in Radar Imaging Using Joint Time–Frequency Analysis and Polarimetric Coherent Decompositions*", IET Proc. Radar Sonar and Navigation, Vol.4, No.3, à paraître, Jun. 2010.

Dans un premier temps, pour des nécessités de validation à partir d'une véritable connaissance de la vérité terrain, elles ont été appliquées sur des signaux très bien connus provenant des mesures de chambre anéchoïque effectuées sur la maquette *Cyrano* présentée à la figure (Fig. 2.14). Cette figure montre également l'analyse par ondelettes de l'énergie totale provenant des quatre voies polarimétriques (SPAN). Cette information énergétique peut être alors synthétisée à l'aide de différentes représentations (par exemple, marginales sur la fréquence, marginales sur le domaine angulaire) pour qualifier les non-stationnarités spectrale et angulaire des points (figure (Fig. 2.15)) : la dispersivité et l'anisotropie des réflecteurs. Lorsqu'on utilise les décompositions cohérentes pour classifier le type de mécanisme polarimétrique, on peut

également caractériser le caractère stationnaire ou non du comportement polarimétrique des réflecteurs. Les décompositions les plus couramment utilisées sont celles de Pauli (simple rebond, double rebond, diffusion de volume) donnée en exemple à la figure (Fig. 2.16-a), celles de Krogager [Krog90] (dièdre, hélice, sphère) à la figure (Fig. 2.16-b). Les décompositions de Cameron [Came90] permettent, quant à elles, de décrire le diffuseur par sa nature (voir figure (Fig. 2.17-a)), son orientation dans le plan vertical (orientation de Huynen [Huyn65], voir (Fig. 2.17-b)) et son orientation relative dans le plan horizontal. La figure (Fig. 2.19) donne le schéma des différentes interprétations physiques du comportement des diffuseurs.

On peut donner alors une interprétation physique pour chaque diffuseur, par exemple, pour les bords d'attaque et de fuite des ailes :

- Le bord d'attaque est marqué par une réponse anisotrope qui se traduit par un comportement directif centré sur les angles d'observation $\theta = 20\,\text{deg}$ ou $\theta = -20\,\text{deg}$. Le comportement polarimétrique sur l'hyperimage de Pauli, est un mélange de contribution de mécanismes de simple diffusion et de double rebond. Cette réponse polarimétrique est constante dans tout le domaine où le diffuseur est actif. Ainsi, le comportement polarimétrique est stationnaire pour la décomposition de Pauli. Sur l'hyperimage polarimétrique de Krogager, le comportement polarimétrique est un mélange de contribution de mécanismes de contribution sphère et de contribution dièdre constant sur toute la réponse du diffuseur. Les résultats de l'hyperimage de Pauli et celle de Krogager sont concordants. Dans la décomposition de Cameron, le bord d'attaque des ailes est caractérisé par une réponse en dipôle, ce qui confirme l'aspect diffraction du phénomène. Les bords d'attaque des ailes sont très affûtés. Au niveau de l'orientation de Huynen, on trouve une inclinaison $\Psi = 10\,\text{deg}$ ou $\Psi = -10\,\text{deg}$. Cette inclinaison, correspond à l'orientation dans le plan vertical du diffuseur. Dans le cas présent, les hyperimages polarimétriques issues de la décomposition de Cameron permettent de décrire globalement les phénomènes de rétrodiffusion (orientation relative dans le plan horizontal, orientation absolue dans le plan vertical et nature du phénomène de rétrodiffusion). De plus, tout comme les hyperimages de Pauli et Krogager, ce diffuseur est polarimétriquement stationnaire sur l'hyperimage de Cameron.

- Tout comme le comportement des bords d'attaque des ailes, la réponse fréquence/angle des bords de fuite est représentée par un comportement anisotrope (réponse directive centrée sur $\theta = 10\,\text{deg}$ ou $\theta = -10\,\text{deg}$). L'interprétation de ce comportement est identique à celles des bords d'attaque, c'est à dire, que ce phénomène directif traduit une diffraction (c'est l'arête de l'aile qui répond) et l'angle d'observation central $\theta = 10\,\text{deg}$ ou $\theta = -10\,\text{deg}$ est à relier à l'orientation du bord de fuite de l'aile dans le plan horizontal. Le phénomène directif se retrouve sur les décompositions de Pauli, de Krogager et de Cameron de l'hyper-matrice de diffusion. Sur cette réponse directive, le comportement polarimétrique sur l'hyperimage polarimétrique de Pauli composé d'un mélange de contributions de mécanismes de simple diffusion ou de double rebond est constant. Dans la décomposition de Krogager, le comportement polarimétrique composé d'un mélange de contributions de mécanismes de type sphère et de type dièdre est également constant. Ce résultat est concordant avec ceux obtenus par la décomposition de Pauli. Sur l'hyperimage polarimétrique de Cameron, le comportement polarimétrique dipôle est constant. Le comportement polarimétrique est donc stationnaire, ce qui correspond aux résultats obtenus par les décompositions de Pauli et Krogager. Au niveau de l'orientation de Huynen, cette réponse est marquée par une inclinaison à $\Psi = \pi$ ou $\Psi = -\pi$. Ceci est explicable par le fait que le radar ne voit pas l'inclinaison des ailes.

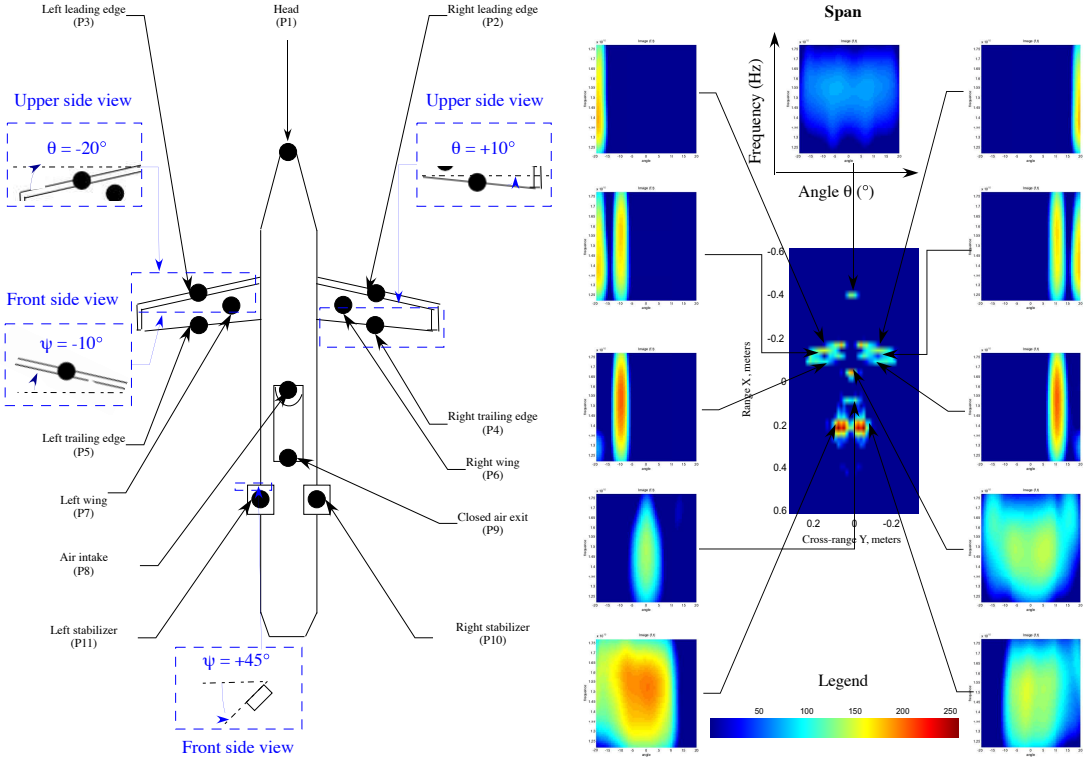


FIGURE 2.14 – a) Maquette Cyrano. b) Évolution du SPAN en fonction de l'angle et de la fréquence

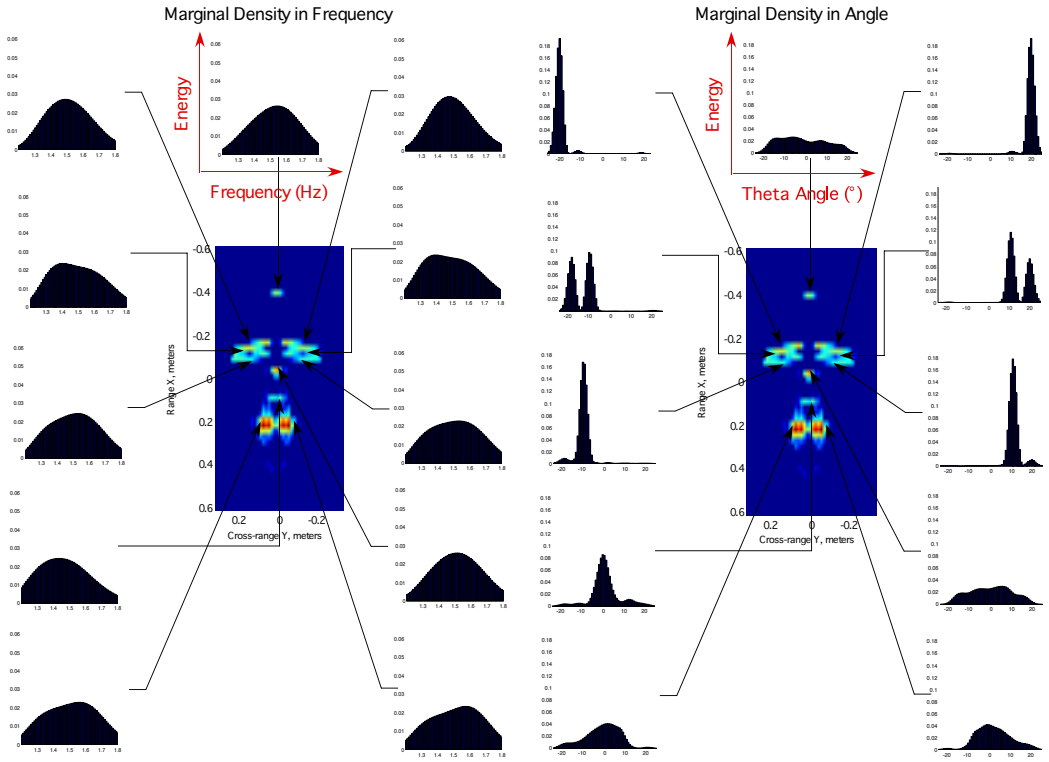


FIGURE 2.15 – a) Évolution de l'énergie avec la fréquence émise. b) Évolution de l'énergie avec la direction d'illumination.

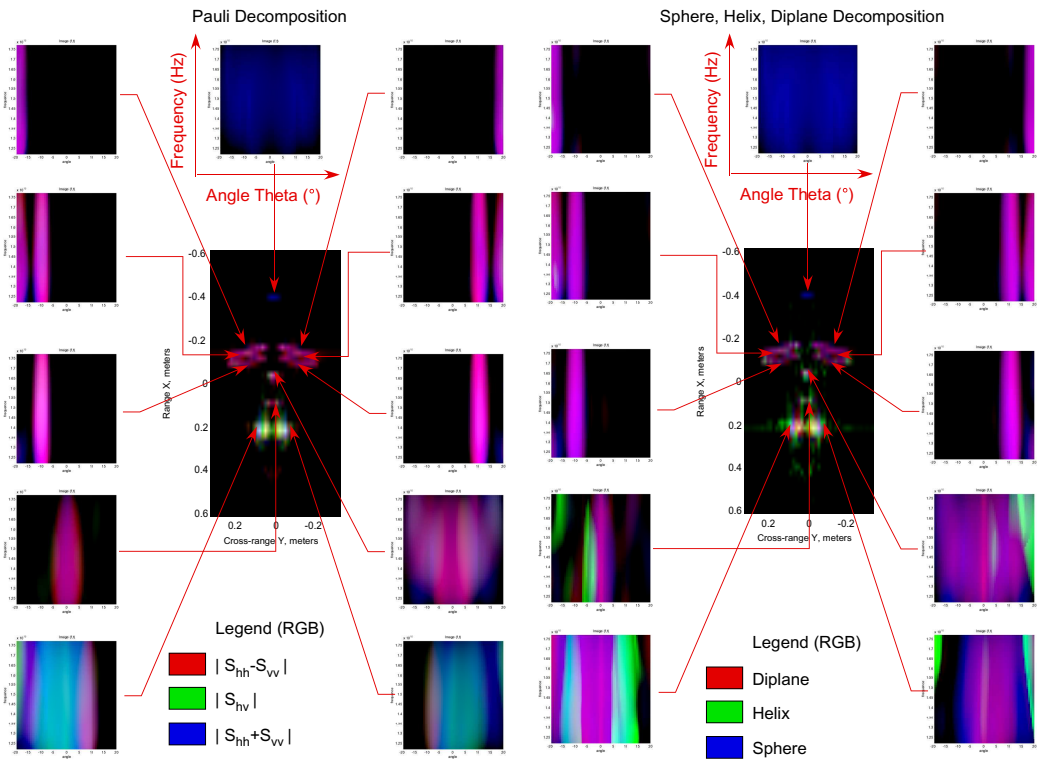


FIGURE 2.16 – Évolution de la réponse polarimétrique des diffuseurs, en fonction de l'angle d'observation et de la fréquence émise, a) Décomposition de Pauli (Codage RGB : simple rebond, double rebond, diffusion de volume). b) Décomposition de Krogager (Codage RVB : trièdre, sphère, hélice)

.

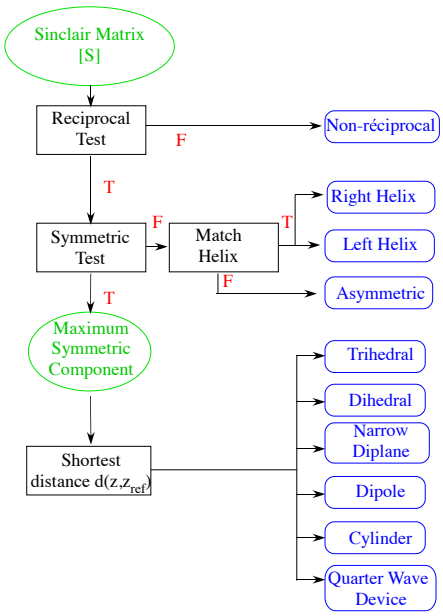


FIGURE 2.17 – Technique de classification basée sur la décomposition de Cameron

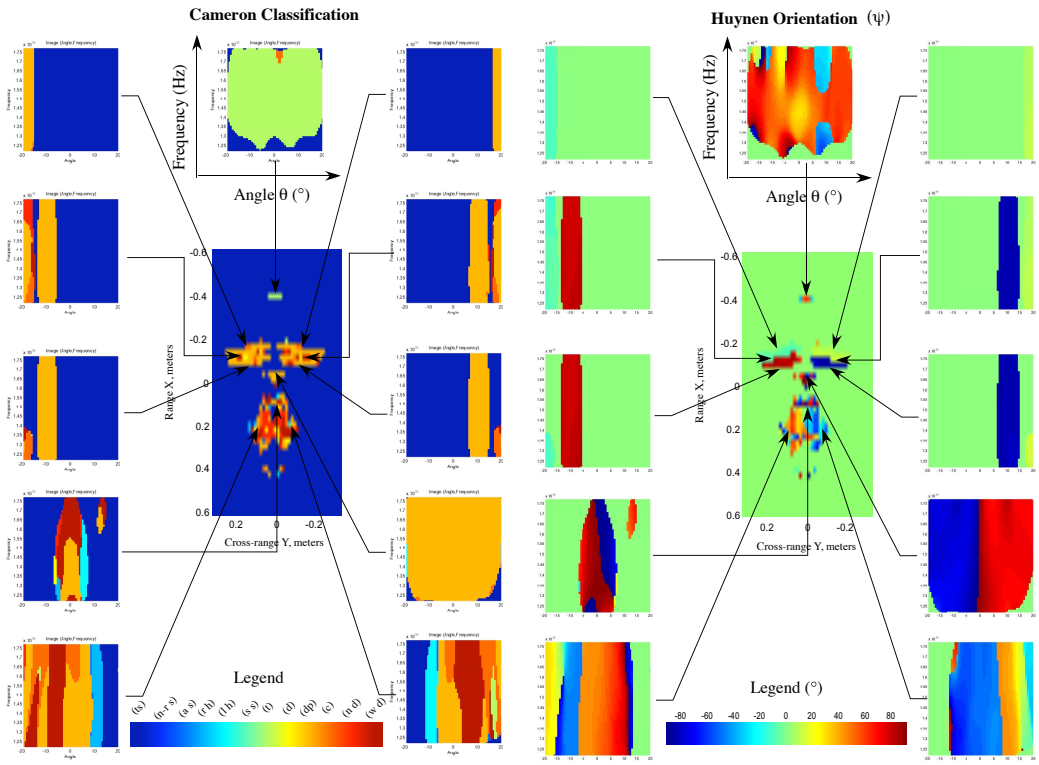


FIGURE 2.18 – Évolution de la réponse polarimétrique des diffuseurs, en fonction de l'angle d'observation et de la fréquence émise. a) Décomposition de Cameron b) Évolution de l'orientation de Huynen

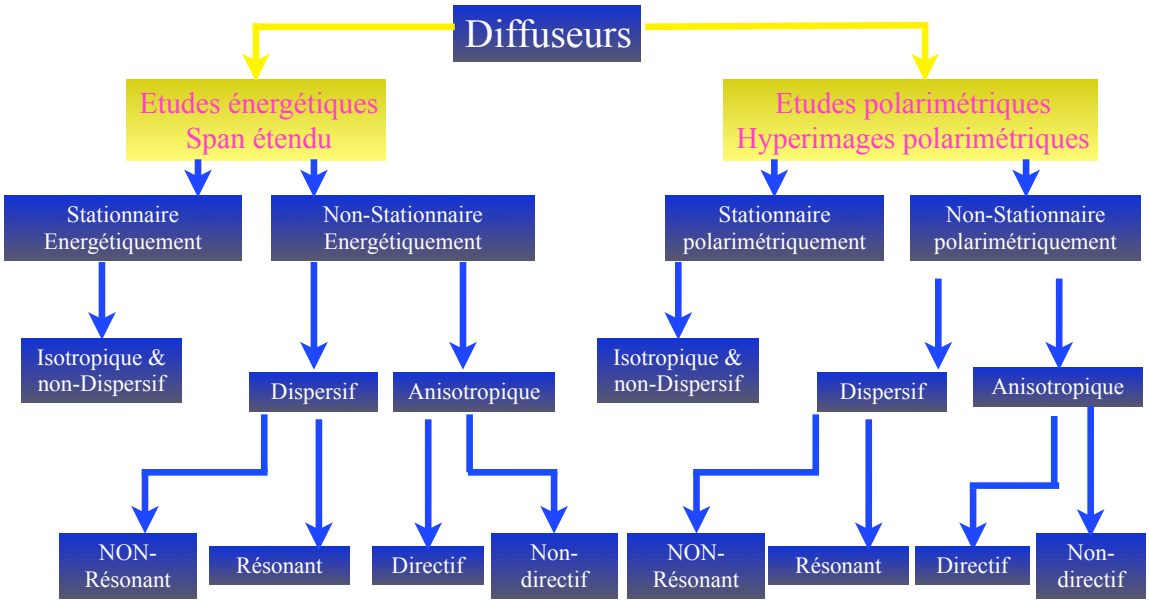


FIGURE 2.19 – Qualification des différents comportements des diffuseurs

2.1.2.5 Décompositions Polarimétriques non Cohérentes

Les techniques Temps-Fréquence ont également été étendues aux méthodes de décompositions non cohérentes de type H-α-A et Freeman-Durden [Free98]. Dans ces types de décomposition, dédiées aux cibles étendues aléatoires comme les fonds des images SAR (forêt, glaciers, neige, herbe, milieu urbain, ...), les décompositions ne se font plus sur l'analyse du vecteur polarimétrique noté $\mathbf{k}$ de chaque pixel de l'image comme dans les décompositions cohérentes mais plutôt sur une statistique du comportement de ces pixels. Idéalement, cette statistique devrait être calculée sur l'axe temporel. Malheureusement, dans la plupart des cas, on n'a à disposition qu'un seul jeu d'images polarimétriques. La statistique d'ordre deux (matrice de covariance $[\mathbf{C}]$ dans la base de Sinclair ou matrice de cohérence $[\mathbf{T}]$ dans la base de Pauli) est donc estimée de manière empirique (estimateur SCM, Sample Covariance Matrix) par simple moyennage sur l'axe spatial distance et azimut de l'image pour K pixels :

$$[\mathbf{T}]_{SCM} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{k}_i \, \mathbf{k}_i^H \tag{2.8}$$

Dans une zone homogène supposée gaussienne $\mathcal{CN}(\mathbf{0}, [\mathbf{T}])$, la matrice $K\,[\mathbf{T}]_{SCM}$, estimateur du Maximum de Vraisemblance de $K\,[\mathbf{T}]$, est distribué selon la loi de Wishart $\mathcal{CW}(K, m, [\mathbf{T}])$ dont la densité est définie par :

$$p([\mathbf{A}]) = \frac{|[\mathbf{A}]|^{K-m}}{\pi^{m(m-1)/2} \, |[\mathbf{T}]|^{K} \, J(K,m)} \exp\left(-\mathrm{tr}([\mathbf{T}]^{-1}\,[\mathbf{A}]\right) \tag{2.9}$$

où l'opérateur tr(.) est la trace, $|.|$ le déterminant et où $J(K,m) = \Gamma(K) \ldots \Gamma(K-m+1)$.

On peut se rendre compte immédiatement de la difficulté d'estimer ce genre de matrice sur des zones hétérogènes ou des zones contenant des points brillants forts qui peuvent biaiser l'information statistique de la covariance. Cette discussion sera reprise dans la section 2.2.9 concernant la robustesse d'estimation de cette matrice de covariance ainsi que dans la section 2.3 concernant la classification des images SAR hautement texturées avec les approches de type SIRV.

La complexité des mécanismes de rétrodiffusion rend également extrêmement difficile l'étude physique des diffuseurs à travers l'analyse directe des matrices de covariance ou de cohérence. Par conséquent, l'objectif des décompositions non-cohérentes est de décomposer les matrices de covariance ou de cohérence en une combinaison de N matrices correspondant à des diffuseurs canoniques ou à des mécanismes de rétrodiffusion simples. Ainsi, le principe des décompositions non-cohérentes se résume suivant :

$$[\mathbf{T}]_{SCM} = \sum_{i=1}^{N} q_i \, [\mathbf{T}]_i \tag{2.10}$$

où les N réponses élémentaires sont représentées par les matrices $[\mathbf{T}]_i$ et où les coefficients q_i représentent le poids de chacune de ces contributions. Comme dans le cas des décompositions cohérentes, il est souhaitable que ces décompositions présentent certaines propriétés. En effet, on souhaite que les contributions élémentaires soient physiquement interprétables et qu'elles soient indépendantes entre elles, voire orthogonales.

Il existe une multitude de décompositions non-cohérentes, cependant il y en a des plus pertinentes que d'autres. Parmi les décompositions non-cohérentes les plus utilisées, il y a la décomposition valeurs propres/vecteurs propres, plus connue sous le nom de décomposition H/α/A (entropie, anisotropie, paramètre alpha) et celle de Freeman-Durden [Free98].

La technique H/α/A est basée sur l'analyse des valeurs propres λ_1, λ_2 et λ_3 de la matrice de cohérence. À partir des valeurs propres λ_k et des vecteurs propres $\mathbf{v}_k$, des paramètres secondaires peuvent être déterminés. En effet, l'ensemble des valeurs propres indique la répartition de la puissance totale sur les différentes composantes de la décomposition. Cette répartition est définie par un élément p_k représentant

la part de puissance associée au mécanisme. L'ensemble des p_k correspond à une normalisation des valeurs propres λ_k [Clou96], [Clou97] :

$$p_k = \frac{\lambda_k}{\sum_{k=1}^{3} \lambda_k} \tag{2.11}$$

Ces valeurs propres normalisées sont associées à des pseudo-probabilités liées aux propriétés statistiques des phénomènes de rétrodiffusion des milieux naturels. Il est donc possible de décrire l'ensemble des valeurs propres au moyen de deux paramètres réels : l'entropie et l'anisotropie.

L'entropie de la cible, H est définie comme l'indicateur du caractère aléatoire du phénomène de rétrodiffusion global. Elle se calcule de la manière suivante :

$$H = -\sum_{k=1}^{3} p_k \log_3(p_k) \tag{2.12}$$

L'entropie H varie entre 0 et 1. Une entropie nulle indique que la cible observée est pure et la rétrodiffusion est déterministe. Ceci se traduit par la présence d'une seule valeur propre normalisée non nulle et égale à 1. Lorsque l'entropie est égale à 1, elle traduit le caractère complètement aléatoire de la cible observée. Ceci se produit quand les pseudo-probabilités sont identiques.

L'anisotropie qui traduit l'importance relative des mécanismes de diffusion secondaires, est défini par :

$$A = \frac{p_2 - p_3}{p_2 + p_3} \tag{2.13}$$

L'anisotropie est un paramètre complémentaire à l'entropie. L'anisotropie mesure l'importance relative de la seconde et de la troisième valeur propre de la décomposition. D'un point de vue pratique, l'anisotropie peut être employée comme une source de discrimination pour des valeurs d'entropie supérieures à 0.7.

Le paramètre α est associé à la nature du mécanisme de diffusion et varie entre 0 deg et 90 deg. Il dépend de la constante diélectrique du milieu étudié, de l'angle d'incidence, et de la fréquence de l'onde. Ce paramètre n'est pas modifié par les variations de l'angle de rotation du radar. Une interprétation phénoménologique peut être attribuée. Si alpha est nul alors le mécanisme est celui d'une diffusion de surface canonique. Dans l'autre cas extrême, c'est-à-dire si alpha prend la valeur de 90 deg, le mécanisme de rétrodiffusion est celui d'un dièdre ou d'une hélice. Toutes autres valeurs intermédiaires représentent un mécanisme de diffusion anisotrope. Par exemple, si α prend la valeur de 45 deg, le mécanisme est celui d'un dipôle canonique. Dans le cas d'un milieu naturel, cette valeur indique une diffusion de volume [Clou97].

Dans le cadre d'une analyse statistique, on n'utilise pas directement le paramètre alpha des vecteurs propres, mais on utilise du paramètre alpha moyen défini par une pondération :

$$\alpha = \sum_{k=1}^{3} p_k \alpha_k \tag{2.14}$$

où $\alpha_k = \arccos v_{k,1}$ et où $v_{k,1}$ est la première composante du vecteur propre $\mathbf{v}_k$ associé à la valeur propre λ_k.

À partir des paramètres entropie et alpha, une procédure de classification peut être envisagée. En effet, en considérant l'espace bidimensionnel H/α/A, tous les mécanismes de diffusion aléatoire peuvent être représentés [Clou97]. Ainsi, ce plan peut être segmenté en huit zones et donner une interprétation du mécanisme moyen de rétrodiffusion global pour chacune des régions. L'axe de l'entropie est partitionné en trois zones traduisant, soit un comportement polarimétrique quasiment déterministe, soit modérément

aléatoire, soit fortement aléatoire. De même, l'axe du paramètre alpha, est partagé en trois régions qui correspondent, soit à une réflexion de surface, soit à une double réflexion, soit à une réflexion de volume. Le plan H/α est représenté sur la figure (Fig. 2.20).

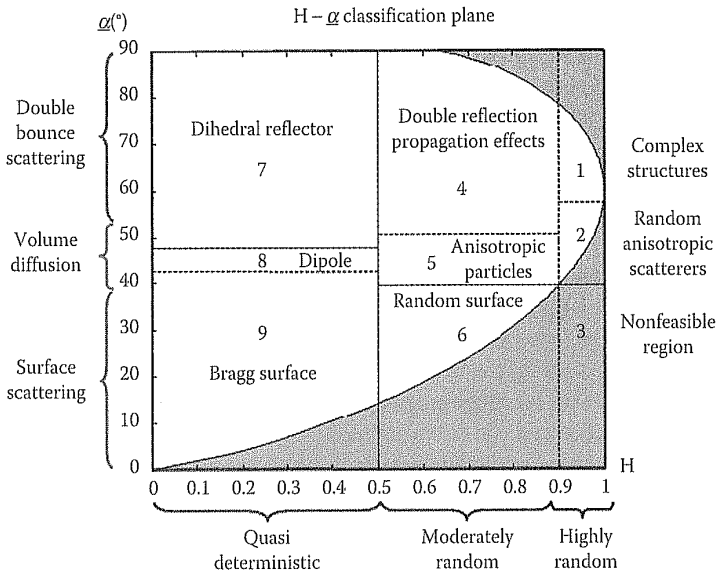


FIGURE 2.20 – Plan H/Alpha segmenté en huit régions.

L'interprétation des huit régions est la suivante [Clou97] :

- Région 1 : Cette zone caractérise une double réflexion avec une forte entropie. Ce phénomène de rétrodiffusion est observé sur des forêts ou sur des zones de végétation qui présentent des branches et un feuillage développés,

- Région 2 : Cette région est le siège d'une diffusion de volume avec une forte entropie. Ce phénomène est le résultat, en général, de la diffusion sur des particules anisotropes et orientées de manière totalement aléatoire,

- Région 3 : Cette zone caractérise une double réflexion avec une entropie moyenne. Ce type de phénomène est observé lors de la mesure d'une forêt en bande L ou en bande P. La double réflexion sur le sol et sur les troncs d'arbres est perturbée par le passage à travers la canopée, entraînant une augmentation du caractère aléatoire,

- Région 4 : Cette région est le siège d'une diffusion de volume par des particules anisotropes à entropie modérée. Ce phénomène est rencontré lors de l'étude de zones recouvertes de végétaux qui présentent des particules anisotropes dont l'orientation est moyennement corrélée,

- Région 5 : Cette zone caractérise une réflexion de surface modérément aléatoire. Ce type de mécanisme de rétrodiffusion peut être rencontré lors de la mesure de surface dont la rugosité et la longueur de corrélation varient.,

- Région 6 : Cette région est le siège d'une double réflexion à faible entropie. Elle caractérise les cibles qui sont généralement des dièdres métalliques ou des diélectriques isolés,

- Région 7 : Cette zone caractérise une diffusion par des dipôles avec une faible entropie. Cette

région est caractéristique de milieux naturels formés de particules anisotropes dont l'orientation est fortement corrélée entraînant une entropie faible,

- Région 8 : Cette région correspond à une réflexion de surface à faible entropie dont le comportement est caractéristique des modèles de réflexion sur des surfaces peu rugueuses comme les modèles de Bragg, de l'optique physique ou de l'optique géométrique. Les milieux naturels correspondants peuvent être la mer en bande P ou L et des surfaces assez lisses en général.

Ainsi, l'appartenance d'un pixel de l'image à une région du plan H/α permet une interprétation physique du mécanisme de diffusion moyen qui intervient.

En général, la segmentation H/α/A sert de point de départ au classifieur de Wishart, développé par Lee et al [Lee99]. Cette méthode de classification est similaire à la méthode des K plus proches voisins (K-mean) :

1. On applique la décomposition H/α/A sur les matrices de cohérence $[\mathbf{T}]$ associées à chaque pixel de l'image,

2. On classifie initialement l'image en 8 classes $\{\omega_i\}_{i=1,8}$ grâce aux zones du plan H/α,

3. À l'itération k, le nouveau centre de chaque classe ω_i est défini comme la moyenne des matrices de cohérence $[\mathbf{T}]_j$ de tous les n_i pixels associés à cette classe :

$$\mathbf{V}_i^{(k)} = \frac{1}{n_i}\sum_{j=1}^{n_j}[\mathbf{T}]_j \tag{2.15}$$

4. On calcule la distance de chaque pixel à chacun des centres ω_i comme la distance minimale $\min_i d_W\left([\mathbf{T}],[\mathbf{V}]_i^{(k)}\right)$ de la matrice de cohérence $[\mathbf{T}]$ du pixel à celles des centre de classes $\mathbf{V}_i^{(k)}$. En supposant que les matrices des pixels et les matrices des centres de classe soient distribuées selon une loi de Wishart donnée en (Eq. 2.9), ce qui implique *de facto* une distribution gaussienne de tous les pixels de l'image SAR, cette distance d_W entre deux matrices, définie comme un Maximum de Vraisemblance est donnée par :

$$d_W\left([\mathbf{T}],[\mathbf{V}]_i^{(k)}\right) = \log\left[\mathbf{V}\right]_i^{(k)} + \mathrm{tr}\left(\left([\mathbf{V}]_i^{(k)}\right)^{-1}[\mathbf{T}]\right) \tag{2.16}$$

5. On vérifie si le critère d'arrêt est atteint (nombre de pixels changeant de classes, la somme des distances intra-classes, un nombre d'itérations spécifié, ...). Sinon l'itération reprend au point 3.

Le papier suivant, présenté en Annexe D, décrit la méthodologie de ces décompositions dans le cadre des distributions temps-fréquence :

[C48] M. Duquenoy, **J.P. Ovarlez**, L. Ferro-Famil and E. Pottier, "*Supervised Classification of Scatterers on SAR Imaging Based on Incoherent Polarimetric Time-Frequency Signatures*", Proc. EUSIPCO 2009, Glasgow, Scotland, Aug. 2009 with Best Student Paper Award.

Ces techniques permettent alors de pouvoir analyser les paramètres liés aux décompositions incohérentes (valeurs propres de la matrice de covariance, entropie, anisotropie, ...) en fonction de la fréquence émise et de l'angle d'illumination. Basées uniquement sur une estimée de matrice de covariance de type SCM, il est tout à fait envisageable, pour améliorer les résultats sur la caractérisation des fonds, d'exploiter l'information donnée par l'axe spectral ou angulaire pour améliorer les effets de moyennage spatial comme pour le cas de l'interférométrie présentée plus haut. On peut également appliquer encore les techniques développées sur les modèle SIRV, plus généraux et en particulier pour la matrice du point fixe que

l'on présentera dans la section 2.3.

2.1.3 Analyse Temps-Fréquence pour le LIDAR

Je participe actuellement à l'encadrement de la thèse **[T7]** de Julien Totems dont la soutenance est prévue pour mars 2011. La thématique est ici la mise en œuvre de lidars cohérents émettant une forme d'onde de type radar (doublet, triplet, ... d'impulsions permettant une estimation de phase différentielle) sur des cibles ou des objets pour analyser leur vibrations propres (vibrométrie). L'analyse Temps-Fréquence apporte ici ses outils en vue de la caractérisation et de l'extraction d'une information de modulation de phase permettant de remonter aux caractéristiques vibratoires de la cible. L'originalité de la méthodologie Temps Fréquence est ici surtout basée sur l'estimation et l'extraction de fréquence instantanée de la vibration. Son travail a donné lieu à plusieurs publications **[J16]**, **[C42]**, **[C50]**. La figure (Fig. 2.21) donne un aperçu des types de modulation extraites.

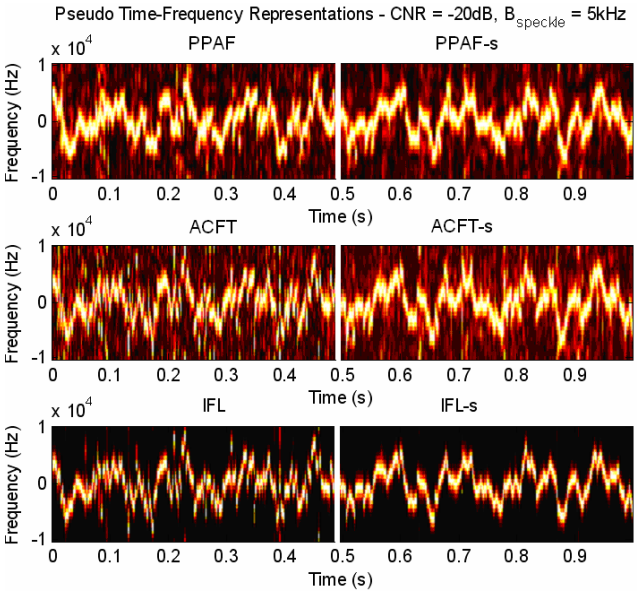


FIGURE 2.21 – Extraction de modulations par Pseudo Représentations Temps-Fréquence obtenues par simulation (forme d'onde à 6 impulsions) : Poly-Pulse Adapted Filter (PPAF), Autocorrelation Fourier Transform (ACFT), Instantaneous Frequency Likelihood (IFL)

2.2 La Détection et l'Estimation pour le Radar

2.2.1 Origine de l'Axe de Recherche

L'origine de mes recherches sur la détection en environnement non-gaussien provient en particulier d'une étude effectuée en ressources internes à l'ONERA à partir de 1993 sur l'analyse de la détectabilité des cibles volant à basse altitude (missile de pénétration par exemple). Cette problématique intéressait fortement la DGA. Sur un théâtre d'opérations, la pénétration à basse altitude d'un missile tiré à distance de sécurité face à un système de défense adverse dépend en grande partie de sa détectabilité vis à vis des radars de veille. Pour rendre cette détection plus difficile et plus tardive :

- on fait souvent voler le missile à basse altitude (masquage naturel par le relief du terrain, case distance de la cible contenant une part importante d'échos parasites dus aux réflecteurs du sol, interaction des ondes électromagnétiques avec le sol, réfraction),

- on cherche également à réduire la signature électromagnétique du missile (SER).

Compte tenu de la diversité et l'importance des phénomènes physiques rencontrés dans un tel duel, l'ONERA a entrepris en 1993 de développer un logiciel de simulation afin d'étudier la détectabilité d'un missile volant à basse altitude face à un radar de veille au sol et en particulier pour analyser la sensibilité de ces performances :

- à la signature électromagnétique du missile,
- aux caractéristiques du radar et du traitement de signal associé,
- à l'altitude, à la vitesse de vol, au type de pilotage du missile,
- à la variété de la nature du terrain (relief, végétation, météo, etc...).

Ce logiciel, dénommé MODEM (Modélisation de l'Observation et de la Détection Électromagnétique de Missiles) fut le fruit d'une coopération entre plusieurs divisions de la Direction des Études de Synthèse de l'époque. Le logiciel MODEM **[I4]**, **[I7]**, **[I8]**, **[I13]**, **[I14]**, qui a fait l'objet d'un dépôt logiciel, est composé principalement de trois modules permettant le calcul de :

- la génération de trajectoires d'un missile donné volant sur un terrain réel défini par un fichier altimétrique du type DLMS, avec prise en compte des mouvements d'attitude induits par le guidage-pilotage. Une réalisation simplifiée mais néanmoins réaliste du mouvement d'attitude de l'engin a été adoptée, par le biais d'un modèle comportemental, du type « pseudo 6 degrés de libertés », défini à partir de catégories de consignes de guidage (loi de guidage avec asservissement sur trajectoire ou navigation proportionnelle sur cible) et de consignes de pilotage (lois de pilotage bank-to-turn ou skid-to-turn),
- la SER (Section Efficace Radar) du missile selon la fréquence, les angles de vue et la polarisation du radar de veille, qui est obtenue soit par la fourniture d'un fichier de mesures, soit par calcul analytique, à partir de sa décomposition en éléments géométriques simples et une sommation incohérente des SER de chaque sous ensemble (modèle de points brillants, optique géométrique, optique physique et théorie géométrique de la diffraction),
- la propagation radar, le rapport signal à bruit et la probabilité de détection le long de la trajectoire.

Le dernier module détermine de manière analytique la probabilité de détection en fonction de la probabilité de fausse alarme choisie et du rapport signal à bruit + fouillis calculé. Il calcule en particulier le bilan de liaison en mettant l'accent sur l'aspect de la modélisation de la propagation sur terrain réel (multitrajets spéculaires, diffraction d'arêtes, diffraction sphérique) et comprenant :

- la puissance de signal reçue par le radar après traitements radar (traitements MTI, filtrage Doppler, fluctuation de cibles, post-intégration non cohérente),
- la puissance de bruit thermique provenant du récepteur radar,
- la puissance de fouillis de sol intercepté par le faisceau radar. Les paramètres régissant le comportement du fouillis de sol (réflectivité surfacique, densité spectrale, lois de probabilité, etc...) sont décrits à partir de modèles décrits issus de la littérature où à partir de l'analyse de mesures réelles provenant de diverses expérimentations menées à l'ONERA.

L'analyse par le logiciel MODEM a ainsi permis de montrer l'efficacité relative de différentes configurations d'implantation de matériaux absorbants ou encore l'analyse de sensibilité pour diverses configurations de vols, d'implantations de défenses **[I5]**, **[I9]**, **[I12]**, **[I16]** sur différents missiles.

Cette étude sur laquelle j'ai pratiquement fait mes armes d'ingénieur de recherche fut sans aucun doute l'initiateur de mes futurs axes de recherche concernant la détection. Après avoir lu la littérature, assez pauvre à l'époque, consacrée à la modélisation du fouillis de sol et à son impact sur les performances de systèmes radar, je me suis donc très vite intéressé à la problématique de la détection en environnement non gaussien.

2.2.2 Analyse des Performances des Détecteurs Radar Conventionnels en Environnement Non Gaussien

Le premier axe de recherche sur lequel je me suis penché est l'analyse de l'impact de la méconnaissance des caractéristiques du fouillis de sol ou de mer sur les performances des détecteurs radar conventionnels de l'époque (je précise, de l'époque, mais le temps montre, 20 ans plus tard, qu'ils sont restés assurément conventionnels !). En analysant la maigre bibliographie sur l'analyse du fouillis de sol (statistique, densité spectrale, valeurs moyenne de rétrodiffusion, dispersion de ces valeurs en fonction de la fréquence, de l'angle sous lequel il est illuminé, la polarisation, le type de fouillis), la dispersion de ces caractéristiques fut assez décourageante. Un rapport du MIT de Billingsley [Bill93], suivi par un ouvrage [Bill02] attira mon attention : il s'agissait d'un rapport analysant une campagne d'expérimentation effectuée aux États Unis et recensant les caractéristiques spectrales, statistiques et autres de mesures de fouillis de sol sur 42 sites différents, de la bande base UHF-VHF à la bande Ku. Celui-ci expliquait la nature du comportement non gaussien du fouillis, dû aux masquages éventuels des zones de fouillis par l'altimétrie et le sursol du terrain mais également par la petitesse de la case de résolution radar contenant un nombre plus petit, plus aléatoire de réflecteurs. Il modélisa dans son rapport la statistique de la puissance des échos de fouillis de sol par des lois de type Weibull et montra le grand écart de ses mesures avec la loi gaussienne.

Une expérimentation **[I8]** fut montée en 1995 à l'ONERA pour tenter de comprendre et valider les résultats de Billingsley et conduisit à la même constatation : la distribution d'amplitude des échos de fouillis de sol n'était pas gaussienne pour certaines configurations notamment à site-bas (angle rasant d'illumination du faisceau radar sur le sol) mais également pour des cases de plus en plus petites (50cm).

Pour pouvoir correctement répondre à nos interlocuteurs de la DGA concernant l'analyse des performances des radars face à des cibles volant à basse altitude, les questions de type *Comment prendre en compte cette information de statistique non gaussienne ?*, ou encore *Quel est l'impact de cette non gaussianité sur les performances de détection ?* dans l'évaluation des performances se firent rapidement sentir.

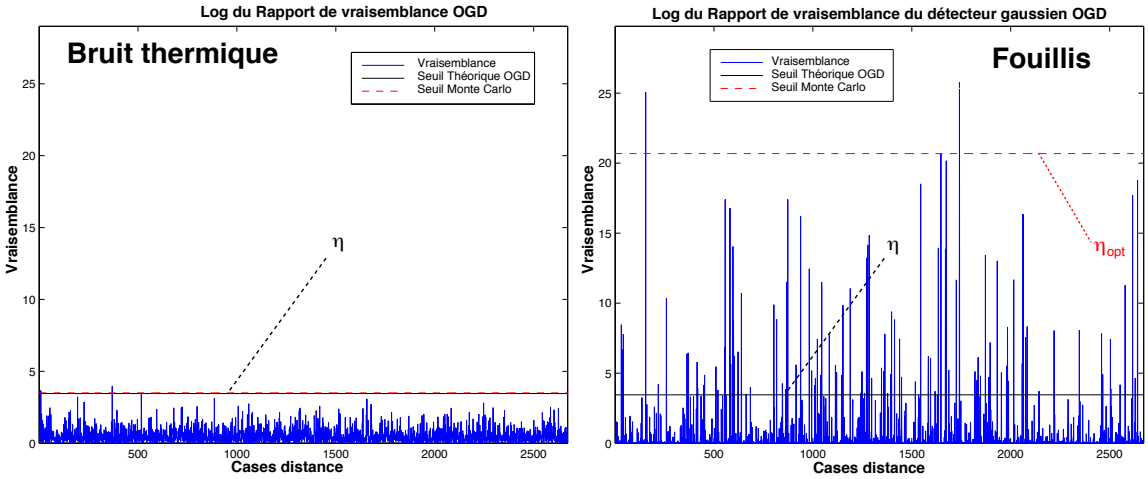
Le problème fondamental auquel il faut alors répondre, consiste généralement à détecter, dans un vecteur d'observation $\mathbf{y}$ de dimension m (correspondant par exemple aux m impulsions émises par le radar), un vecteur $\mathbf{s} = A\,\mathbf{p}$, d'amplitude complexe A généralement inconnue et de *steering vector* $\mathbf{p}$ caractérisant les paramètres inconnus de la cible (position, vitesse, réponse polarimétrique, direction, etc) corrompu par un bruit de fouillis $\mathbf{c}$ additif non gaussien de densité $p_{\mathbf{c}}(.)$. Ce problème se formalise généralement par un test d'hypothèses binaires :

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \ldots, K \\ H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \ldots, K \end{cases} \tag{2.17}$$

où les $\mathbf{y}_i$ sont les K vecteurs d'observations indépendants appelés *données secondaires*, contenant uniquement le bruit de fouillis et permettant d'estimer les paramètres du fouillis (matrice de covariance, puissance, densité, ...), puis à terme de construire un détecteur optimal. Ce détecteur optimal, construit selon la théorie de Neymann Pearson, est basé sur le test du rapport de vraisemblance :

$$\Lambda(\mathbf{y}) = \frac{p_{\mathbf{c}}(\mathbf{y} - \mathbf{s})}{p_{\mathbf{c}}(\mathbf{y})} \tag{2.18}$$

La figure (Fig. 2.22) donne un aperçu du problème du choix crucial de la connaissance des caractéristiques du bruit additif $\mathbf{c}$ et de la mise en œuvre du détecteur optimal associé. Sur cette figure sont représentées les enveloppes (rapport de vraisemblance du détecteur gaussien OGD (Optimum Gaussian Detector) de deux processus de même moment d'ordre deux, l'un gaussien (enveloppe distribuée selon la loi de Rayleigh) et l'autre, non gaussien, distribué selon une loi K de paramètre $\nu = 0.1$. Pour réguler son taux de fausse alarme, un détecteur radar conventionnel connaît parfaitement, sous hypothèse gaussienne, le seuil de détection η à appliquer. Ce seuil de détection ne dépend que de la probabilité de fausse alarme fixée ainsi que de la puissance du bruit. Pour un bruit non gaussien, distribué ici en loi K, ce seuil de



(a) Bruit thermique (b) Fouillis impulsionnel

FIGURE 2.22 – Mise en défaut de l'AMF dans du fouillis non gaussien, de même puissance que le bruit thermique - Ajustement du seuil de détection

détection est faussé par l'impulsivité du bruit. Il doit donc être rehaussé à η_{opt} pour garantir le taux de fausse alarme désiré mais ce faisant, les performances de détection se dégradent.

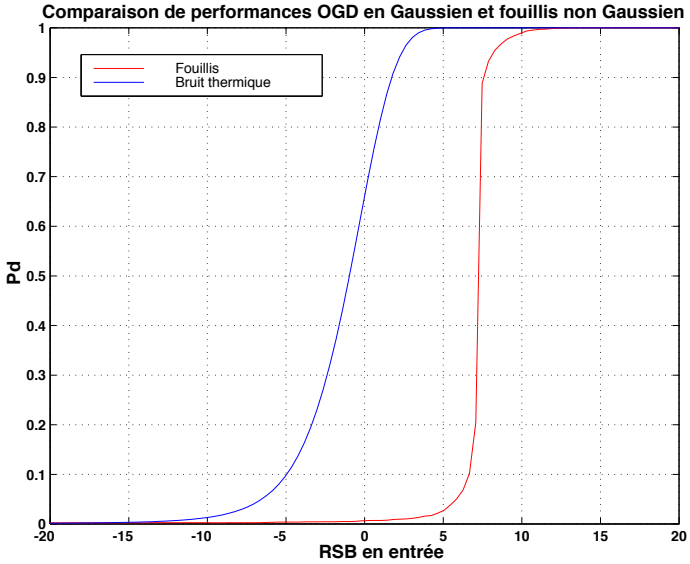


FIGURE 2.23 – Mise en défaut de l'OGD dans du fouillis non gaussien, de même puissance que le bruit thermique - Probabilité de détection

Le seuil optimal étant réglé sous ces deux hypothèses de bruit additif, la figure (Fig. 2.23) met en valeur la dégradation des performances de l'OGD. La courbe bleue représente la probabilité de détection de l'OGD dans le bruit thermique tandis que la courbe rouge représente cette même probabilité dans le fouillis non gaussien. Le constat est sans appel : pour un même RSB, 0 dB par exemple, la P_d dans le bruit thermique est proche de 0.7 mais ne vaut que 10^{-2} dans le fouillis K-distribué.

Les dégradations des performances de détection sont alors conséquentes sous des hypothèses d'environnement non valides. Plus grave, ne pas connaître l'environnement dans lequel on évolue, sur-évalue les performances de détections annoncées des radar conventionnels basés sur la détection d'enveloppe, puisque ces dernières sont généralement annoncées pour le seul cas du bruit thermique gaussien ! Il est donc indispensable de bien connaître la statistique de cet environnement, ou dans une certaine mesure, d'utiliser une modélisation de fouillis proche de ce que révèlent les mesures réelles afin d'une part de qualifier les performances réelles des radars mais d'autre part de développer des détecteurs optimisés relativement à la statistique des bruits rencontrés.

Une première approche fut de se donner des hypothèses *a priori* sur la statistique du fouillis de sol avec des lois très connues de la littérature comme les lois de Weibull ou encore les K-distributions, et d'évaluer les performances des détecteurs d'enveloppe conventionnels dans ces différents bruits. Cette technique, pouvant techniquement être mise en œuvre théoriquement ou par tirage Monte Carlo, souffrait néanmoins d'un calcul fastidieux voire impossible mais également d'un temps de calcul assez prohibitif. Elle était également tributaire de la parfaite connaissance des caractéristiques statistiques du fouillis qui pouvait changer d'un terrain à un autre.

L'encadrement du stage de DEA et de la thèse d'Emmanuelle Jay **[T1]** en 1998 (soutenue en 2002) a permis de répondre à ces interrogations et a véritablement ouvert la voie de cette thématique de recherche. Le premier objectif était **[I10]**], **[I13]** d'établir les relations liant le seuil de détection à la probabilité de fausse alarme mais également de caractériser les performances de détection d'enveloppe d'un radar dans n'importe quel processus de bruit additif. Nous avons ainsi établi une relation liant la densité d'enveloppe d'un bruit b complexe de densité donnée $p_b(r)$ à la densité de probabilité $p_{A+b}(r)$ de l'enveloppe d'un signal connu A noyé dans ce même bruit b :

$$p_{A+b}(r) = \int_0^{+\infty} \int_0^{+\infty} r\,\rho\,J_0(\rho\,A)\,J_0(\rho\,r)\,J_0(\rho\,r')\,p_b(r')\,d\rho\,dr' \tag{2.19}$$

L'approche suivante fut de modéliser la statistique $p_b(.)$ de l'enveloppe des échos de fouillis par Approximants de Padé [Amin91], [Amin94a], [Amin94b]. Cette modélisation est l'analogue de la modélisation paramétrique ARMA des processus stationnaires discrets pour les processus aléatoires continus. La théorie des approximants Padé consiste à construire une approximation de la fonction génératrice de moments du bruit sous forme de fraction rationnelle avec un dénominateur de degré M et un numérateur de degré $L < M$ telle que son développement en série de puissance croissante coïncide avec celui de la fonction génératrice de moments jusqu'à l'ordre $L + M$. Par transformée de Laplace inverse, l'approximation de la densité de l'enveloppe du bruit $p_b(r)$ est donnée par une somme finie d'exponentielles complexes de la forme :

$$p_b(r) = \sum_{k=1}^{M} \lambda_k\,e^{\alpha_k\,r} \tag{2.20}$$

qui permet ainsi d'expliciter également la densité $p_{A+b}(r)$ donnée par (Eq. 2.19) :

$$p_{A+b}(r) = \sum_{k=1}^{M} \lambda_k \int_0^{+\infty} \rho\,r\,\frac{J_0(\rho\,A)\,J_0(\rho\,r)}{\sqrt{\rho^2+\alpha_k^2}}\,d\rho \tag{2.21}$$

On peut alors calculer la probabilité de détection P_d par :

$$P_d = 1 - \sum_{k=1}^{M} \lambda_k \int_0^{+\infty} r_b\frac{J_0(\rho\,A)\,J_1(\rho\,r_b)}{\sqrt{\rho^2+\alpha_k^2}}\,d\rho \tag{2.22}$$

avec r_b définissant le seuil de détection à P_{fa} fixée :

$$P_{fa} = -\sum_{k=1}^{M} \frac{\lambda_k}{\alpha_k}\,e^{\alpha_k\,r_b} \tag{2.23}$$

Cette procédure **[J3]**, **[C12]**, **[C13]**, **[C14]**, **[C15]** permet alors de calculer, de manière somme toute assez simple, les performances de détection radar d'une cible non fluctuante d'amplitude constante dans un bruit de densité d'enveloppe *a priori* quelconque mais parfaitement défini par les jeux de coefficients $\{\lambda_k\}$ et $\{\alpha_k\}$. Le papier suivant donné en Annexe E présente en détail cette méthodologie :

[J3] E. Jay, **J.P. Ovarlez** and P. Duvaut, "*New Methods of Radar Detection Performances Analysis*", Signal Processing, Vol.80, No.12, pp.2527-2540, Dec. 2000.

Le travail de thèse d'Emmanuelle Jay **[T1]** a également contribué à l'exploration et l'élaboration d'autres types de détecteurs radar dits optimaux basés sur des familles de bruit non gaussien, les processus SIRV, très élégante que nous présentons dans le paragraphe suivant.

2.2.3 Processus SIRP et Vecteurs SIRV

Parallèlement à ces recherches sur l'analyse des performances radar, les questions essentielles de *Comment modéliser de manière générale des processus non gaussiens ?* ou encore *Comment modéliser le fouillis radar ?* nous ont conduit à étudier de nombreux papiers sur les lois de Weibull, les lois K-distributions, les loi gamma, ... En fait, toutes ces lois peuvent être, de manière élégante, englobées dans une immense famille de processus appelée SIRP (Spherically Invariant Random Process). Ces processus [Yao73], [Jake80], [Jao84], [Barn96], également appelés, gaussiens composés, sont des processus aléatoires gaussiens dont la variance est également aléatoire. On comprend alors immédiatement l'intérêt de ces processus pour la modélisation du fouillis radar. Le fouillis radar provient de la sommation amplitude-phase des différents réflecteurs du sol présents dans la cellule analysée. De par la variabilité de l'environnement, les réflecteurs changent, d'une cellule à l'autre, d'orientation, de nature, de réflectivité et varient en nombre. La puissance du signal rétrodiffusé par la case devient alors aléatoire. C'est cet aspect aléatoire qui rend complexe l'analyse du fouillis. Les modèles SIRP permettent alors de prendre en compte cet aspect aléatoire par un double processus : un processus gaussien appelé *speckle* dont la puissance, appelée *texture* est elle-même aléatoire.

L'exemple de la K-distribution, processus SIRP dont la puissance est gamma-distribuée, illustre en quelque sorte la connexion du modèle avec la physique du radar : si la case d'analyse contient, de par sa taille, une infinité de réflecteurs supposés identiquement distribués, le Théorème Central Limite nous assure un comportement statistique de type gaussien. Si la distribution du nombre de réflecteurs est une loi binomiale négative, l'écho rétrodiffusé sera K-distribué.

Il est possible d'étendre la théorie des SIRP au cas multivarié [Rang93]. La théorie nous conduit alors à la définition des SIRV (Spherically Invariant Random Vectors). Plus précisément, un SIRV $\mathbf{c}$ est le produit de la racine carré d'une variable aléatoire τ scalaire positive, appelée la texture, et d'un vecteur aléatoire complexe gaussien, circulaire, $\mathbf{x}$, appelé *speckle* de dimension m, centré et ayant pour matrice de covariance $\mathbf{M} = \mathbb{E}\left[\mathbf{x}\,\mathbf{x}^H\right]$ où $\mathbb{E}\left[.\right]$ désigne l'espérance mathématique ou moyenne statistique d'une variable aléatoire :

$$\mathbf{c} = \sqrt{\tau}\,\mathbf{x}\,. \tag{2.24}$$

D'autre part, la PDF $p_{\mathbf{c}}(.)$ de $\mathbf{c}$ est connue et son expression est la suivante :

$$p_{\mathbf{c}}(\mathbf{c}) = \int_0^{+\infty} \frac{1}{(\pi\,\tau)^m\,|\mathbf{M}|}\,\exp\left(-\frac{\mathbf{c}^H\,\mathbf{M}^{-1}\,\mathbf{c}}{\tau}\right)\,p_\tau(\tau)\,d\tau\,. \tag{2.25}$$

Conditionnellement à la case d'analyse ou à τ, le vecteur $\mathbf{c}$ de données collectées est supposé gaussien et la matrice de covariance $\mathbf{M}$ gère ainsi le degré de corrélation qui existe dans ce vecteur. De case d'analyse à case d'analyse, la texture modélise le fait que la norme du vecteur d'observation varie aléatoirement. On peut comprendre l'interprétation physique de cette matrice et de la texture dans les cas différents suivants :

- pour un radar émettant périodiquement des impulsions et découpant l'axe distance en cases de résolution, la matrice de covariance, gère, dans une case distance donnée, la corrélation existant

entre les différentes impulsions. Cette information de corrélation est reliée au caractère spectral (et Doppler) du fouillis (vent sur une forêt, fouillis dynamique de la mer) et influe sur les performances de détection. La texture gère la variation de la norme du vecteur sur l'axe des distances,

- pour un radar polarimétrique, cette matrice de covariance peut gérer le degré de corrélation existant entre les différentes composantes polarimétrique du signal reçu. La texture gère la variation de la norme du vecteur sur l'axe des distances,

- pour un réseau de capteurs, la matrice de corrélation gère la corrélation existant sur l'axe angulaire. La texture peut représenter la variation temporelle de la puissance,

- pour un traitement de type STAP, la matrice de covariance gère à la fois l'aspect spectral et angulaire du fouillis, la texture pilote plutôt la variation de puissance des échos collectés sur l'axe distance,

- pour une image SAR polarimétrique, interférométrique voire les deux ensemble, la matrice de covariance (dite de cohérence), calculée sur le domaine spatial des pixels de l'image SAR, gère la corrélation existant sur ces différents canaux. La texture représente dans ce cas, la variation spatiale de la puissance de vecteurs collectés de pixel à pixel.

Cette modélisation des processus SIRV est donc très générale et peut s'adapter à de nombreux problèmes radar. L'intérêt de cette modélisation est qu'elle englobe également les processus gaussiens, ce qui la rend encore plus intéressante dans le sens où tout schéma de détection basé sur ce modèle pourra encore s'appliquer dans le cas classique.

2.2.4 Détection Dans les Processus SIRV

Pour de nombreux modèles de texture du processus SIRV, on peut déterminer le détecteur optimal associé (rapport de formes intégrales). Cependant, la forme du rapport de vraisemblance associé au test de détection peut rarement s'expliciter sous forme analytique ce qui le rend difficile à mettre en œuvre. La méthode des Approximants de Padé a donc été également utilisée pour exploiter plus facilement ces détecteurs optimaux. Cette technique **[T1]**, **[C16]**, **[C17]** a permis de définir le PEOD (Padé Estimated Optimum Detector) sous deux formes, une forme idéale où la texture du fouillis est directement connue et une forme dérivée de type bayésienne qui fait l'hypothèse de la connaissance *a posteriori* de la loi de la texture. Cette dernière idée a contribué au développement de formes plus générales de détecteurs, comme le BORD (Bayesian Optimum Radar Detector).

2.2.4.1 Le PEOD : Padé Estimated Optimum Detector

L'idée première du détecteur PEOD (Padé Estimated Optimum Detector) proposé en Annexe F :

[C16] E. Jay, **J.P. Ovarlez**, D. Declerq and P. Duvaut, *"PEOD : Padé Estimated Optimum (Radar) Detector"*, IEEE-RADAR Conference, Atlanta, USA, May 2001,

est d'utiliser la modélisation par approximants de Padé de la loi de la texture $p_\tau(.)$ du processus SIRV. Sous une modélisation du processus SIRV général, le Rapport de Vraisemblance généralisé (après estimation au sens du Maximum de Vraisemblance de l'amplitude complexe inconnue de la cible) correspondant au test binaire d'hypothèse classique (H_0 : cible absente, H_1 cible présente) est donné par :

$$\Lambda(\mathbf{y}) = \frac{p(\mathbf{y}/H_1)}{p(\mathbf{y}/H_0)} = \frac{\displaystyle\int_0^{+\infty} \tau^{-m} \exp\left(-\frac{q_1(\mathbf{y})}{\tau}\right) p_\tau(\tau)\, d\tau}{\displaystyle\int_0^{+\infty} \tau^{-m} \exp\left(-\frac{q_0(\mathbf{y})}{\tau}\right) p_\tau(\tau)\, d\tau} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{2.26}$$

où $q_0(\mathbf{y}) = \mathbf{y}^H \mathbf{M}^{-1} \mathbf{y}$, $q_1(\mathbf{y}) = \mathbf{y}^H \mathbf{M}^{-1} \mathbf{y} - \dfrac{|\mathbf{p}^H \mathbf{M}^{-1} \mathbf{y}|^2}{\mathbf{p}^H \mathbf{M}^{-1} \mathbf{p}}$, où le seuil de détection η est défini tel que :

$$P_{fa} = \mathbb{P}(\Lambda(\mathbf{y}) \overset{H_0}{>} \eta) = \int_{\{\mathbf{y}:\Lambda(\mathbf{y})>\eta\}} p(\mathbf{y}/H_0) d\mathbf{y}. \tag{2.27}$$

Ce test, dépendant de la loi de la texture, n'est pas simple à mettre en œuvre. Si l'on remplace la loi de la texture $p_\tau(.)$ par son approximation de Padé, le rapport de vraisemblance correspondant devient une forme plus exploitable :

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{\frac{1-m}{2}} \frac{\displaystyle\sum_{k=1}^{M} \lambda_k \, \alpha_k^{\frac{m-1}{2}} \, K_{1-m}\left(\sqrt{\alpha_k \, q_1(\mathbf{y})}\right)}{\displaystyle\sum_{k=1}^{M} \lambda_k \, \alpha_k^{\frac{m-1}{2}} \, K_{1-m}\left(\sqrt{\alpha_k \, q_0(\mathbf{y})}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{2.28}$$

où $K_\nu(.)$ est la fonction de Bessel modifiée de deuxième espèce d'ordre ν. Pour la mise en œuvre du PEOD, la statistique de l'environnement est estimée au préalable sur données de référence.

2.2.4.2 Le BORD : Bayesian Optimum Radar Detector

Ce détecteur et sa forme asymptotique ont tous deux été proposés et étudiés dans la thèse d'Emmanuelle Jay [T1] et ont été publiés principalement dans cet article donné en Annexe G :

[J4] E. Jay, **J.P. Ovarlez**, D. Declerq and P. Duvaut, "*BORD : Bayesian Optimum Radar Detector*", Signal Processing, Vol.83, No.6, pp.1151–1162, Jun. 2003.

L'idée fut de déterminer *a posteriori* la loi de la texture du modèle SIRV de l'environnement du radar à partir de K mesures de vecteurs référence bruit. Soit $[\mathbf{c}_1, \cdots, \mathbf{c}_K]^T$ une collection de K SIRV de taille m. D'après le modèle SIRV, on a : $\mathbf{c}_i = \mathbf{x}_i \sqrt{\tau}$. Connaissant la loi conditionnelle de $\mathbf{c}_i$ par rapport à τ, à savoir une loi gaussienne multivariée, la règle de Bayes nous fournit une expression de la loi *a posteriori* de la *texture*, en fonction de sa "vraie" loi (inconnue), de la vraisemblance des références (connue) et de la loi du vecteur des références (inconnue). On a donc :

$$p(\tau/\mathbf{c}_i) = \frac{p(\mathbf{c}_i/\tau) \, p_\tau(\tau)}{\displaystyle\int_0^{+\infty} p(\mathbf{c}_i/\tau) \, p_\tau(\tau) \, d\tau}, \tag{2.29}$$

Dans cette expression, seule $p(\mathbf{c}_i/\tau)$ est connue, c'est la densité gaussienne. Afin d'en déterminer une expression analytique, une densité *a priori* $g(\tau)$ est choisie pour représenter $p_\tau(\tau)$ sur les données de références. L'idée est ici de remplacer $p_\tau(\tau)$ par $g(\tau)$:

$$p(\tau/\mathbf{c}_i) = \frac{p(\mathbf{c}_i/\tau) \, g(\tau)}{\displaystyle\int_0^{+\infty} p(\mathbf{c}_i/\tau) \, g(\tau) \, d\tau}, \tag{2.30}$$

Pour un environnement radar inconnu, la seule connaissance apportée par la modélisation SIRV est la positivité de la variable. Dans pareil cas, le prior de Jeffrey non informatif $g(\tau) = 1/\tau$, proportionnel à la racine carrée de l'information de Fisher, est choisi afin d'éviter toute restriction sur le domaine réel de validité de la variable [Robe92], [Robe94]. Malgré le caractère impropre de la densité du prior de Jeffrey définie sur $\mathbb{R}^+$, son expression garantit la positivité de la variable tout en respectant le comportement en $1/\tau$ de la variance dans l'expression de la loi gaussienne. Ainsi défini, le prior de Jeffrey est valable pour les paramètres d'échelle en général. La loi *a posteriori* instantanée de la *texture* prend ainsi la forme :

$$p(\tau/\mathbf{c}_i) = \frac{(\mathbf{c}_i^H \mathbf{M}^{-1} \mathbf{c}_i)^m}{\tau^{m+1} \, \Gamma(m)} \exp\left(-\frac{\mathbf{c}_i^H \mathbf{M}^{-1} \mathbf{c}_i}{\tau}\right). \tag{2.31}$$

Cette loi représente la loi *a posteriori* instantanée de la *texture* et correspond à une loi Inverse Gamma $\mathcal{IG}(\tau; m, 1/\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i)$. D'après la règle de Bayes et une estimation Monte-Carlo d'intégrale, un estimateur bayésien de la loi de la *texture* peut être déterminé. Pour ce faire, la loi de la *texture* peut s'exprimer sous la forme d'une intégrale sur l'ensemble des données de référence $\mathbf{c}_i$:

$$p_\tau(\tau) = \int p_\tau(\tau/\mathbf{c}_i)\, p(\mathbf{c}_i)\, d\mathbf{c}_i. \tag{2.32}$$

Connaissant les K vecteurs de référence $\mathbf{c}_i$, une approximation Monte-Carlo de (Eq. 2.32) s'écrit :

$$\hat{p}_K(\tau) = \frac{1}{K}\sum_{i=1}^{K} p(\tau/\mathbf{c}_i). \tag{2.33}$$

Après quelques manipulations, on obtient l'estimateur bayésien de $p_\tau(\tau)$:

$$\hat{p}_K(\tau) = \frac{\tau^{-m-1}}{\Gamma(m)\,K}\sum_{i=1}^{K}\left(\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i\right)^m \exp\left(-\frac{\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i}{\tau}\right). \tag{2.34}$$

L'estimateur ainsi déterminé à partir de données de référence permet de calculer le Rapport de Vraisemblance, que nous avons appelé BORD :

$$\Lambda_K(\mathbf{y}) = \frac{\displaystyle\sum_{i=1}^{K}\left[\frac{\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i}{(q_1(\mathbf{y}) + \mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i)^2}\right]^m}{\displaystyle\sum_{i=1}^{K}\left[\frac{\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i}{(q_0(\mathbf{y}) + \mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i)^2}\right]^m} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{2.35}$$

où $q_0(\mathbf{y}) = \mathbf{y}^H\,\mathbf{M}^{-1}\,\mathbf{y}$, $q_1(\mathbf{y}) = \mathbf{y}^H\,\mathbf{M}^{-1}\,\mathbf{y} - \dfrac{|\mathbf{p}^H\,\mathbf{M}^{-1}\,\mathbf{y}|^2}{\mathbf{p}^H\,\mathbf{M}^{-1}\,\mathbf{p}}$.

Sur des considérations probabilistes, et connaissant les K données de référence $\mathbf{c}_i$, l'expression du BORD (Eq. 2.35) peut être considérée comme l'approximation Monte-Carlo de l'expression suivante :

$$\frac{\displaystyle\int_0^{+\infty}\frac{z^m}{(q_1(\mathbf{y}) + z)^{2\,m}}\, p_Z(z)\, dz}{\displaystyle\int_0^{+\infty}\frac{z^m}{(q_0(\mathbf{y}) + z)^{2\,m}}\, p_Z(z)\, dz}, \tag{2.36}$$

où $Z = \mathbf{c}^H\,\mathbf{M}^{-1}\,\mathbf{c}$, forme quadratique d'un SIRV complexe de loi $p_Z(z)$ donnée :

$$p_Z(z) = \int_0^{+\infty}\frac{z^{m-1}}{\Gamma(m)}\,\tau^{-m}\exp\left(-\frac{z}{\tau}\right)\, p_\tau(\tau)\, d\tau.$$

La loi de Z représente la loi de la forme quadratique des données de référence sur lesquelles le prior $g(\tau) = 1/\tau$ a été choisi. La loi de Z est donc déterminée ici en intégrant non pas sur $p(\tau)$, *a priori* inconnue, mais sur le prior $g(\tau)$. On obtient alors :

$$p_Z(z) = \int_0^{+\infty}\frac{z^{m-1}}{\Gamma(m)}\,\tau^{-m}\exp\left(-\frac{z}{\tau}\right)\, g(\tau)\, d\tau = \frac{1}{z}. \tag{2.37}$$

Le calcul de (Eq. 2.36) s'effectue en intégrant le numérateur et le dénominateur par parties et le membre de gauche de l'équation (Eq. 2.35) converge alors en loi vers (Eq. 2.36), ce qui s'écrit :

$$\lim_{K\to+\infty}\Lambda_K(\mathbf{y}) \overset{\mathcal{L}}{=} \left(\frac{q_0(\mathbf{y})}{q_1(\mathbf{y})}\right)^m. \tag{2.38}$$

Le test associé peut alors se réécrire sous la forme suivante :

$$\frac{q_0(\mathbf{y}) - q_1(\mathbf{y})}{q_0(\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda = \frac{\sqrt[m]{\eta} - 1}{\sqrt[m]{\eta}}, \tag{2.39}$$

soit encore sous la forme plus connue :

$$\Lambda(\mathbf{y}, \mathbf{M}) = \frac{|\mathbf{p}^H \mathbf{M}^{-1} \mathbf{y}|^2}{(\mathbf{p}^H \mathbf{M}^{-1} \mathbf{p})(\mathbf{y}^H \mathbf{M}^{-1} \mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{2.40}$$

Contrairement au détecteur OGD bien connu dont l'expression est donnée par :

$$\frac{|\mathbf{p}^H \mathbf{M}^{-1} \mathbf{y}|^2}{\mathbf{p}^H \mathbf{M}^{-1} \mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{2.41}$$

le détecteur BORD asymptotique (Eq. 2.40) est homogène en toutes les variables ($\mathbf{p}$, $\mathbf{M}$ et $\mathbf{y}$) ce qui le rend invariant à tout changement d'échelle de ces quantités. Il représente en fait le cosinus de l'angle entre $\mathbf{p}$ et $\mathbf{y}$ pour un certain type de produit scalaire (relié à $\mathbf{M}$). Quelle que soit $\mathbf{M}$, la quantité $\Lambda(\mathbf{y}, \mathbf{M})$ est donc toujours comprise entre 0 et 1. De manière surprenante, ce détecteur était déjà connu dans la littérature sous des noms différents et avait également été établi de différentes manières :

- l'ANMF (Adaptive Normalized Matched Filter) proposé par Scharf [Scha71], [Kraut01] : son objectif était, dans un contexte gaussien, de définir un détecteur invariant au paramètre d'échelle. Dans le cas des SIRV, ce paramètre d'échelle est ici de manière logique la texture. Antérieurement, Korado dans [Kora68], Picinbono et Vezzosi dans [Pici70] avaient déjà obtenu cette expression dans des hypothèses de bruit Gaussien,

- le GLRT-LQ (Generalized Likelihood Ratio Test – Linear Quadratic) défini par Conte et Gini [Cont95], [Gini97]. Les détecteurs optimaux basés sur les processus SIRV étant définis par des formes intégrales et dépendantes de la densité de probabilité de la texture, le GLRT-LQ est défini comme approximation asymptotique du détecteur optimal dans un environnement de bruit K-distribué,

- K. J. Sangston *et al.* dans [Sang99] l'obtiennent en considérant la texture du SIRV comme un paramètre déterministe et en la remplaçant par son estimée au sens du MV sous chacune des hypothèses.

Emmanuelle Jay **[J4]** a donc, durant ses travaux de thèse, défini la loi statistique du test 2.40 et étudié les propriétés très intéressantes de ce détecteur, considérant la matrice de covariance du noyau gaussien du SIRV connue. Cette loi a la remarquable propriété d'être,

- invariante à la loi de la texture, c'est à dire que le détecteur reste invariant quelle que soit la loi de la texture du SIRV. Cette propriété appelée CFAR-texture (Constant False Alarm rate) garantit que le niveau de fausse alarme est toujours bien maintenu pour ce détecteur. Cette propriété est mise en évidence à la figure (Fig. 2.24). Celle-ci représente, en échelle logarithmique, des simulations Monte-Carlo des courbes de seuils de détection en fonction de la probabilité de fausse alarme, pour 4 différents SIRV : un processus gaussien, une K-distribution de paramètre $\nu = 0.1$, une texture distribuée selon une loi de Weibull et un Student-t. La relation théorique (Eq. 2.42) liant le seuil de détection λ à la probabilité de fausse alarme P_{fa} a également été tracée sur ce graphique.

- invariante à la matrice de covariance (comme pour l'AMF d'ailleurs), c'est à dire que la loi du test ne dépend pas de la valeur de la matrice. Cette propriété appelée CFAR-matrice garantit que le niveau de fausse alarme est toujours bien maintenu pour ce détecteur quelle que soit la valeur de la matrice de covariance du processus SIRV.

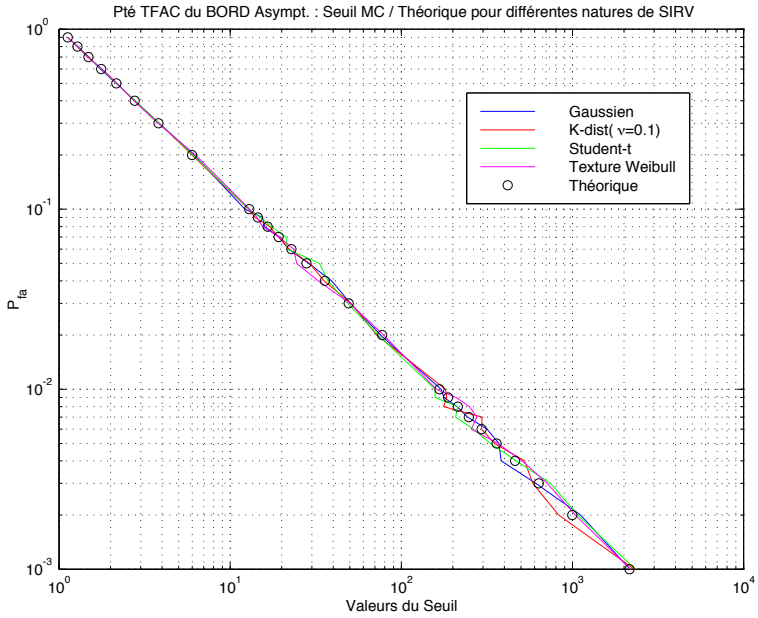


FIGURE 2.24 – Propriété CFAR-texture du BORD asymptotique

Ces deux propriétés confèrent au détecteur BORD Asymptotique la qualité d'être SIRV-CFAR. La loi statistique du test $\Lambda(\mathbf{y}, \mathbf{M})$ donné en (Eq. 2.40) a été obtenue, sous l'hypothèse de bruit seul, dans la thèse [T1]. Ce rapport de vraisemblance suit une loi Beta de première espèce. Une relation analytique entre le seuil de détection et la probabilité de fausse alarme P_{fa} fixée a pu ainsi être établie :

$$\lambda = 1 - P_{fa}^{\frac{-1}{1-m}} . \tag{2.42}$$

Cependant, ces résultats pourtant encourageants à l'époque ne se sont pas révélés parfaits sur données expérimentales car cette relation Pfa-seuil, sous l'hypothèse H_0 *bruit seul* n'était pas correctement respectée. Ceci s'explique par le fait, premièrement, du choix de l'estimateur de la matrice de covariance $\mathbf{M}$, inconnue sur données réelles, mais également par le fait que l'aspect aléatoire de cet estimateur devait également être pris en compte dans l'élaboration de la loi du test.

2.2.5 Estimation des Paramètres du SIRV

Les détecteurs étant définis, nous nous intéressons dans ce paragraphe aux méthodes d'estimation par Maximum de Vraisemblance de la matrice de covariance du processus SIRV dans deux cas généraux, tout d'abord, en la supposant aléatoire et inconnue, ce qui est le cas dans la modélisation SIRV puis en la supposant déterministe mais inconnue. Les deux méthodes conduisent à la définition d'un nouvel estimateur de la matrice de covariance du SIRV : l'estimateur FP du point fixe donnée par la solution d'une équation implicite au point fixe.

Le problème de l'estimation est récurrent dans la théorie adaptative du radar où les paramètres inconnus de l'environnement sont généralement estimés. La théorie du GLRT (Generalized Likelihood Ratio Test) permet de définir un détecteur optimal construit par le rapport des maxima des densités par rapport au vecteur $\mathbf{\Theta}$ de tous les paramètres inconnus :

$$\Lambda(\mathbf{y}) = \frac{\max\limits_{\mathbf{\Theta}} p_{\mathbf{c}}(\mathbf{y} - \mathbf{s})}{\max\limits_{\mathbf{\Theta}} p_{\mathbf{c}}(\mathbf{y})} \tag{2.43}$$

Ce test peut être dans certains cas difficilement déterminé de manière analytique. Une solution simple consiste alors à remplacer une estimée de $\mathbf{\Theta}$ dans l'expression du test mais le détecteur ainsi construit n'est plus l'optimal. Dans le cas du modèle de bruit SIRV, il est quasi-impossible de déterminer le GLRT. Nous nous intéresserons par la suite aux détecteurs construits en remplaçant les estimateurs de paramètres inconnus dans le rapport de vraisemblance (*Two Step GLRT*). En ce qui concerne plus particulièrement l'estimateur de la matrice de covariance, tout estimateur $\widehat{\mathbf{M}}$ de $\mathbf{M}$ pourra donc par la suite être reinjecté dans le GLRT-LQ (respectivement dans l'OGD), pour donner une version adaptative du GLRT-LQ, appelée ANMF Adaptive Normalized Matched Filter (respectivement une version adaptative de l'OGD, appelée AMF Adaptive Matched Filter) donnés par :

$$\Lambda_{ANMF}(\mathbf{y}, \widehat{\mathbf{M}}) = \frac{\left|\mathbf{p}^H \widehat{\mathbf{M}}^{-1} \mathbf{y}\right|^2}{\left(\mathbf{p}^H \widehat{\mathbf{M}}^{-1} \mathbf{p}\right)\left(\mathbf{y}^H \widehat{\mathbf{M}}^{-1} \mathbf{y}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{2.44}$$

et

$$\Lambda_{AMF}(\mathbf{y}, \widehat{\mathbf{M}}) = \frac{\left|\mathbf{p}^H \widehat{\mathbf{M}}^{-1} \mathbf{y}\right|^2}{\mathbf{p}^H \widehat{\mathbf{M}}^{-1} \mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{2.45}$$

Bien entendu, le choix de l'estimateur $\widehat{\mathbf{M}}$ est ici primordial. Si l'on souhaite, par exemple, conserver la propriété texture CFAR du détecteur donné en (Eq. 2.44), il est nécessaire de trouver un estimateur de matrice de covariance invariant lui aussi à cette texture.

Le premier type d'estimateur de la matrice de covariance du processus SIRV que nous avons rencontré est évidemment l'estimateur empirique $\widehat{\mathbf{M}} = \mathbf{M}_{SCM}$ de la covariance (Sample Covariance Matrix) construit à partir de K m-vecteurs $\mathbf{c}_k$ de données secondaires :

$$\mathbf{M}_{SCM} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{c}_k \, \mathbf{c}_k^H \,. \tag{2.46}$$

Cet estimateur, la solution du Maximum de Vraisemblance du cas gaussien, distribué selon la loi de Wishart, n'est pourtant pas adapté au cas général des SIRV car sa construction fait intervenir l'information de texture qui biaise l'estimateur. Le GLRT-LQ adaptatif construit avec cet estimateur n'est donc pas texture-CFAR. Une solution pour s'affranchir de l'information de texture a été définie par Conte et Gini : un nouvel estimateur, la NSCM (Normalized Sample Covariance Matrix), construit sur l'estimateur empirique des données normalisées par leur puissance, s'est avéré invariant à cette texture :

$$\mathbf{M}_{NSCM} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{c}_k \, \mathbf{c}_k^H}{\mathbf{c}_k^H \, \mathbf{c}_k} \,. \tag{2.47}$$

Le détecteur $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{NSCM})$ est donc texture-CFAR. Malheureusement, il n'est pas matrice CFAR car l'estimateur NSCM est biaisé et non consistant **[J8]**.

L'encadrement du stage de DEA et de la thèse ONERA de Frédéric Pascal **[T3]** en 2003 (soutenue en 2006) se sont révélés très fructueux et furent encore un nouveau déclencheur pour mes recherches. La loi de la matrice de covariance NSCM estimée étant non connue à K fixé, la loi du GLRT-LQ adaptatif donnée par (Eq. 2.44) n'était pas très simple à établir. L'idée fut de se dire que, puisque le détecteur était texture-CFAR, une solution était de se placer dans le cas purement gaussien (cas où la loi de l'estimateur SCM du sens du Maximum de Vraisemblance est distribué selon la loi de Wishart). Frédéric Pascal a alors établi lors de son stage de DEA la loi de $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{SCM})$ conduisant à l'expression de la relation entre le seuil de détection λ et la probabilité de fausse alarme P_{fa} :

$$P_{fa} = (1-\lambda)^{a-1} \, {}_2F_1(a, a-1; b-1; \lambda) \,. \tag{2.48}$$

avec $a = K - m + 2$ et $b = K + 2$ et où ${}_2F_1$ est la fonction hypergéométrique.

Le détecteur $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{SCM})$ était donc matrice CFAR mais pas texture-CFAR.

Lors de ce travail de thèse, nos recherches se sont alors axées sur un autre type d'estimateur proposé dans la littérature : l'estimateur $\widehat{\mathbf{M}} = \mathbf{M}_{FP}$ du point fixe :

$$\mathbf{M}_{FP} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{c}_k \, \mathbf{c}_k^H}{\mathbf{c}_k^H \, \mathbf{M}_{FP}^{-1} \, \mathbf{c}_k} \, . \tag{2.49}$$

Cet estimateur, solution d'une équation implicite au point fixe, a été proposé par Conte et Gini. Notons, que cet estimateur est invariant par changement d'échelle des données $\mathbf{c}_k$. Ainsi, quelle que soit la puissance de ces données (texture) régissant les données secondaires, l'estimateur reste invariant et *ne s'occupe que du speckle*. Comme pour le GLRT-LQ, il peut s'obtenir également de différentes façons :

- **Amélioration empirique de l'approche NSCM :**

Connaissant $\mathbf{M}$, la meilleure estimation de la puissance n'est pas $\mathbf{c}_k^H \, \mathbf{c}_k$ mais $\mathbf{c}_k^H \, \mathbf{M}^{-1} \mathbf{c}_k$. L'estimateur empirique des données normalisées par cette nouvelle estimée de puissance nous conduit alors à l'estimateur du Point Fixe (Eq. 2.49),

- **Cas général de la texture aléatoire**

Dans le cas général des SIRV, on peut déterminer l'estimateur $\mathbf{M}_{MV}$ du Maximum de Vraisemblance de la matrice $\mathbf{M}$. Pour estimer la matrice de covariance $\mathbf{M}$ inconnue du processus SIRV, la fonction de vraisemblance des N vecteurs $\mathbf{c}_i$ supposés indépendants à maximiser est donnée par :

$$p_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M}) = \prod_{i=1}^{N} \int_0^{+\infty} \frac{1}{(\pi \, \tau_i)^m \, |\mathbf{M}|} \exp\left(-\frac{\mathbf{c}_i^H \, \mathbf{M}^{-1} \, \mathbf{c}_i}{\tau_i} \right) p_\tau(\tau_i) \, \mathrm{d}\tau_i \, . \tag{2.50}$$

En annulant le gradient de (Eq. 2.50) par rapport à $\mathbf{M}$, Gini et al. [Gini02] ont ainsi obtenu la solution du Maximum de Vraisemblance suivante :

$$\widehat{\mathbf{M}}_{MV} = \frac{1}{N} \sum_{i=1}^{N} \frac{h_{m+1}(\mathbf{c}_i^H \, \widehat{\mathbf{M}}_{MV}^{-1} \, \mathbf{c}_i)}{h_m(\mathbf{c}_i^H \, \widehat{\mathbf{M}}_{MV}^{-1} \, \mathbf{c}_i)} \, \mathbf{c}_i \mathbf{c}_i^H = \frac{1}{N} \sum_{i=1}^{N} c_m(\mathbf{c}_i^H \, \widehat{\mathbf{M}}_{MV}^{-1} \, \mathbf{c}_i) \, \mathbf{c}_i \mathbf{c}_i^H \, , \tag{2.51}$$

où $h_m(q) = \displaystyle\int_0^{+\infty} \tau^{-m} \exp(-q/\tau) \, p_\tau(\tau) \, \mathrm{d}\tau$ et $c_m(q) = \dfrac{h_{m+1}(q)}{h_m(q)}$. Cette fonction $c_m(.)$ a été représentée à la figure (Fig. 2.25) pour différentes fonctions h_m, en particulier, celles des lois de type K-distribution de paramètre ν. L'allure des courbes est la même, de type décroissante en $1/x$.

Après plusieurs approximations, Gini [Gini02] a obtenu un Maximum de Vraisemblance Approché $\widehat{\mathbf{M}}$ (AML pour Approximate Maximum Likelihood) solution de l'équation au point fixe $\widehat{\mathbf{M}} = f(\widehat{\mathbf{M}})$ avec f définie par :

$$f(\widehat{\mathbf{M}}) = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{c}_i \, \mathbf{c}_i^H}{\mathbf{c}_i^H \, \widehat{\mathbf{M}}^{-1} \, \mathbf{c}_i} \, . \tag{2.52}$$

Le choix particulier $c_m(x) = d_m(x) = m/x$ qui correspond en fait à utiliser le prior impropre de Jeffrey [Robe92] comme densité de la texture $p_\tau(.)$, conduit exactement à l'AML, solution de l'équation (Eq. 2.52). Cet AML a l'avantage de ne plus dépendre de h_m qui dépend de la loi de la texture.

- **Cas de la texture déterministe**

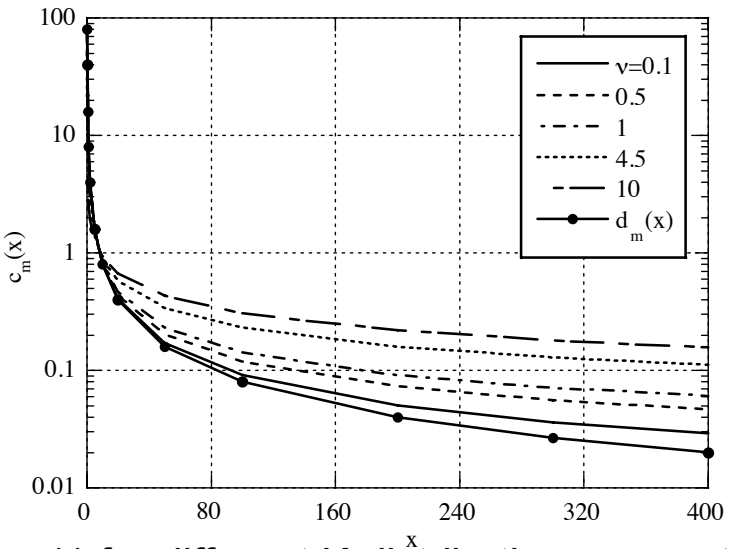


FIGURE 2.25 – Allure générale des fonctions $c_m(x) = \dfrac{h_{m+1}(x)}{h_m(x)}$ pour différentes lois K de paramètre ν. Comparaison avec le cas particulier $c_m(x) = d_m(x) = m/x$.

En supposant que le paramètre τ est déterministe inconnu, la solution du Maximum de Vraisemblance Généralisée pour l'estimation de $\mathbf{M}$ est exactement donnée l'estimateur du Point Fixe (Eq. 2.49). Cette approche très générale a été développée dans [Cont02] où les τ_i sont supposés être des paramètres inconnus. Conditionnellement aux τ_i, la densité des données est gaussienne. Pour des vecteurs $\mathbf{c}_i$ indépendants, la fonction de vraisemblance correspondante à maximiser en $\mathbf{M}$ et en τ_i est donc donnée par :

$$p_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M}, \tau_1, \ldots, \tau_N) = \frac{1}{(\pi)^{mN}\,|\mathbf{M}|^N} \prod_{i=1}^{N} \frac{1}{\tau_i^m} \exp\left(-\frac{\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i}{\tau_i}\right). \tag{2.53}$$

Maximiser cette fonctionnelle en en τ_i, pour un $\mathbf{M}$ fixé (mais inconnu), conduit à

$$\hat{\tau}_i = \frac{\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i}{m}. \tag{2.54}$$

En remplaçant les τ_i's dans (Eq. 2.53) par leur estimateur du Maximum de Vraisemblance $\hat{\tau}_i$'s défini par (Eq. 2.54), la fonction de vraisemblance réduite est donnée par :

$$\hat{p}_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M}) = \frac{1}{(\pi)^{mN}\,|\mathbf{M}|^N} \prod_{i=1}^{N} \frac{m^m\,\exp(-m)}{(\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i)^m}.$$

De ce fait, maximiser $\hat{p}_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M})$ en $\mathbf{M}$ revient à maximiser la fonction suivante F :

$$F(\mathbf{M}) = \frac{1}{|\mathbf{M}|^N} \prod_{i=1}^{N} \frac{1}{(\mathbf{c}_i^H\,\mathbf{M}^{-1}\,\mathbf{c}_i)^m}.$$

L'estimateur du Maximum de Vraisemblance de $\mathbf{M}$ est le $\widehat{\mathbf{M}}$ qui annule le gradient de F, ce qui revient encore à l'équation (Eq. 2.52) en $\widehat{\mathbf{M}}$ suivante :

$$\widehat{\mathbf{M}} = f(\widehat{\mathbf{M}}), \tag{2.55}$$

où la fonction $f(.)$ est donnée par (Eq. 2.52).

Les travaux de thèse de Frédéric Pascal **[T3]**, **[J9]** ont permis de démontrer un certain nombre de propriétés intéressantes, notamment dans la publication suivante, présentée en Annexe H :

[J9] F. Pascal, Y. Chitour, **J.P. Ovarlez**, P. Forster and P. Larzabal, "*Covariance Structure Maximum Likelihood Estimates in Compound Gaussian Noise : Existence and Algorithm Analysis*", IEEE Trans. on Signal Processing, Vol. 56, No.1, pp.34-48, Jan. 2008.

Frédéric Pascal a montré que l'estimateur du Point Fixe (Eq. 2.49), solution d'une équation implicite au point fixe, existait, était unique (à un facteur d'échelle près) et pouvait être très facilement atteinte par une approche récursive du type

$$\mathbf{M}_{n+1} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{c}_k \, \mathbf{c}_k^H}{\mathbf{c}_k^H \, \mathbf{M}_n^{-1} \, \mathbf{c}_k} \, . \tag{2.56}$$

quel que soit le point de départ $\mathbf{M}_0$. Le choix particulier et judicieux $\mathbf{M}_0 = \mathbf{I}$, nous donne, à la première itération, $\mathbf{M}_1 = \mathbf{M}_{NSCM}$, soit la matrice empirique des données normalisées NSCM rencontrée plus haut !

Cet estimateur possède en outre de remarquables propriétés statistiques **[J10]** :

(1) $\mathbf{M}_{FP}$ est un estimateur consistant de $\mathbf{M}$,
(2) $\mathbf{M}_{FP}$ est un estimateur non biaisé de $\mathbf{M}$,
(3) la distribution asymptotique de $\mathbf{M}_{FP}$ est gaussienne,
(4) cette distribution est la même que la loi asymptotique de Wishart à $\left(\frac{m}{m+1}\right) K$ degrés de liberté.

2.2.6 Propriétés SIRV-CFAR des Détecteurs

Le détecteur $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ possède des propriétés vraiment très intéressantes **[T3]**, **[C21]**, **[C22]**, **[C23]**, **[C25]**, **[C26]**, **[C31]**. La première est la propriété texture-CFAR illustrée à la figure (Fig. 2.26) qui confirme l'invariance du détecteur à la texture. La deuxième est la propriété matrice-CFAR illustrée à la figure (Fig. 2.27) pour différentes matrice de covariance (ici matrices de Toeplitz du type $\mathbf{M}_{i,j} = \rho^{|i-j|}$). Enfin, la loi du test a pu être établie et donne explicitement la valeur du seuil de détection λ nécessaire à la régulation de fausse alarme fixée par la probabilité de fausse alarme P_{fa} :

$$P_{fa} = (1-\lambda)^{a-1} \, {}_2F_1(a, a-1; b-1; \lambda) \, . \tag{2.57}$$

avec $a = \frac{m}{m+1} K - m + 2$ et $b = \frac{m}{m+1} K + 2$ et où ${}_2F_1$ est la fonction hypergéométrique.

Ces résultats sont démontrés dans l'article proposé en Annexe I :

[C31] F. Pascal, **J.P. Ovarlez**, P. Forster and P. Larzabal, "*On a SIRV-CFAR Detector With Radar Experimentations in Impulsive Noise*", Proc. EUSIPCO 2006, Firenze, Italy, Sep. 2006.

Pour maintenant illustrer la puissance de ce détecteur construit avec l'estimateur du point fixe, nous présentons les résultats issus de la régulation de la fausse alarme sur des données réelles provenant de THALES Air Defence [1] **[I30]**. La figure (Fig. 2.28) représente une carte "distance-azimut" d'échos de fouillis de sol collectés par un radar de TAD. Le radar, positionné à 13 mètres au-dessus du niveau de sol, éclaire la zone avec un angle rasant. Les échos du sol ont été collectés pour $N = 868$ cases distances, pour 70 angles d'azimut et pour $m = 8$ impulsions. Près du radar, les échos caractérisent un fouillis de sol hétérogène en puissance et non gaussien tandis qu'au delà de l'horizon radio-électrique du radar (environ 15 kilomètres), seul le bruit thermique gaussien est présent (partie de droite en bleue sur l'image).

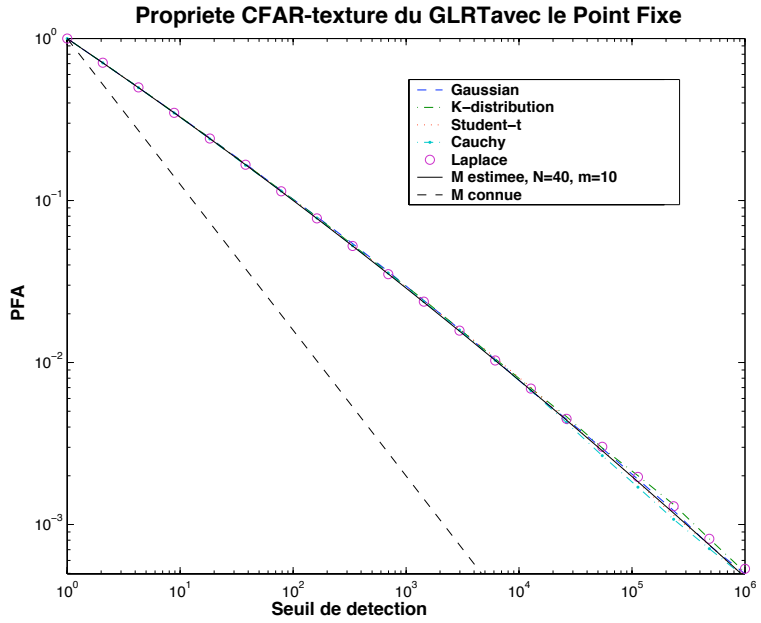


FIGURE 2.26 – Propriété CFAR-texture de $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$

Le seuil de détection est réalisé par comptage, en déplaçant un masque CFAR carré de dimension 5×5. Pour chaque case centrale $\mathbf{y}$ du masque (i.e. la case testée), la valeur de $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ est calculée. La matrice de covariance $\mathbf{M}_{FP}$ est estimée sur les $K = 24$ autres cases, considérés ainsi comme les données secondaires $\mathbf{c}_k$. Ce processus est réitéré pour chaque pixel sur la carte de fouillis, ici $868 \times 70 = 60760$. Le comptage est ainsi effectué et est présenté à la figure (Fig. 2.29). Notons que la courbe du GLRT-LQ construit avec l'estimateur NSCM, $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{NSCM})$ ne garantit pas la bonne régulation de fausse alarme puisqu'il n'est pas matrice CFAR. La courbe rouge du comptage des données effectué avec le GLRT-LQ construit avec le Point Fixe est en excellente adéquation avec la théorie. La courbe verte est la courbe idéale que l'on obtiendrait si la meilleure des estimations (cas $K \to \infty$) avait été réalisée (en fait, les performances pour $\mathbf{M}$ connue). Cet écart avec l'estimation idéale définit ce que l'on appelle en radar les *pertes CFAR* (CFAR loss).

Ce détecteur montre ici sa capacité de robustesse vis à vis de l'hétérogénéité de la zone sur laquelle sont collectées les données secondaires permettant d'estimer la matrice de covariance du fouillis.

Cette capacité de robustesse aux transitions de fouillis est encore ici renforcée. Des simulations ont été réalisées pour différentes zones de fouillis, allant du bruit gaussien à un fouillis K-distribué très impulsif. Pour chaque zone, le détecteur $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ est comparé à $\Lambda_{AMF}(\mathbf{y}, \mathbf{M}_{SCM})$. La figure (Fig. 2.30-a) présente l'étape de régulation de fausses alarmes. La taille des vecteurs est $m = 10$ et le nombre de données secondaires utilisées pour estimer la matrice de covariance est $K = 20$. L'axe des ordonnées correspond au seuil de détection, l'axe des abscisses représente l'axe spatial (80 cases distance) correspondant à différentes zones de fouillis : de 1 à 10, le bruit est gaussien pour ensuite être K-distribué avec les paramètres d'échelle ν suivants : de la case 10 à 30, $\nu = 2$, puis de 30 à 50, $\nu = 1$, de 50 à 70, $\nu = 0,5$, et enfin, de 70 à 80, $\nu = 0.1$. En fait, plus le paramètre d'échelle diminue, plus le fouillis devient impulsif.

On peut immédiatement remarquer que le détecteur $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$, possédant la propriété SIRV-CFAR, garantit la même relation "P_{fa}-seuil" dans toutes les zones de fouillis et est de ce fait robuste aux transitions. L'AMF a un comportement totalement différent : le seuil de détection doit être rehaussé quand le bruit devient plus impulsif pour garantir le même taux de fausses alarmes. La figure (Fig.

1. Je remercie THALES Air Defence pour l'exploitation de ses données

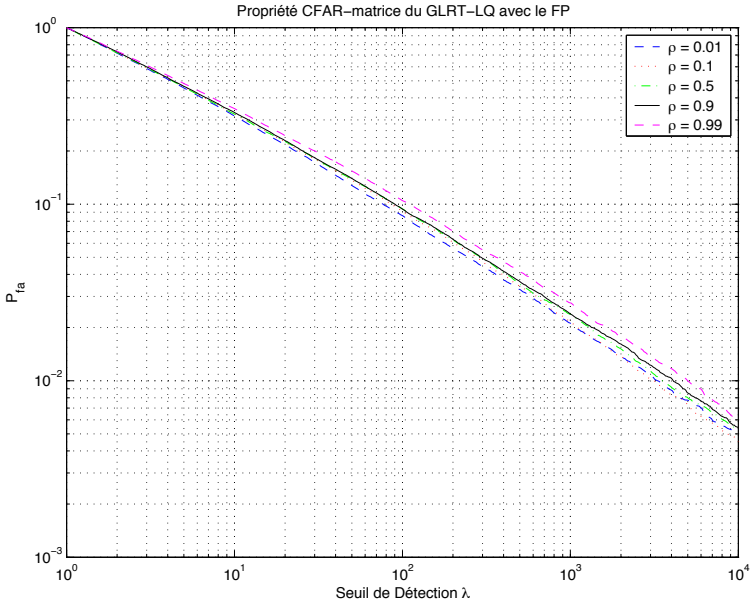


FIGURE 2.27 – Propriété CFAR-matrice de $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$

2.30-b) présente les performances de détection des deux détecteurs. L'axe des ordonnées correspond au rapport signal-à-bruit (RSB) tandis que l'axe des abscisses représente à nouveau les 80 cases distances. La probabilité de détection P_d pour une P_{fa} réglée à 10^{-3} est, elle aussi codée en couleur.

2.2.7 Apport de la Connaissance *a priori* sur la Structure de la Matrice de Covariance

Les techniques de détection présentées précédemment ont pu être étendues (thèse 2007-2009 de Guilhem Pailloux) pour répondre au problème crucial de la réduction du nombre de cases distance (réduction du nombre K de données secondaires) nécessaires à l'estimation de la matrice de covariance du fouillis. Nous nous sommes ainsi appuyés sur l'information *a priori* que l'on pouvait apporter à la matrice de covariance. En utilisant la propriété dite de persymétrie (symétrie par rapport à l'antidiagonale) de cette matrice que l'on rencontre en STAP latéral conventionnel, dans tout type de radar à période de répétition constante ou dans toute application d'antenne de type ULA, il a été possible, à nombre K de cases distance donné, d'augmenter le nombre de degrés de liberté, d'améliorer de ce fait l'estimation de la matrice de covariance et d'améliorer enfin les performances des détecteurs. Ceci a été rendu possible, grâce à la propriété de persymétrie caractérisant la matrice de covariance des signaux, par une transformation des données par une matrice unitaire $\mathbf{T}$ donnée par :

$$\mathbf{T} = \begin{cases} \dfrac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{m/2} & \mathbf{J}_{m/2} \\ i\,\mathbf{I}_{m/2} & -i\,\mathbf{J}_{m/2} \end{pmatrix} & \text{pour } m \text{ pair} \\ \\ \dfrac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{(m-1)/2} & 0 & \mathbf{J}_{(m-1)/2} \\ 0 & \sqrt{2} & 0 \\ i\,\mathbf{I}_{(m-1)/2} & 0 & -i\,\mathbf{J}_{(m-1)/2} \end{pmatrix} & \text{pour } m \text{ impair.} \end{cases} \tag{2.58}$$

où $\mathbf{I}_m$ et $\mathbf{J}_m$ sont respectivement la matrice unité de dimension m et la matrice de dimension m constituée de 1 sur l'antidiagonale. Cette transformation unitaire permet de réduire le nombre de composantes à estimer de la matrice de covariance. Elle nous a conduits à définir deux nouveaux estimateurs : la matrice $\mathbf{R}_{PSCM}$, estimateur au sens du Maximum de Vraisemblance du cas gaussien sous hypothèse de persymétrie (PSCM) et la matrice $\mathbf{R}_{PFP}$ du point fixe persymétrique (PFP), qui elle, n'est pas solution du Maximum

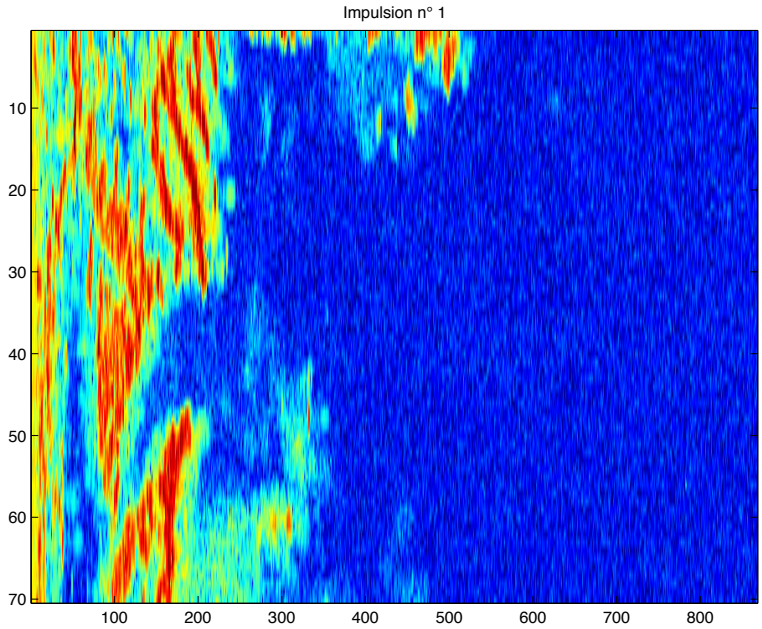


FIGURE 2.28 – Carte azimut-distance des échos radar de fouillis de sol

de Vraisemblance. Ces deux matrices sont définies comme :

$$\mathbf{R}_{PSCM} = \mathcal{R}e\left(\mathbf{T}\,\mathbf{M}_{SCM}\,\mathbf{T}^H\right) \quad \text{et} \quad \mathbf{R}_{PFP} = \mathcal{R}e\left(\mathbf{T}\,\mathbf{M}_{FP}\,\mathbf{T}^H\right) \tag{2.59}$$

et ont permis de définir différents type de détecteurs :

- le P-AMF, le *Persymmetric Adaptive Matched Filter*, qui est l'extension de l'AMF pour la matrice PSCM :

$$\Lambda_{P-AMF}(\mathbf{y}, \mathbf{R}_{PSCM}) = \frac{\left|\mathbf{p}^H\,\mathbf{T}^H\,\mathbf{R}_{PSCM}^{-1}\,\mathbf{T}\,\mathbf{y}\right|^2}{\mathbf{p}^H\,\mathbf{T}^H\,\mathbf{R}_{PSCM}^{-1}\,\mathbf{T}\,\mathbf{p}}\,, \tag{2.60}$$

  et dont la loi a pu être établie.

- le P-ANMF, le *Persymmetric Adaptive Normalized Matched Filter*, qui est l'extension du GLRT-LQ (ou ANMF) la matrice PFP :

$$\Lambda_{P-ANMF}(\mathbf{y}, \mathbf{R}_{PFP}) = \frac{\left|\mathbf{p}^H\,\mathbf{T}^H\,\mathbf{R}_{PFP}^{-1}\,\mathbf{T}\,\mathbf{y}\right|^2}{\left(\mathbf{y}^H\,\mathbf{T}^H\,\mathbf{R}_{PFP}^{-1}\,\mathbf{T}\,\mathbf{y}\right)\left(\mathbf{p}^H\,\mathbf{T}^H\,\mathbf{R}_{PFP}^{-1}\,\mathbf{T}\,\mathbf{p}\right)}\,, \tag{2.61}$$

À nombre de vecteurs secondaires donné, ces techniques améliorent encore les performances des détecteurs conventionnels de type AMF + SCM (nouveau détecteur P-AMF) ou les détecteurs de type GLRT-LQ+FP (nouveau détecteur P-ANMF). La publication suivante est présentée en Annexe J :

[**J13**] G. Pailloux, P. Forster, **J.P. Ovarlez** and F. Pascal, "*Persymmetric Adaptive Radar Detectors*", IEEE Trans. on AES, à paraître, 2010.

La figure (Fig. 2.31) donne un exemple de résultat obtenu sur les données de THALES de la figure (Fig. 2.28) pour $m = 8$ et $K = 8$, soit un nombre K de données secondaires égal à la dimension m de la matrice de covariance. En bleu foncé, sont présentées la courbe de fausses alarmes pour une covariance

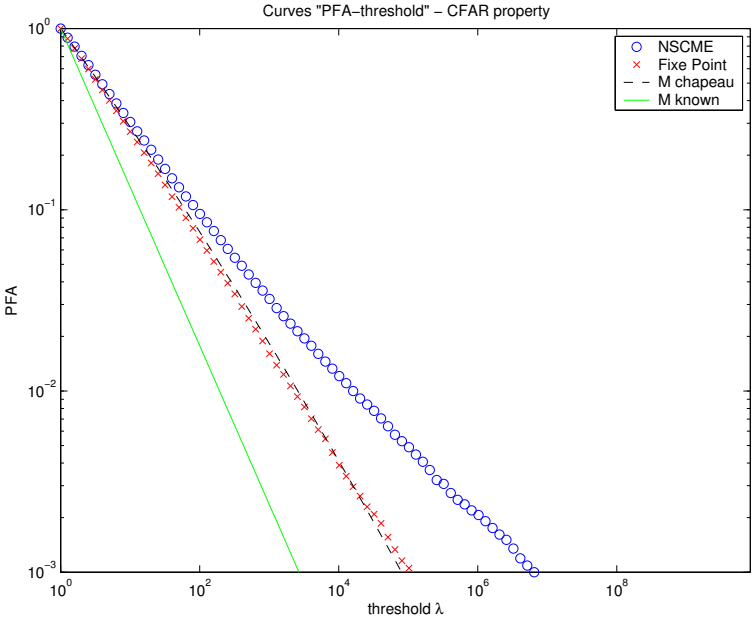


FIGURE 2.29 – Comparaison des relations "P_{fa}-seuil" pour le GLRT-LQ construit avec différents estimateurs. a) Comptage Monte-Carlo de $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{NSCM})$ en bleu b) Courbe théorique de l'équation (Eq. 2.57) en noir. c) Courbe théorique correspondant à l'idéal (équation (Eq. 2.42)) en vert. d) Comptage Monte-Carlo de $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ en rouge

théoriquement connue, en rose et rouge, les courbes de fausses alarmes théorique et estimée (simulation) du détecteur ANMF n'utilisant pas la propriété de persymétrie de la matrice, en noir et bleu clair, les performances apportées par l'information *a priori* de la persymétrie dans le détecteur P-ANMF. La figure (Fig. 2.32) donne le même genre d'exemple pour $m = 8$ et $K = 20$, soit un nombre K de données secondaires supérieur au double de la dimension m de la matrice de covariance. Sur chaque figure, sont représentés les gains de performance de détection. Il est clair qu'à un nombre de données secondaires K égal à la dimension m du vecteur de données (voir figure (Fig. 2.31)), les performances sont grandement améliorées.

La thèse proposée en 2007 à Guilhem Pailloux **[T5]** a contribué au développement de ces nouveaux algorithmes **[J13]**, **[C36]**, **[C37]**, **[C47]** et a permis également de pouvoir les appliquer au problème de la détection des cibles mobiles à partir de signaux aéroportés (STAP) que nous allons décrire dans le paragraphe suivant.

2.2.8 Application à la Problématique du STAP

La détection radar de cibles mobiles grâce à un capteur aéroporté de surveillance du sol (Ground Moving Target Indicator) ou grâce à une antenne de pointe avant est un sujet de recherche qui suscite un intérêt grandissant. À l'aide d'un réseau d'antennes se déplaçant sur une trajectoire, le principe du STAP (Space Time Adaptive Processing) est alors d'éliminer conjointement, par traitement temporel et spatial, la réponse des échos fixes du sol (fouillis) pouvant considérablement réduire les performances de détection d'une cible d'intérêt. Une analyse bibliographique montre que tous les algorithmes proposés sont majoritairement basés sur l'hypothèse d'un fouillis gaussien spatialement homogène. Leurs traitements consistent à *blanchir* les signaux collectés sur le réseau de capteurs en estimant classiquement la matrice de covariance du fouillis situé autour de la cible d'intérêt (SCM, Sample Covariance Matrix). Cette hypothèse simplificatrice n'est plus satisfaisante dès lors que la zone de fouillis n'est plus gaussienne (voire non homogène) et engendre naturellement une mauvaise régulation de la fausse alarme et une dégradation

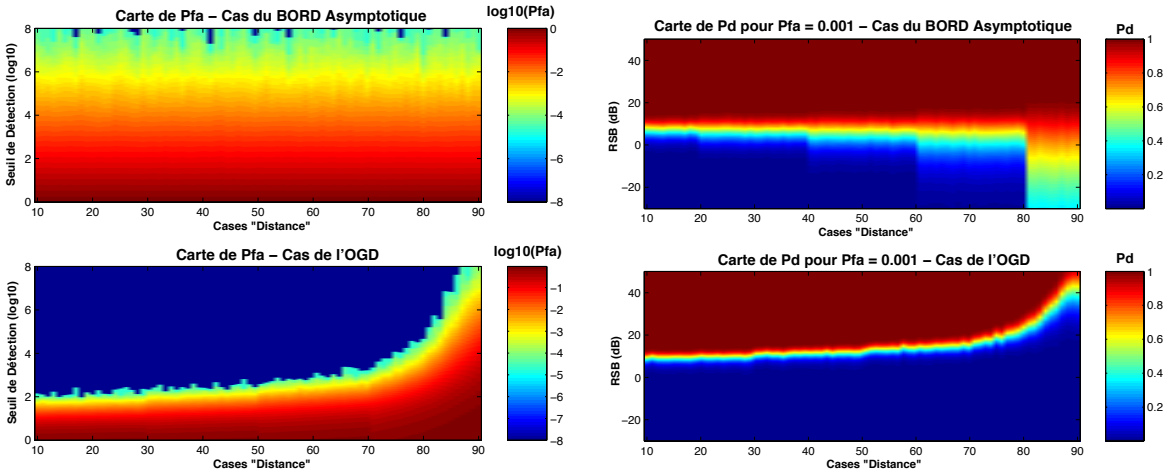


FIGURE 2.30 – Analyse des performances de régulation du taux de fausses alarmes et de détection pour l'AMF $\Lambda_{AMF}(\mathbf{y}, \mathbf{M}_{SCM})$ et le GLRT-LQ $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ dans 5 différentes zones de fouillis, allant du bruit gaussien à un bruit très impulsif. a) à gauche, régulation du taux de fausses alarmes. b) à droite, performances de détection.

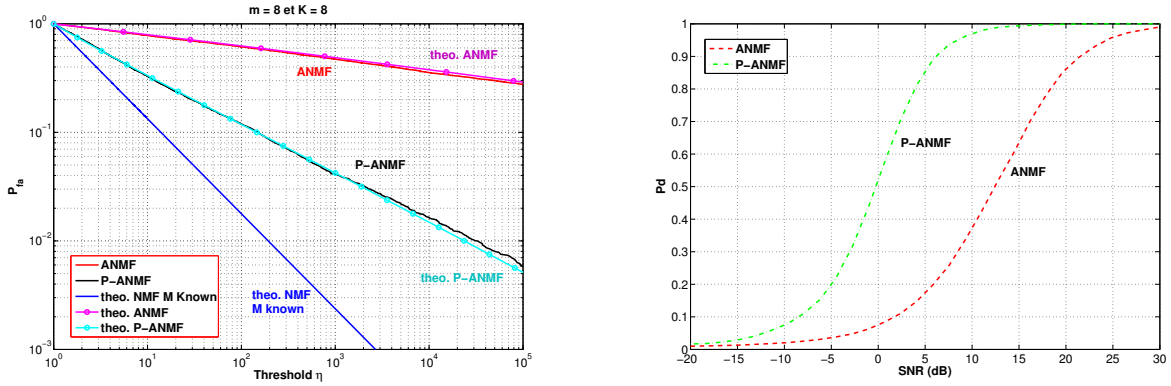
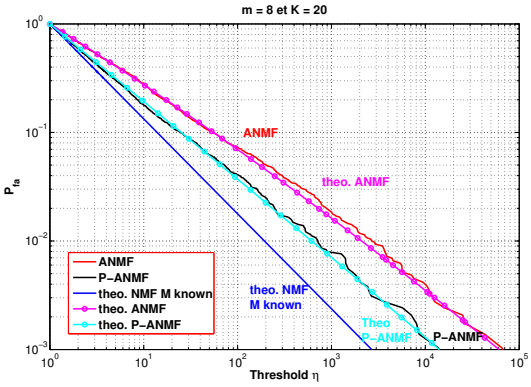


FIGURE 2.31 – Courbes de régulation de fausse alarme et de performance de détection sur les données de THALES (seuil de détection $\eta = (1-\lambda)^{-m}$ en fonction de la probabilité de fausse alarme) pour les détecteurs $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ et $\widehat{\Lambda}_{P-ANMF}(\mathbf{y}, \mathbf{R}_{PFP})$. Cas $P_{fa} = 10^{-2}$, $m = 8$ et $K = 8$

des performances de détection.

Les résultats précédemment présentés peuvent, dans la théorie générale de la détection, s'appliquer également au STAP. La formulation de la théorie du STAP dans la communauté radar est presque toujours présentée par la détermination d'un filtre $\mathbf{w}$ maximisant le rapport signal à bruit et on peut oublier que celle-ci peut également se comprendre comme un simple problème de détection adaptative toujours basée sur une estimation de matrice de covariance et un blanchiment. Cet approche est vraie également en MIMO, en détection polarimétrique, interférométrique, multi-bandes, multi-looks !

Le problème général inhérent au STAP est d'estimer cette matrice de covariance. Dans le plupart des cas, cette matrice est estimée en utilisant des données secondaires dans le domaine spatial des cases distance (mais on peut également trouver des estimations sous-optimales effectuées dans le domaine spectral, c'est le cas du post ou pré Doppler STAP). Pour correctement estimer cette covariance, il faut généralement un nombre de cases distance supérieur à la taille de la matrice (pour garantir l'inversibilité ce qui



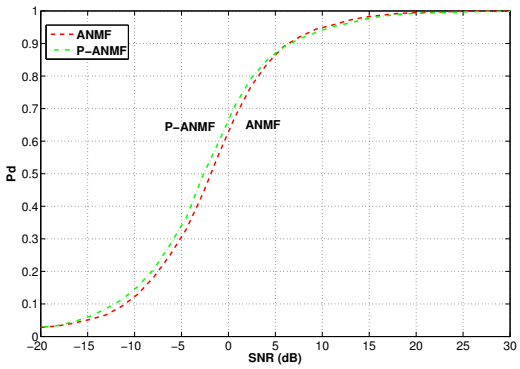


FIGURE 2.32 – Courbes de régulation de fausse alarme et de performance de détection sur les données de THALES (seuil de détection $\eta = (1-\lambda)^{-m}$ en fonction de la probabilité de fausse alarme) pour les détecteurs $\Lambda_{ANMF}(\mathbf{y}, \mathbf{M}_{FP})$ et $\Lambda_{P-ANMF}(\mathbf{y}, \mathbf{R}_{PFP})$. Cas $P_{fa} = 10^{-2}$, $m = 8$ et $K = 20$

n'est pas une condition nécessaire et suffisante), voire le double (règle de Brennan spécifiant une perte CFAR de moins de 3 dB). Ce nombre de vecteurs de données collectées dans le domaine spatial peut alors être important. Ceci conduit alors à s'interroger sur la pertinence de l'hypothèse d'homogénéité de la zone spatiale caractérisant ces données secondaires. On peut aussi réfléchir sur la manière de réduire ce nombre de données secondaires (en exploitant par exemple la structure de la matrice de covariance).

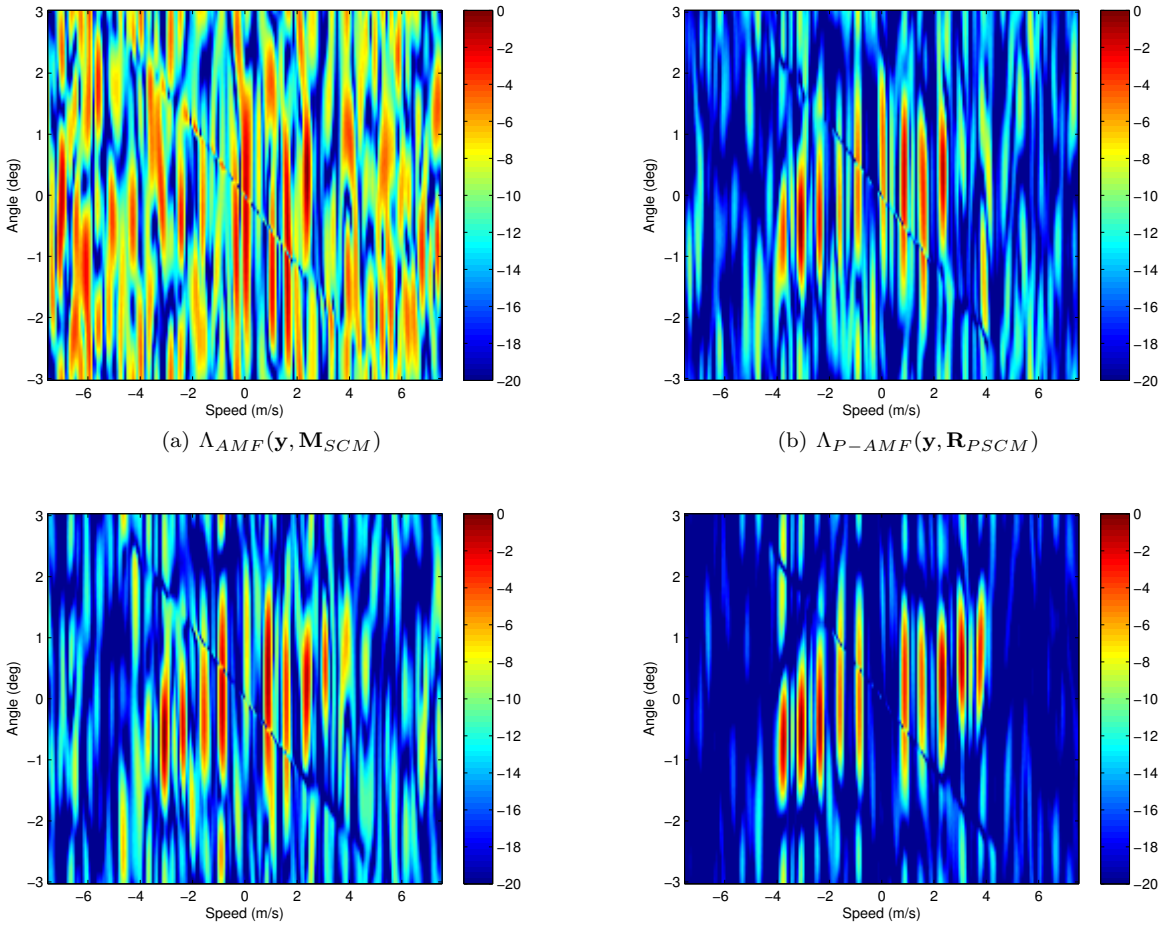
Les nouveaux détecteurs et processus d'estimation des matrices de covariance sur lesquels nous avons travaillé ont alors été étendus dans le cadre du traitement STAP et apportent, dans le cadre d'un fouillis non gaussien ou non homogène, une régulation fiable de la fausse alarme et une très nette amélioration des performances de détection par rapport à celles d'un traitement classique de type AMF. La thèse proposée à Guilhem Pailloux **[T5]** faisait partie de cet objectif **[J13]**, **[C36]**, **[C37**, **[C47]**.

2.2.8.1 Mise en Œuvre sur Données STAP

Ces travaux alimentent les échanges dans le club STAP que j'ai eu l'honneur de piloter depuis 2007 et qui a pour but de fédérer les activités de recherche autour de la thématique STAP menée par les laboratoires universitaires (ENS, ISAE, LSS, IETR, ...), les établissement étatiques (CELAR, ONERA,...) et industriels (THALES, ...). Le club STAP créé en janvier 2007 met à disposition des données synthétiques ou expérimentales provenant des différents partenaires et sur lesquelles peuvent être mis en œuvre les différents algorithmes développés par la communauté scientifique. Environ quatre réunions annuelles sont organisées. Le club STAP a été récemment étendu à la communauté scientifique européenne dans le cadre d'un appel à proposition de l'Agence de Défense Européenne (EDA) en juin 2009 initié par l'ONERA et THALES avec le partenariat d'un certain nombre de laboratoires universitaires. Une publication commune dans la revue française *Traitement du Signal* sur les travaux de groupe devrait d'ailleurs voir le jour avant la fin de cette année 2010.

La figure (Fig. 2.33) montre les résultats obtenus **[C47]** sur des données STAP synthétisées par le CELAR [2] (dans le cadre des échanges de données du club STAP) à partir d'images réelles SAR du capteur SETHI de l'ONERA. Dix cibles artificielles dans la même case distance mais à des vitesses différentes ont ainsi été ajoutées dans les données synthétisées. La figure (Fig. 2.34) présente la carte Doppler-distance d'un des capteurs (simple transformation de Fourier des données acquises au cours du temps sur un seul capteur) des données contenues dans la case distance.

2. Je remercie la DGA Maîtrise de l'information (ex CELAR) pour l'exploitation de ses données



(a) $\Lambda_{AMF}(\mathbf{y}, \mathbf{M}_{SCM})$

(b) $\Lambda_{P-AMF}(\mathbf{y}, \mathbf{R}_{PSCM})$

(c) $\Lambda(\mathbf{y}, \mathbf{M}_{FP})$

(d) $\Lambda_{P-ANMF}(\mathbf{y}, \mathbf{R}_{PFP})$

FIGURE 2.33 – Résultats de détection Doppler-azimut dans une case distance pour 10 cibles de vitesse différentes

2.2.8.2 STAP à Rang Réduit

Une manière de n'utiliser que peu de données secondaires est de faire appel aux techniques dites *de rang réduit* qui exploitent le fait que la matrice de covariance des données STAP possède des valeurs propres séparant l'espace signal (le fouillis) de l'espace bruit (bruit thermique). Connaissant la géométrie de visée, il est également possible de déterminer, à l'avance (règle de Brennan), quel sera le rang de chaque sous-espace. En projetant sur l'espace orthogonal au fouillis, on peut ainsi le rejeter. Cette projection nécessite alors au moins autant de données secondaires que la taille de l'espace fouillis, soit en fait nettement moins que la la taille de la matrice. Cette approche a été appliquée dans **[T5]** pour les détecteurs AMF construits avec les matrices de projection $\mathbf{\Pi}$ basées sur la covariance SCM et PSCM.

Les figures (Fig. 2.35) et (Fig. 2.36) présentent ainsi les résultats de détection (données CELAR) dans une case distance d'une cible dans la direction azimutale 0 deg et de vitesse 4m/s cible pour les deux détecteurs AMF et PAMF. La détection se fait premièrement avec un ensemble de $K = 200$ données secondaires de taille $m = 256$ bien au dessus du rang de Brennan du fouillis donné par $r = 46$. On s'assure ainsi une détection avec moins de 3 dB de pertes par rapport au traitement optimum. La persymétrie n'apporte dans ce cas que peu d'amélioration par rapport aux détecteurs classiques, les résultats du PAMF sont donc très similaires à ceux de l'AMF (cf. (Fig. 2.35)). La figure (Fig. 2.36) présente les

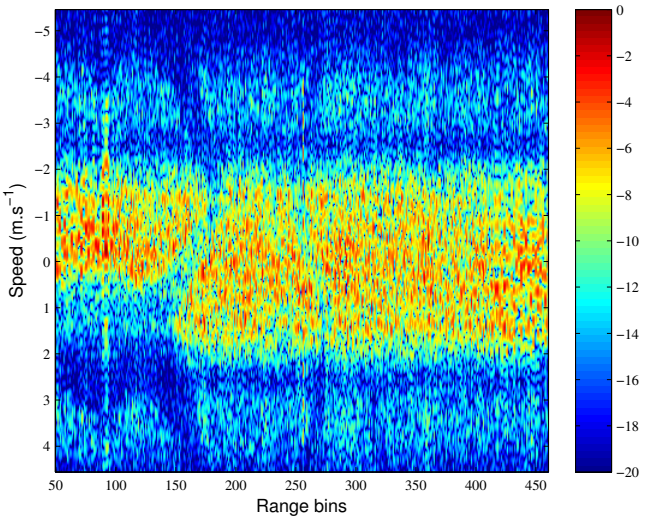


FIGURE 2.34 – Carte Doppler-distance d'un des capteurs

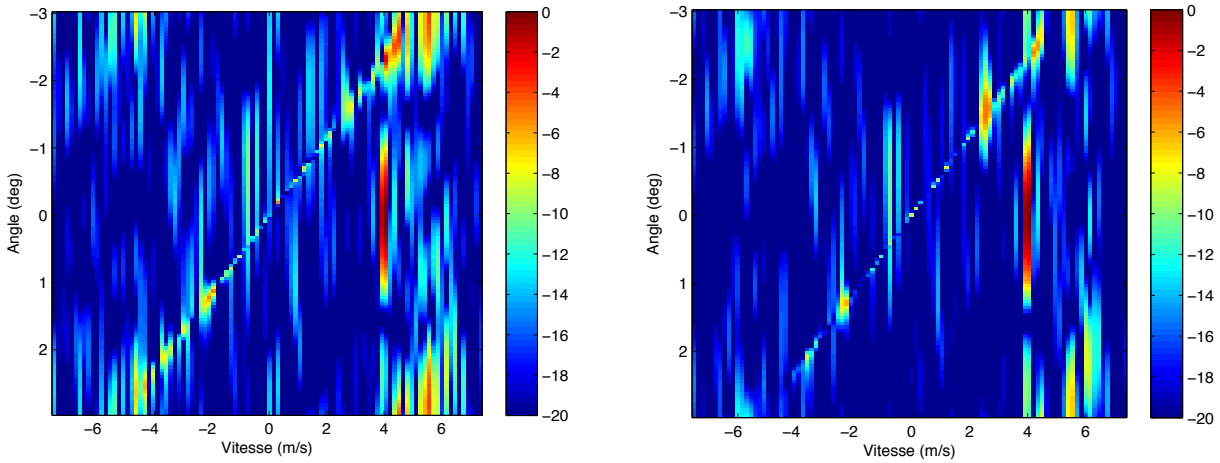


FIGURE 2.35 – Résultats de détection en rang réduit dans la case distance d'une cible de vitesse 4 m/s. Cas des détecteurs gaussiens AMF (à gauche) et PAMF (à droite) avec $K = 200$ et $r = 46$

résultats de détection pour les même détecteurs mais avec seulement $K = 46$ données secondaires ce qui correspond donc au rang r même du clutter. Dans ce cas, la SCM est mal estimée et le fouillis est donc mal rejeté par l'AMF. Dans le cas du PAMF, le doublement virtuel du nombre de données secondaires fait que le faible nombre de données secondaires d'origine est suffisant pour une bonne estimation de la matrice PSCM. Le fouillis est correctement rejeté et la cible, bien visible, est parfaitement détectée. Enfin, la figure (Fig. 2.37) montre que, même dans le cas d'un nombre de données secondaires $K = 400$ très supérieur à $r = 46$, l'emploi de la persymétrie avec le PAMF améliore encore les résultats en comparaison avec l'AMF.

En ce qui concerne le cas des détecteurs non-gaussiens, nous fûmes confrontés au problème de comment estimer le sous-espace fouillis à partir des matrices NSCM ou FP des données STAP. Un des résultats assez surprenant est que la construction de la matrice NSCM garantit la conservation du sous-espace fouillis ([**J8**]. Plus encore, elle assure l'ordre des valeurs propres liées au fouillis. Ces propriétés n'ont pas encore été démontrées pour la matrice du point fixe.

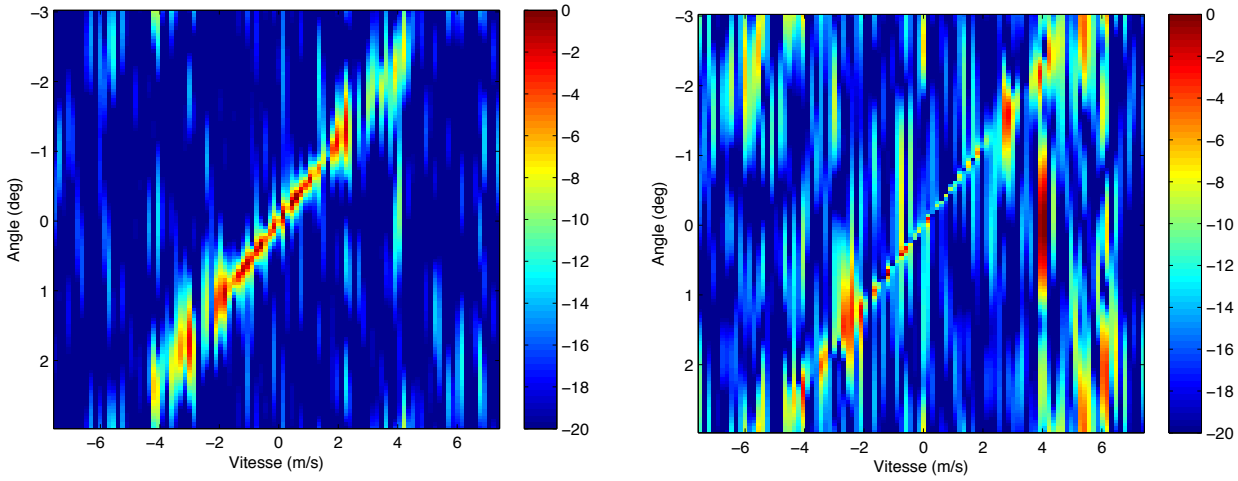


FIGURE 2.36 – Résultats de détection en rang réduit dans la case distance d'une cible de vitesse 4 m/s. Cas des détecteurs gaussiens AMF (à gauche) et PAMF (à droite) avec $K = 46$ et $r = 46$

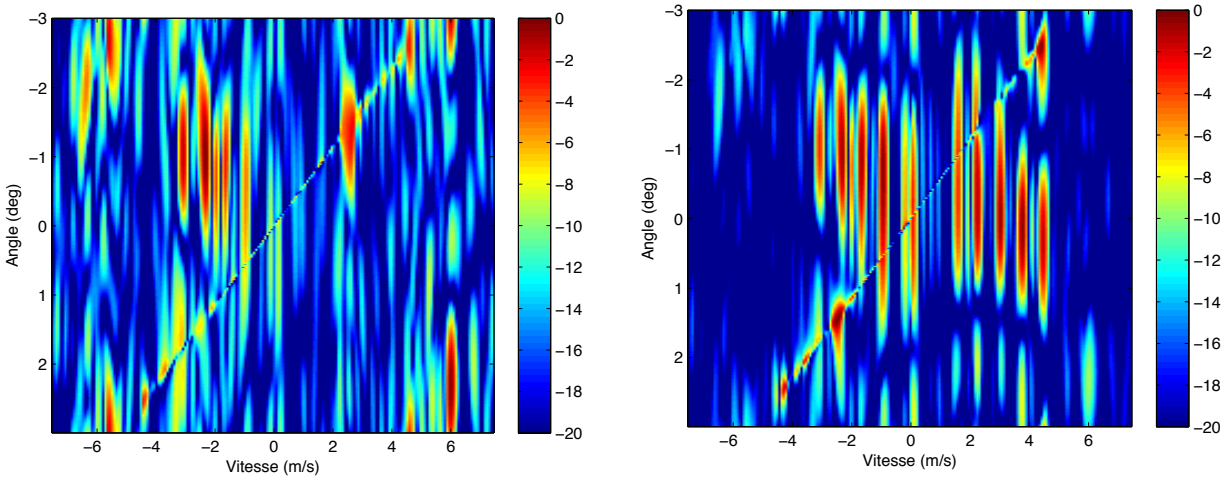


FIGURE 2.37 – Résultats de détection STAP en rang réduit dans la case distance de 10 cibles de vitesses et azimuts différents. Cas des détecteurs gaussiens AMF (à gauche) et PAMF (à droite) avec $K = 400$ et $r = 46$

Ces travaux ([**J20**], présentés dans l'Annexe K,

[**C41**] G. Ginolhac, P. Forster, **J.P. Ovarlez**, and F. Pascal, "*Spatio-Temporal Adaptive Detector in Non-Homogeneous and Low-Rank Clutter*", Proc. IEEE-ICASSP09, Taipei, Taiwan, Apr. 2009,

ont montré également que, même si la matrice NSCM, n'assurait pas le caractère SIRV-CFAR, son projecteur sur le sous espace fouillis, lui, le garantissait (asymptotiquement pourtant pour le caractère matrice-CFAR). Nous avons également constaté une propriété très intéressante qui fait actuellement l'objet de perspectives de recherche : la robustesse de cet estimateur. Si les données secondaires contiennent des brouilleurs, d'autres cibles, des ponctuels de fouillis forts (pylônes, ...), les détecteurs construits avec les matrices NSCM ou FP n'y sont pas sensibles. La figure (Fig. 2.38) illustre d'une part les performances de détection STAP avec ces techniques de rang réduit respectivement sur la matrice SCM (détecteur LR-AMF-SCM) et NSCM (détecteur LR-AMF-NSCM) mais également leur robustesse. Dans les données

secondaires collectées dans les cases distance adjacentes à la case distance de test, deux cibles fortes (dont l'une est de même vitesse, même azimut que la cible à détecter) sont présentes dans les données secondaires. Lorsqu'on blanchit par les techniques conventionnelles de type SCM, la matrice de covariance *apprend* donc ces cibles et le détecteur les blanchit. Le fait d'utiliser la matrice SCM des données normalisées permet de s'affranchir du fort niveau potentiel des données aberrantes, *ouliers* ou encore cibles secondaires dans les données secondaires.

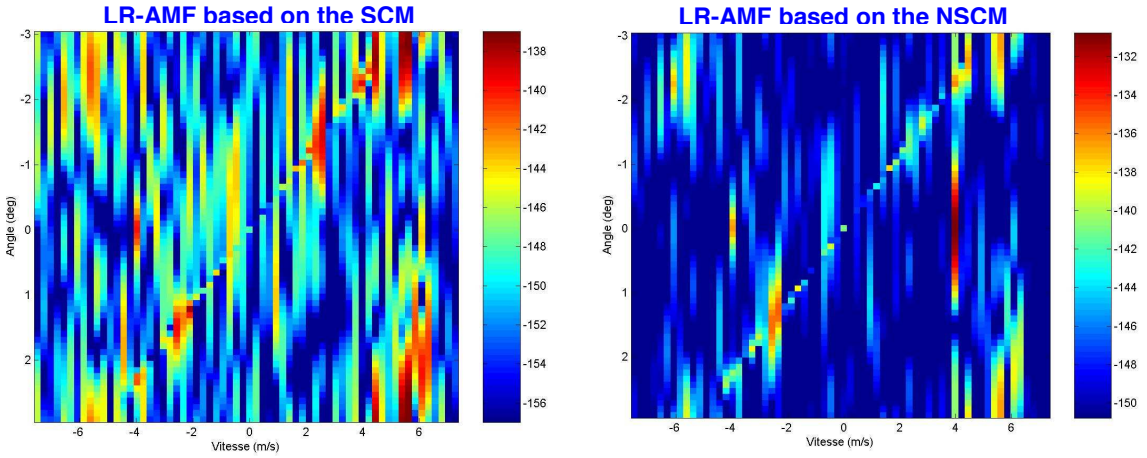


FIGURE 2.38 – Résultats de détection STAP en rang réduit dans la case distance d'une cible de vitesse 4m/s. Les données secondaires contiennent deux cibles possédant des vitesses de 4m/s et -4m/s. Cas des détecteurs gaussiens LR-AMF-SCM (à gauche) et LR-AMF-NSCM (à droite) avec $K = 410$ et $r = 46$

2.2.9 Robustesse des Estimateurs

Ces nouveaux détecteurs sont dits robustes car ils ne perdent pas leurs propriétés CFAR lorsque l'on s'écarte légèrement des hypothèses sous lesquelles ils ont été construits : robustesse à l'hétérogénéité spatiale et la non-gaussianité du fouillis, robustesse aux "*outliers*" c'est à dire à la présence éventuelles de cibles et de brouilleurs dans les cases adjacentes à la case de test. Le stage de Master et la nouvelle thèse de Mélanie Mahot ([T8] sur financement DGA 2009-2012) ont eu et auront ainsi pour but d'étudier ces phénomènes de robustesse et de non-stationnarité. Le stage de Mélanie Mahot a pu mettre en évidence l'intérêt d'emploi de la matrice FP du point fixe (même en environnement gaussien) de par son incroyable robustesse aux données aberrantes dans les algorithmes de type Music ou Esprit ou dans la mise en œuvre des détecteurs radar [C58].

Les données secondaires sont supposées toujours respecter le modèle, c'est à dire que les données secondaires dans le cas de la détection contiennent toujours un bruit respectant les hypothèses de départ. On considère ici le problème d'estimation de la matrice de covariance du processus étant donné N observations secondaires indépendantes $\{\mathbf{y}_k\}_{k=1..N}$ contenant du bruit et quelques perturbations additives. Quand K observations $\{\mathbf{y}_k\}_{k=1..K}$ contiennent de fortes perturbations déterministes $\{\mathbf{p}_k\}_{k=1..K}$ et un bruit additif $\{\mathbf{c}_k\}_{k=1..K}$, on peut analyser le biais entre l'estimation de la matrice de covariance contaminée et l'estimation de la matrice non contaminée. Supposons ici, pour des raisons de compréhension, que $||\mathbf{p}_k|| >> ||\mathbf{c}_k||$ et que les $\{\mathbf{c}_k\}$ soient gaussiens.

Analyse du biais de la SCM en présence de perturbations :

La différence Δ_{SCM} entre l'espérance statistique de la SCM contaminée et l'espérance statistique

$E[\widehat{\mathbf{M}}_{\text{SCM}}] = \mathbf{M}$ de la SCM non contaminée, soit le biais dû à la contamination, s'écrit :

$$\Delta_{SCM} = \frac{m}{N} \sum_{k=1}^{K} \mathbf{p}_k \, \mathbf{p}_k^H - \frac{K}{N} \mathbf{M} \tag{2.62}$$

On peut noter que plus la norme $||\mathbf{p}_k||$ des perturbations est forte, plus le biais est important. La matrice estimée $\widehat{\mathbf{M}}_{\text{SCM}}$ n'est donc pas robuste.

Analyse du biais de la NSCM en présence de perturbations :

La différence Δ_{NSCM} entre l'espérance statistique de la NSCM contaminée et l'espérance statistique $E[\widehat{\mathbf{M}}_{\text{NSCM}}]$ de la NSCM non contaminée ($\neq \mathbf{M}$ car l'estimateur NSCM est biaisé), soit le biais dû à la contamination, s'écrit :

$$\Delta_{NSCM} = -\frac{K}{N} E[\widehat{\mathbf{M}}_{\text{NSCM}}] + \frac{m}{N} \sum_{k=1}^{K} \frac{\mathbf{p}_k \, \mathbf{p}_k^H}{\mathbf{p}_k^H \, \mathbf{p}_k}. \tag{2.63}$$

Analyse du biais de la FP en présence de perturbations :

Dans ce cas, le biais $\Delta_{FP} = E[\widehat{\mathbf{M}}_{\text{FP}}] - \mathbf{M}$ de la matrice du point fixe contaminée s'écrit

$$\Delta_{FP} = \frac{m+1}{N} \left(\sum_{k=1}^{K} \frac{\mathbf{p}_k \, \mathbf{p}_k^H}{\mathbf{p}_k^H \, \mathbf{M}^{-1} \, \mathbf{p}_k} - \frac{K}{m} \mathbf{M} \right) \tag{2.64}$$

Dans les deux derniers cas, on peut remarquer que la force $||\mathbf{p}_k||$ de la contamination n'influe pas dans le calcul de ces biais. Tout facteur d'échelle scalaire sur les vecteurs $\{\mathbf{p}_k\}$ laisse le bais invariant. Les estimateurs NCCM et FP sont donc robustes à la présence de perturbations déterministes.

La figure (Fig. 2.39) montre l'erreur relative que l'on commet pour l'estimateur classique SCM et l'estimateur robuste du point fixe lorsqu'on estime la covariance avec des données de référence contaminées (en nombre et en puissance). La figure (Fig. 2.40) donne une comparaison des performances entre une technique conventionnelle et une technique robuste pour une seule cellule contaminée dans les données de référence contenant une cible très forte de 20 dB. L'axe des abscisses définit la puissance des contaminations et l'axe des ordonnées définit le nombre de cellules contaminées (sur 200 cases).

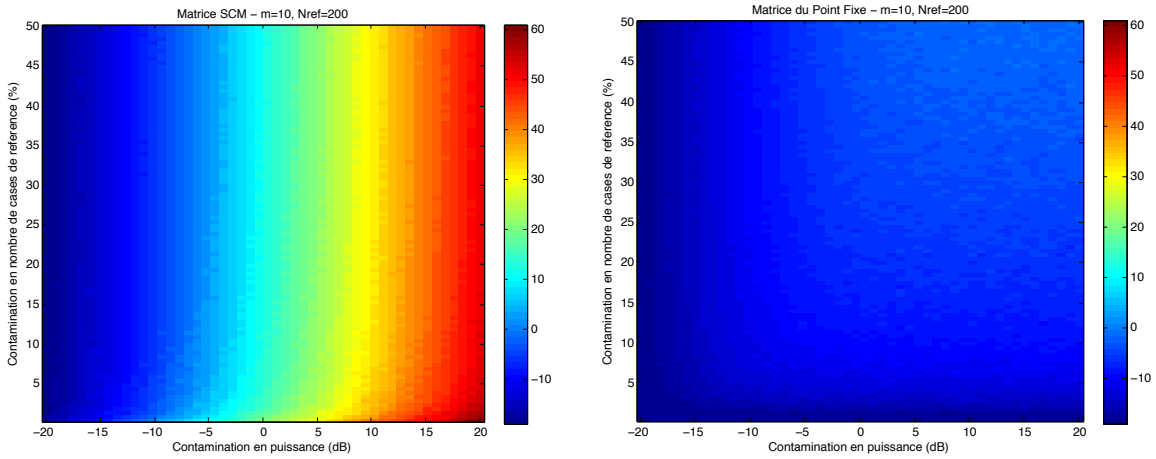


FIGURE 2.39 – Erreur relative (en dB) entre la matrice de covariance estimée non contaminée et la matrice de covariance estimée contaminée. a) à gauche, matrice SCM conventionnelle. b) à droite, matrice du point fixe.

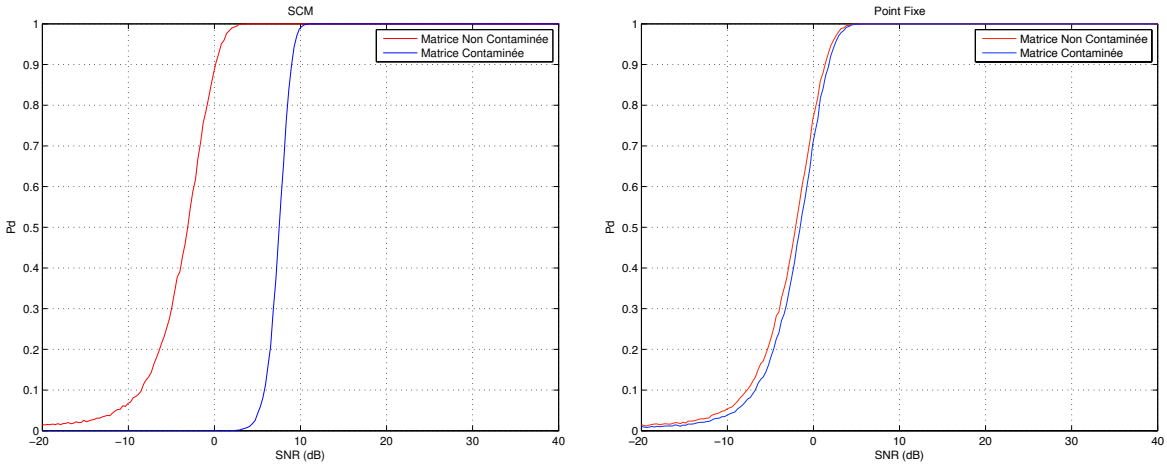


FIGURE 2.40 – Performances de détection (Pfa=0.01) d'une cible pour une cellule de référence fortement contaminée (20 dB) ou pas. a) à gauche, performances de l'AMF avec SCM biaisée ou pas par la contamination. b) à droite, performances de l'ANMF avec l'estimateur robuste du point fixe

Les recherches menées actuellement portent sur la prise en compte de la non-stationnarité de case distance à case distance de cette matrice de covariance que l'on rencontre en STAP pointe avant par exemple ou sur du fouillis dynamique comme la mer.

2.2.10 Détecteurs SIRV et MIMO Radar

Ma participation (non officielle) à l'encadrement de la thèse menée à SONDRA par Chin Yuan Chong [T6]) sous la direction de Frédéric Pascal et Marc Lesturgie nous a permis d'étendre, une fois de plus, l'utilisation ces détecteurs SIRV au problème radar du Multiple Input, Multiple Output (MIMO) qui consiste à exploiter la diversité de fréquence (émission spectrale des codes radar) ou temporelle, d'espace (réseau d'émetteurs et de récepteurs délocalisés), de la rétrodiffusion de la cible existant dans un réseau distribué d'émetteurs et récepteurs radar. Ces travaux, qui apportent un peu de sang neuf dans le problème presque toujours gaussien du MIMO radar ont fait l'objet de publications récentes ([J14], [C49], [C53], [C57]), montrent également, du fait de leur robustesse, un gain significatif de performance mais également en terme de régulation de fausse alarme par rapport aux techniques classiques dès lors que l'environnement est hétérogène ou que les données secondaires sont polluées.

Le MIMO radar est généralement caractérisé par $\tilde{N}$ émetteurs radar et $\tilde{M}$ récepteurs radar. Le nombre d'émetteurs du n-ième radar est noté N_n et le nombre de récepteurs du m-ième radar est noté M_m, pour $n = 1, \ldots, \tilde{N}$ et $m = 1, \ldots, \tilde{M}$. La SER de la cible est supposée différente mais déterministe pour chaque couple émetteur-récepteur. En outre, elle est supposée constante pour chaque récepteur radar (hypothèse de cible cohérente sur le réseau de réception). La matrice $\mathbf{B}$, de taille $\tilde{M} \times \tilde{N}$, contient les amplitudes de la cible pour tous les couple émetteurs-récepteurs. Le *steering vector* $\mathbf{p}_{m,n}$, de taille $M_m N_n \times 1$ et fonction de la géométrie du problème, caractérise la réponse bistatique angulaire théorique de la cible à détecter. Le problème général du détecteur MIMO peut encore simplement se formaliser par un test binaire d'hypothèses du type :

$$\begin{cases} \mathrm{H}_0 : & \mathbf{y}_{m,n} = \mathbf{z}_{m,n} \\ \mathrm{H}_1 : & \mathbf{y}_{m,n} = \mathbf{B}(m,n)\,\mathbf{p}_{m,n} + \mathbf{z}_{m,n} \end{cases} \qquad n = 1, \ldots, \tilde{N},\ m = 1, \ldots, \tilde{M}$$

où $\mathbf{z}_{m,n}$ est le vecteur $M_m N_n \times 1$ contenant les échos de fouillis modélisé ici par un processus SIRV.

Le test de détection devient alors :

$$\Lambda(\mathbf{Y}) = \prod_{m,n} \left[\frac{1}{1 - \frac{\left|\mathbf{p}_{m,n}^H \mathbf{M}_{m,n}^{-1} \mathbf{y}_{m,n}\right|^2}{\left(\mathbf{p}_{m,n}^H \mathbf{M}_{m,n}^{-1} \mathbf{p}_{m,n}\right)\left(\mathbf{y}_{m,n}^H \mathbf{M}_{m,n}^{-1} \mathbf{y}_{m,n}\right)}} \right]^{M_m N_n} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta, \tag{2.65}$$

où $\mathbf{Y}$ la matrice formée par les vecteurs $[\mathbf{y}_{1,1}, \cdots, \mathbf{y}_{\tilde{M},\tilde{N}}]$ des données reçues.

Dans le cas où les matrices $\mathbf{M}_{m,n}$ sont connues et dans le cas où le produit $L = \tilde{M} \times \tilde{N}$ est constant pour tous m et n, on peut exprimer analytiquement la relation liant la probabilité de fausse alarme P_{fa} et le seuil de détection η :

$$P_{fa} = \eta^{-1+1/L} \sum_{k=0}^{K-1} \frac{(L-1)^k}{k!} \left(\frac{\ln \eta}{L}\right)^k. \tag{2.66}$$

Dans le cas où les matrices $\mathbf{M}_{m,n}$ ne sont pas connues, il est possible de les estimer grâce à l'estimateur $\widehat{\mathbf{M}}_{m,n}^{FP}$ du point fixe construit sur L_r données secondaires de fouillis :

$$\widehat{\mathbf{M}}_{m,n}^{FP} = \frac{L}{L_r} \sum_{l=1}^{L_r} \frac{\mathbf{y}_{m,n}(l)\, \mathbf{y}_{m,n}(l)^H}{\mathbf{y}_{m,n}(l)^H \left(\widehat{\mathbf{M}}_{m,n}^{FP}\right)^{-1} \mathbf{y}_{m,n}(l)}. \tag{2.67}$$

Malheureusement, même pour le cas $L = \tilde{M} \times \tilde{N}$ constant, il est très difficile d'exprimer la loi de ce nouveau test et les méthodes de type Monte-Carlo deviennent nécessaires. La publication suivante, proposée en Annexe L, présente de très bons résultats de simulation pour différentes configurations MIMO en environnement hétérogène :

[J14] C.Y. Chong, F. Pascal, **J.P. Ovarlez** and M. Lesturgie, "*MIMO Radar Detection in non-Gaussian and Heterogeneous Clutter*", IEEE Journal of Selected Topics in Signal Processing, Special Issue on MIMO Radar and its Applications, Vol.4, No.1, pp.115-126, Feb. 2010.

Des résultats sont également présentés dans le cas conventionnel purement gaussien et montrent que ces nouveaux détecteurs ont le même type de performances que les détecteurs conventionnels. Cette extension de ces détecteurs SIRV permet alors de pouvoir se placer dans un environnement de fouillis plus complexe, tout en garantissant les mêmes performances que dans le cas conventionnel.

2.3 Classification des Images SAR

Cette thématique est directement issue des idées développées dans le cadre des travaux de thèses menés sur l'estimateur du Point Fixe de la matrice de covariance. La plupart des techniques usuelles de classification des fonds et des sols dans les images SAR utilisent, presque toutes, des classifications de type Wishart et donc basées sur l'hypothèse gaussienne du fouillis. Ce constat fût encore à l'origine du développement de ces nouvelles idées.

La texture d'une image peut être définie comme la variation locale spatiale du niveau d'intensité d'une image. L'analyse de cette texture permet, à une certaine échelle, de caractériser l'image en zones homogènes nécessaires, par exemple, à la mise en œuvre d'un processus de détection et/ou d'une classification. Contrairement à une image en niveau de gris ou à une image en couleur, chaque pixel d'une image provenant d'un capteur SAR (Synthetic Aperture Radar) est caractérisé par une amplitude et une phase. Ces images sont généralement multi-voies (utilisation de canaux de polarisation, interférométrie), multi-vues (même scène pour différents angles de vue) ou multi-spectrales (même scène analysée pour plusieurs bandes de fréquence). Par conséquent, les processus d'analyse de l'image deviennent de plus en plus complexes mais font généralement appel à des outils de classification basés sur des hypothèses

de type gaussien. Ainsi, dans la littérature, la plupart des travaux conduisent aux classifieurs classiques (classification de type H-α-A) construits sur des estimateurs de matrices de covariances conventionnelles $[\mathbf{T}]$ de type Wishart (estimation au sens du Maximum de Vraisemblance de la matrice de covariance à partir de données supposées gaussiennes).

Supposons que l'image soit ici polarimétrique (mais ce type de raisonnement pourrait également s'appliquer sur des données interférométriques, polarimétriques et interférométriques, hyperspectrales,...). Le vecteur d'information polarimétrique associé à chaque pixel de l'image SAR est donc un vecteur, noté $\mathbf{k}$, de dimension $m = 4$ ou $m = 3$ contenant la valeur du pixel de l'image correspondant au signal rétrodiffusé pour les voies polarimétriques HH, HV, VH et VV (le cas $m = 3$ correspond au principe de réciprocité qui impose que le signal reçu sur HV soit le même que celui reçu sur VH). Ces vecteurs sont généralement modélisés comme un processus multivarié gaussien de matrice de covariance $[\mathbf{C}]$ (base de Sinclair). Dans la base dite de Pauli (HH+VV, HH-VV, 2HV), cette matrice que l'on notera $[\mathbf{T}]$ est appelée matrice de cohérence polarimétrique. Il existe bien entendu une matrice unitaire de passage entre la base de Sinclair et celle de Pauli. Ces processus gaussiens sont définis par leur densité de probabilité :

$$p_m(\mathbf{k}) = \frac{1}{\pi^m \, |[\mathbf{T}]|} \exp\left(-\mathbf{k}^H \, [\mathbf{T}]^{-1} \, \mathbf{k}\right), \tag{2.68}$$

où $[\mathbf{T}] = \mathbb{E}\left[\mathbf{k}\,\mathbf{k}^H\right]$ est généralement estimé par la SCM $[\widehat{\mathbf{T}}] = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} \mathbf{k}_i \, \mathbf{k}_i^H$.

La classification non supervisée H/α/A a été présentée dans la section 2.1.2.5. Cette technique est basée principalement sur l'estimation de la matrice de covariance sur l'axe spatial de l'image. La distance associée à la distribution de cette estimée (loi de Wishart) est principalement basée sur l'hypothèse de distribution gaussienne. Cette distance d_W entre une matrice de covariance $[\widehat{\mathbf{T}}]$ et un centre de classe $[\widehat{\mathbf{T}}]_\omega$ peut s'exprimer de la façon suivante :

$$d_W\left([\widehat{\mathbf{T}}], [\widehat{\mathbf{T}}]_\omega\right) = \ln \frac{\left|[\widehat{\mathbf{T}}]_\omega\right|}{\left|[\widehat{\mathbf{T}}]\right|} + \mathrm{Tr}\left([\widehat{\mathbf{T}}]\,[\widehat{\mathbf{T}}]_\omega^{-1}\right) \tag{2.69}$$

Une analyse approfondie de ces techniques montre que leurs performances se dégradent de manière significative sur des zones hétérogènes, non-gaussiennes et/ou texturées. C'est le cas pour des images SAR Très Haute Résolution provenant des dernières générations de capteurs ou pour des images hautement texturées (zones urbaines par exemple). Des outils de classification robustes à ces différents scénarii ont donc été développés lors de l'encadrement d'un travail du post-doc de Gabriel Vasile **[J12]**, **[C38]**, **[C39]**, **[C45**, **[C46]** sur financement CNES et mené à l'ONERA. Ces outils sont encore ici basés sur une généralisation de la statistique gaussienne pour la modélisation des données : les processus aléatoires gaussiens composés (processus SIRV sphériquement invariants, processus SIRV généralisés). Ces processus (loi K, Weibull, etc) sont basés sur une modélisation multidimensionnelle gaussienne définie par une matrice de covariance et caractérisant le speckle des images dont la puissance (appelée texture) est elle-même aléatoire, caractérisant de ce fait les fluctuations de puissance de pixel à pixel. Cette analyse a conduit à développer un estimateur performant de la cohérence polarimétrique, c'est à dire une estimation robuste de la matrice de covariance du bruit de speckle, seule grandeur physique à contenir les informations polarimétriques, insensible à la fluctuation de texture. Cet estimateur est appelé cohérence polarimétrique normalisée du point fixe et est noté $[\widehat{\mathbf{M}}]_{FP}$.

Cette nouvelle approche étend alors les techniques déjà utilisées dans la littérature comme celles basées sur le Polarimetric Whitening Filter (PWF) de Lee [Lee91], [Lee94] par exemple bien connu dans la communauté du *despeckeling* pour filtrer le speckle dans les images SAR et augmenter de ce fait leur contraste. Un nouvel estimateur $\widehat{P}_{PWF}$ du SPAN polarimétrique (puissance totale des voies polarimétriques) filtré du bruit de speckle a ainsi été défini comme :

$$\widehat{P}_{PWF} = \mathbf{k}_i^H \, [\widehat{\mathbf{M}}]_{FP}^{-1} \, \mathbf{k}_i. \tag{2.70}$$

Une nouvelle mesure de distance d_{SIRV} au sens du Maximum de Vraisemblance entre les matrices de covariance du point fixe $[\mathbf{M}]_\omega$ des centres de classe ω et celle des pixels de l'image peut alors également être introduite pour la classification non-supervisée POLSAR (H/α/A par exemple). Cette nouvelle distance, optimale au sens du rapport de vraisemblance généralisé, est définie comme :

$$d_{SIRV}\left([\widehat{\mathbf{M}}]_{FP}, [\mathbf{M}]_\omega\right) = \ln \frac{|[\mathbf{M}]_\omega|}{\left|[\widehat{\mathbf{M}}]_{FP}\right|} + \frac{m}{N}\sum_{n=1}^{N} \frac{\mathbf{k}_n^H\,[\mathbf{M}]_\omega^{-1}\,\mathbf{k}_n}{\mathbf{k}_n^H\,[\widehat{\mathbf{M}}]_{FP}^{-1}\mathbf{k}_n}. \tag{2.71}$$

De manière surprenante, on peut montrer [3] que cette distance GLRT sur les matrices du point fixe peut se mettre sous la forme suivante :

$$d_{SIRV}\left([\widehat{\mathbf{M}}]_{FP}, [\mathbf{M}]_\omega\right) = d_W\left([\widehat{\mathbf{M}}]_{FP}, [\mathbf{M}]_\omega\right). \tag{2.72}$$

La distance SIRV entre deux matrices de covariance n'est que la distance de Wishart sur les estimateurs du point fixe ! Cette dernière relation renforce encore le fait que la matrice de covariance du Point Fixe peut être considérée comme une matrice de Wishart.

En comparant les techniques usuelles de classification sur les différents estimateurs sur des images SAR polarimétriques hautement texturées, nous avons pu mettre en évidence le fait que celles construites en partant d'hypothèses gaussiennes sans tenir compte de la fluctuation de texture étaient fortement biaisées par l'information de texture qui, à notre avis, ne peut pas représenter à elle seule, l'information polarimétrique. En effet, dans des images texturées, l'estimateur SCM classique de la matrice de covariance contient implicitement de l'information de texture. De ce fait, classifier les images juste par leur SPAN (puissance des voies polarimétriques) sans exploiter l'information de phase ou d'amplitude entre les différents canaux (ce qui fait la richesse de la polarimétrie) apporte la même information que celle donnée par les techniques de classification dédiées du type H/α/A. Cette constatation montre ainsi que les techniques conventionnelles sont mises en défaut dans certaines configurations.

La publication suivante proposée en Annexe M explique plus en détail cette méthodologie :

[J12] G. Vasile, **J.P. Ovarlez**, F. Pascal and C. Tison, "*Coherency Matrix Estimation of Heterogeneous Clutter in High Resolution Polarimetric SAR Images*", IEEE Trans. on Geoscience and Remote Sensing, Vol.48, No.4, pp.1809-1826, April 2010.

La figure (Fig. 2.41) montre un certain nombre de résultats. L'image (a) représente l'estimateur de SPAN $\widehat{P}_{PWF}$. L'image (c) représente en niveau de couleur les éléments diagonaux de la matrice de cohérence polarimétrique $[\widehat{\mathbf{T}}]$ non normalisée. La figure (e) représente la même information pour la matrice de cohérence normalisée $[\widehat{\mathbf{M}}]_{FP}$ du point fixe. Il est très clair que l'information donnée par les deux premières figures est très similaire tandis que la troisième image (e) est plus surprenante, moins contrastée du fait de la normalisation. La figure (b) caractérise la classification purement énergétique (classification de type gamma) non supervisée de l'image de la figure (a), c'est à dire que l'information de phase polarimétrique entre canaux a été totalement perdue. Cette image ressemble étrangement à l'image (d) qui résulte de la classification complètement polarimétrique conventionnelle H-α-A de l'image (c). Il est clair que l'information polarimétrique extraite est fortement influencée par les seules variations spatiales de l'intensité de l'image. L'image (f) représente enfin la classification polarimétrique conventionnelle de type H/α/A calculée sur l'image (e), c'est à dire appliquée sur la cohérence normalisée du point fixe. L'information qui en résulte est beaucoup plus riche mais indépendante de l'information de fluctuations de puissance. Nous pensons ainsi que cette image est la seule représentative de l'information polarimétrique. Les recherches menées actuellement portent sur l'exploitation conjointe du SPAN et de cette image de classification.

Une proposition de thèse sur cette thématique a ainsi été proposée à la DGA. Le stage de Master et la thèse **[T8]** de Pierre Formont (sept 2009-2012), encadrés conjointement par Gabriel Vasile, Frédéric Pascal et Laurent Ferro-Famil exploitent cette nouvelle piste de recherche **[J18]**, **[C59]**, **[C60]**. Son

3. Merci à Nicolas Trouvé, doctorant à l'ONERA, d'avoir trouvé cette audacieuse simplification !

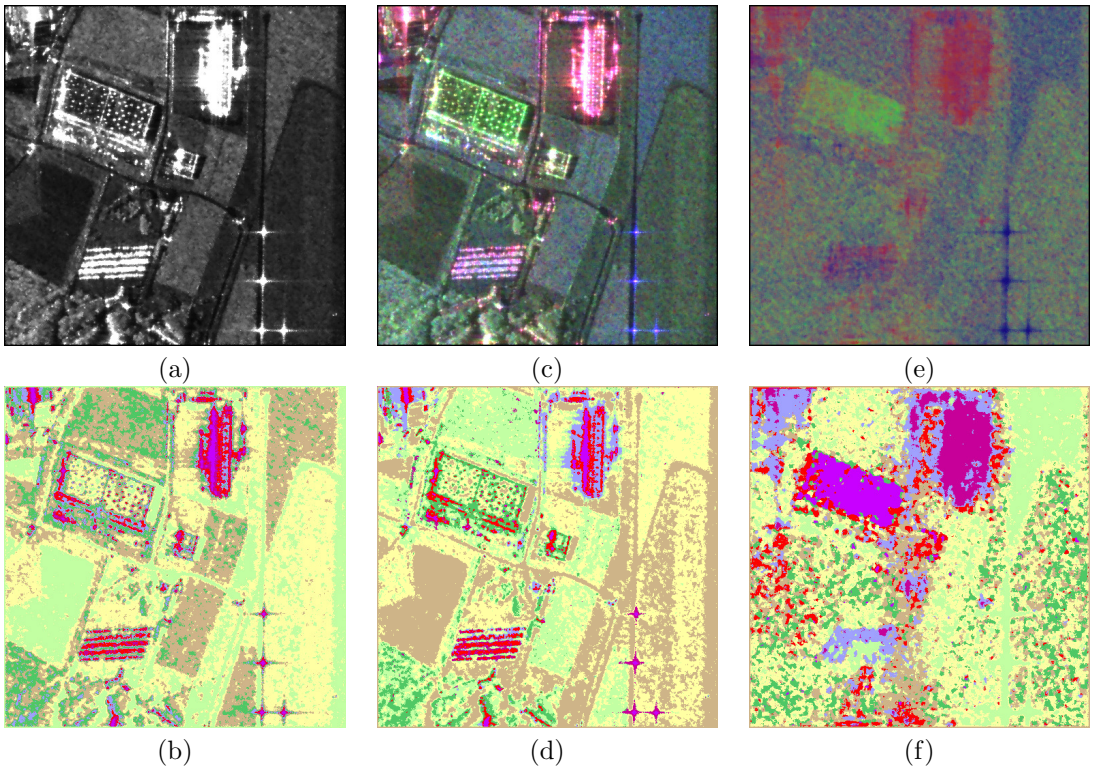


FIGURE 2.41 – SAR RAMSES polarimétriques de la zone de Brétigny, bande X (501 x 501 pixels). Figure (a) : Span $\widehat{P}_{PWF}$ estimé par la cohérence polarimétrique normalisée du point fixe. Figure b) : classification énergétique non supervisée de type Gamma de l'image (a). Figure (c) : décomposition colorée des éléments diagonaux de la matrice de cohérence $[\widehat{\mathbf{T}}]$. Figure (d) : classification non supervisée de type Wishart. Figure e) : décomposition colorée des éléments diagonaux de la matrice de cohérence $[\widehat{\mathbf{M}}]_{FP}$ normalisée du point fixe. Figure (f) : classification non supervisée de type SIRV.

travail actuel consiste à établir des nouvelles méthodes de segmentation et de classification basées sur le test de Box (test d'égalité, à distance finie, de deux matrices de covariance) qui prend en compte le fait que les populations de test n'ont pas le même nombre d'éléments par exemple. Ce test de Box est équivalent, asymptotiquement, au test de Wishart. Dans le cadre non gaussien, il a ainsi été étendu à la matrice de covariance du point fixe qui se comporte également comme une matrice de Wishart. Le test de Box, appliqué au cas gaussien ou au cas SIRV peut être également efficacement appliqué à détection de changement dans les données SAR multi-passes et multivoies.

Les deux images de gauche de la figure (Fig. 2.42) montrent une comparaison de résultats obtenus par la classification de Wishart sur la matrice de covariance SCM avec ou sans initialisation du processus de classification par la méthode H/α/A. La classification reste, à la palette de couleur près, la même. La carte de droite, effectuée avec la matrice du point fixe, montre une meilleure classification. Ces premiers résultats, bien entendu, se doivent d'être validés. Un des principaux défauts de cette méthodologie est l'étape (Eq. 2.15) qui consiste à recalculer le centre de classe par moyennage des matrices des points contenus dans la classe. En effet, rien ne dit que la matrice SCM ou FP du centre de classes est la moyenne des matrices des pixels contenus dans la classe. Comment calculer alors un barycentre de matrices de covariance ? Certainement pas par la technique du moyennage. En effet, on peut montrer que les matrices des centres de classe changent de zones dans le processus itératif de cartographie du plan H/α/A. Une des pistes potentielle de recherche est la géométrie différentielle (travaux actuels de Frédéric Barbaresco [Barb09]) qui exploite la notion de géodésique dans l'espace des matrices hermitiennes définies positives et fait actuellement l'objet d'investigations **[C61]**.

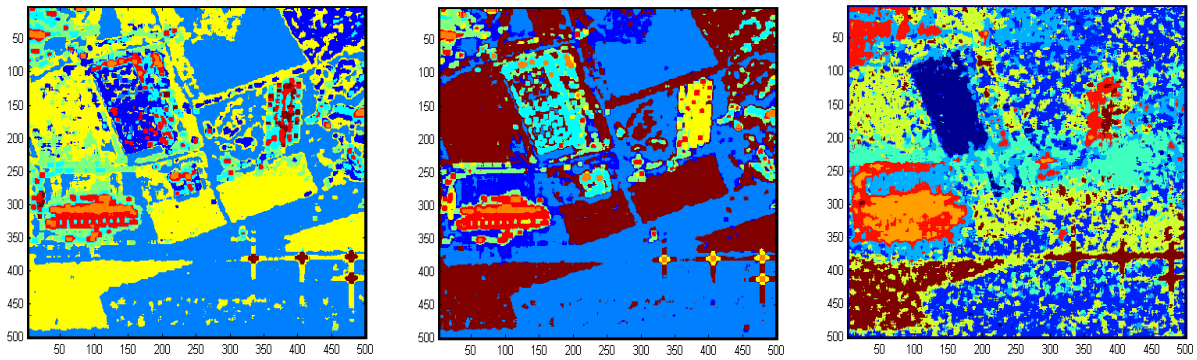


FIGURE 2.42 – a) Classification de type Wishart avec SCM et initialisation H-α. b) Classification de type Wishart avec SCM et initialisation aléatoire. c) Classification de type Wishart avec FP et initialisation H-α

La collaboration active de recherche avec Gabriel Vasile du Gipsa Lab nous a conduit à accueillir à SONDRA pour six mois (octobre 2009 - avril 2010) un post-doctorant, Lionel Bombrun **[PD2]** ayant déjà travaillé durant sa thèse sur la segmentation statistique des images SAR. Nous avons ainsi initié un certain nombre de travaux en commun **[C45]**, **[C53]**, toujours en connexion avec notre problématique actuelle de la classification polarimétrique basée sur le SIRV. Un des résultats prometteurs **[C55]**, **[C54]** de Lionel Bombrun est d'avoir élaboré un détecteur polarimétrique dans les images SAR de cibles basé sur les SIRV et la matrice de covariance du point fixe mais également sur un modèle de *steering vector*, appelé TSVM (Target Scattering Vector Model [Touz07]) adapté à l'invariance de présentation de la cible dans l'axe de visée. Lionel Bombrun a montré que les performances de détection peuvent être également tributaires du choix des paramètres des caractéristiques de cibles. Ces travaux font encore activement l'objet de recherches.

2.4 Autres Thématiques en Bref

Dans le cadre de mes études contractuelles, il m'est arrivé, de publier quelques résultats intéressants ou prometteurs. J'en citerai ici deux :

- le calcul des bornes de Ziv-Zakaï **[C35]** pour l'estimation de retard construit sur un GLRT qui a permis de mieux cerner la zone de SNR (voir figure (Fig. 2.43)) permettant à des techniques de type Track Before Detect (Intégration longue radar, filtrage particulaire **[I38]**, **[I39]**) de mieux converger. Le calcul de cette borne a été mené en utilisant des techniques intéressantes de calcul de densité de probabilité de type Point-Selle (Saddlepoint). Cette technique est également actuellement utilisée dans la thèse de Julien Totems **[T7]** pour comparer, dans la zone de transition du décrochement de la borne de Cramèr Rao, les techniques d'estimation de fréquence qui montrent une supériorité de la vibrométrie impulsionnelle par rapport à la vibrométrie continue.

- l'utilisation de la technique du Saddlepoint pour calculer des lois de rapports de formes quadratiques (encore appelé Quotient de Rayleigh) de processus gaussiens aléatoire **[C18]** de la forme :

$$\frac{\mathbf{x}^H \mathbf{A}\, \mathbf{x}}{\mathbf{x}^H \mathbf{B}\, \mathbf{x}} \tag{2.73}$$

  avec $\mathbf{A}$ et $\mathbf{B}$ deux matrices quelconques et $\mathbf{x}$ processus multidimensionnel gaussien.

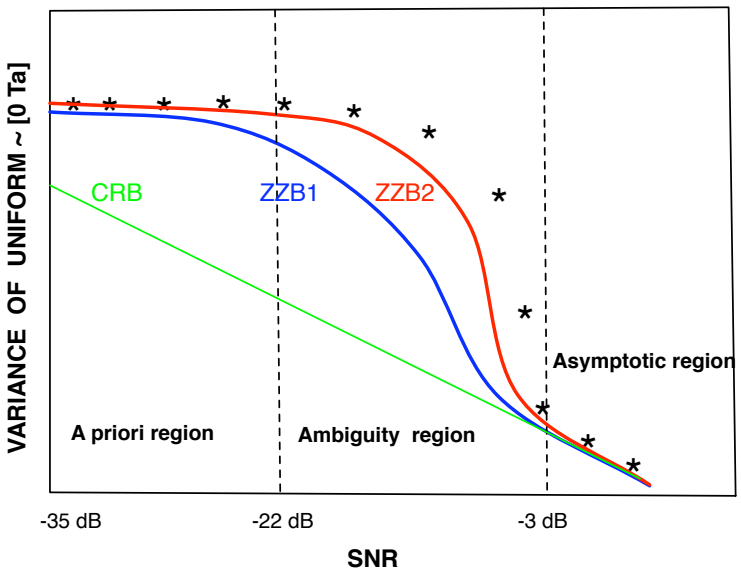


FIGURE 2.43 – Comparaison entre la Borne de Cramèr Rao (en vert), la borne de Ziv-Zakaï (BZZ) classique (en bleu), la borne de Ziv-Zakaï (en rouge) construite sur le GLRT et l'Erreur Quadratique Moyenne ($*$)

Chapitre 3

Perspectives de recherche

3.1 Contexte Scientifique

L'évolution technologique grandissante des radars (veille, poursuite, ou encore imageur) et des capteurs leur permet aujourd'hui d'obtenir une résolution des cellules à analyser de plus en plus fine, selon les axes "Distance - Azimut", "Distance – Distance", "Distance - Vitesse" ou encore "Distance - Vitesse - Azimut". Cette résolution de plus en plus petite est un atout essentiel pour la détection, le pistage, la caractérisation et/ou la reconnaissance de cibles. Cependant, la haute résolution induit une plus grande hétérogénéité et impulsivité du fouillis dans lequel est noyée la cible, et dès lors, l'hypothèse gaussienne de ce bruit additif n'est plus valide. En conséquence, la détection d'objet devient plus difficile et il est impératif de mettre en place de nouveaux outils tels que des détecteurs robustes, performants et adaptés à cet environnement non-gaussien, des nouveaux estimateurs de paramètres ou encore de nouveaux types de classifieurs permettant de caractériser les cibles et les fonds. De nombreuses applications sont concernées par cette avancée technologique :

- Le radar de veille en général : radar maritime pour la détection de petites cibles en mer agitée, radar de surveillance pour la détection basse altitude,
- Le radar MIMO (Multiple Inputs, Multiple Outputs), généralisation à plusieurs émetteurs et récepteurs de la fonction radar,
- la détection GMTI (Ground Moving Target Indicator) de cibles mobiles à partir de capteurs aéroportés et les techniques de type STAP (Space Time Adaptive Processing),
- la détection de cibles sous couvert (FOPEN), détection de mines,
- la détection et la reconnaissance des cibles et des fonds dans les scènes SAR (Synthetic Aperture Radar) ou SAS (Synthetic Aperture Sonar),
- la détection de changement, la détection de cibles mobiles dans les images SAR.

3.2 Les Objectifs du Projet

Dans le contexte général de la détection et de l'estimation pour le radar, la plupart des méthodes classiques, basées sur l'hypothèse gaussienne, sont souvent mis en défaut dès lors que le bruit additif environnant (fouillis) devient hétérogène, non stationnaire, impulsionnel, s'écartant très vite du modèle gaussien. Des modèles physiques de bruit (modèles de type SIRV, Spherically Invariant Random Vectors) basés sur les modèles de processus gaussiens composés permettent de mieux représenter la réalité (variations spatiales de la puissance du processus, transitions, ...). Ces processus dépendent cependant

de paramètres (matrice de covariance, loi de texture, paramètres de nuisance) qu'il devient nécessaire d'estimer. Une fois ces paramètres estimés, il est possible de construire, selon l'application, des détecteurs optimaux, des distances optimales. Mon travail de recherche s'est jusqu'à présent focalisé sur l'élaboration de détecteurs optimaux pour différentes applications radar (radar de veille, STAP, ...) ainsi que sur l'estimation de la matrice de covariance du processus et ses propriétés. Un estimateur particulier (appelé M-estimateur) de cette matrice de covariance a été étudiée depuis quelques temps et a montré véritablement ses potentialités en terme de performance, de propriétés statistiques pour les applications de détection radar. Cet estimateur a été également utilisé pour améliorer la classification polarimétrique et la segmentation statistique des images SAR (Synthetic Aperture Radar) hautement texturées. Mon travail de recherche actuel se concentre sur l'analyse de la robustesse de cet estimateur pour les applications de traitement du signal (robustesse vis à vis de brouilleurs, d'outliers ou cibles présentes dans les données et qui peuvent perturber ou biaiser les estimateurs conventionnels) et a permis de dégager de nouvelles thématiques de recherche.

3.2.1 Techniques Robustes pour la Détection Radar en Environnement de Fouillis Non Gaussien, Hétérogène et Non Stationnaire

La détection des cibles est un sujet de recherche qui suscite encore un grand intérêt opérationnel. La théorie de la détection radar adaptative repose principalement sur l'élaboration de tests optimaux (LRT, GLRT ou autres) mais également via le biais de l'estimation d'une matrice de covariance de signaux collectés par les capteurs radar (signaux reçus par une antenne élémentaire, par un réseau de capteurs, sur des voies polarimétriques, interférométriques,...). Ces signaux sont généralement entachés de bruit, interférences d'autres cibles ou encore de fouillis présent autour de la cible d'intérêt. La qualité d'estimation de cette matrice conditionne fortement les performances attendues des différents détecteurs. Une analyse bibliographique a montré que tous les algorithmes proposés sont majoritairement basés sur l'hypothèse d'un fouillis gaussien, homogène spatialement. Cette hypothèse simplificatrice n'est plus respectée lorsque la zone de fouillis n'est plus homogène et lorsque la cellule d'analyse tend à devenir de plus en plus petite. Ceci entraîne une dégradation des performances de réjection de fouillis et implicitement de détection. Or, actuellement, tous les traitements radar (radar haute résolution, imagerie SAR, STAP, ...) tendent à analyser des cases de résolution de plus en plus petite.

De nouveaux détecteurs (ANMF, BORD, ...) et processus d'estimation des matrices de covariance (NSCM, FP) ont été étendus pour prendre en compte de manière plus élaborée et plus efficace la complexité du problème. Ils apportent une amélioration très nette des performances de détection par rapport à celles d'un traitement classique. Le travail de recherche s'attaque maintenant à deux problèmes majeurs :

- Le premier concerne la présence éventuelle d'une ou plusieurs cibles, voire des brouilleurs ou des interférences dans les données secondaires nécessaires à l'estimation de la matrice de covariance. L'estimateur classique de la matrice de covariance s'avère alors complètement biaisé par la présence de ces cibles ou brouilleurs et dégrade fortement les performances attendues du détecteur. L'objectif de ces travaux est d'étudier et de caractériser la propriété de robustesse de la matrice de covariance du point fixe. Cette robustesse caractérise le pouvoir d'invariance à un écart sur les hypothèses faites pour le modèle : ici, les données secondaires sont polluées par la présence de cibles et s'écartent de ce fait de l'hypothèse d'homogénéité sur la densité de probabilité les régissant.
- Le deuxième problème concerne plus spécifiquement la stationnarité du processus de fouillis. Par exemple, en STAP pointe avant (air-air ou air-sol) ou en STAP bistatique, la matrice de covariance n'est plus stationnaire sur les cases distance. Des techniques de type *Focusing Matrix* ou encore des analyses à court terme, peuvent permettre de *stationnariser* les processus et gagner encore en terme de performances.

3.2.2 Classification Polarimétrique, Segmentation Statistique dans les Images SAR Hautement Texturées

La texture d'une image peut être définie comme la variation locale spatiale du niveau d'intensité d'une image. L'analyse de cette texture permet, à une certaine échelle, de caractériser l'image en zones

homogènes nécessaires, par exemple, à la mise en œuvre d'un processus de détection et/ou d'une classification. Contrairement à une image en niveau de gris ou à une image en couleur, chaque pixel d'une image provenant d'un capteur SAR (Synthetic Aperture Radar) est caractérisé par une amplitude et une phase. Ces images sont généralement multi-voies (utilisation de canaux de polarisation, interférométrie), multi-vues (même scène pour différents angles de vue) ou multi-spectrales (même scène analysée pour plusieurs bandes de fréquence). Par conséquent, les processus d'analyse de la texture de l'image deviennent de plus en plus complexes mais font généralement appel à des outils de classification basés sur des hypothèses de type gaussiennes. Ainsi, dans la littérature, la plupart des travaux conduisent aux classifieurs classiques construits sur des matrices de covariances de type Wishart (estimation au sens du Maximum de Vraisemblance de la matrice de covariance à partir de données supposées gaussiennes). Une analyse approfondie de ces méthodes montre que leurs performances se dégradent de manière significative sur des zones hétérogènes, non gaussiennes et/ou texturées. C'est le cas pour des images SAR Très Haute Résolution provenant des dernières générations de capteurs ou pour des images hautement texturées (zones urbaines par exemple). Le travail mené a permis d'établir un certain nombres de lacunes liés à la classification (remise en cause de la robustesse des méthodes itératives de classification de type H/α/A à l'initialisation, influence de la texture dans les classifieurs, calcul du centre de classe des classifieurs, ...). Le travail de recherche envisagé est d'exploiter ces nouvelles pistes de recherche en utilisant ou en construisant des détecteurs, estimateurs et classifieurs robustes dans des hypothèses d'environnements un peu plus complexes qu'auparavant :

- développement et améliorations d'outils statistiques de type SIRV : estimation conjointe de la matrice de covariance du noyau gaussien et de la densité de probabilité de la loi de texture,
- extension aux SIRP généralisés [Barn96],
- optimisation de la cohérence polarimétrique et interférométrique pour l'estimation de la hauteur des réflecteurs (POLinSAR) dans le cadre des estimateurs robutes de matrices de covariance de type Point-Fixe,
- mise en place d'outils d'analyse de la texture, segmentation polarimétrique,
- développement de nouvelles distances adaptées, nouveaux outils (ex : géométrie différentielle pour calculer un barycentre de matrices de covariance **[C61]**),
- développement de détecteurs optimaux adaptés aux zones hétérogènes et texturées,
- élaboration de nouvelles méthodes de classification polarimétrique adaptées à la texture.

Les retombées de ces travaux peuvent donner lieu à de nombreuses applications, qu'elles soient civiles ou militaires :

- segmentation (spatiale, polarimétrique et spectrale) de zones de terrain (agricoles ou autres) pour l'analyse des paramètres du sol ou la reconnaissance de type de sols à partir de données aéroportées et/ou satellitaires de type SAR,
- identification de zones texturées homogènes pour la détection d'objets ou de cibles (cas du SAR FOPEN),
- mise en œuvre de détecteurs adaptés aux zones texturées non homogènes,
- détection de changement dans les données SAR polarimétriques et/ou interférométriques.

3.2.3 Détection de Cibles ou de Changement dans les Images SAR Mono-Voie

Dans les images mono-voie comme les images SAR, par exemple, les techniques mentionnées plus haut ne permettent pas d'être mises en œuvre puisqu'il est impossible de définir un vecteur d'observation pour chaque pixel de l'image. Il est alors nécessaire de définir d'autres degrés de liberté pour construire ces *Spherically Invariant Random Vectors*. Les techniques Temps-Fréquence, déjà utilisées en imagerie radar pour l'analyse de la dispersivité et de l'anisotropie des cibles peuvent être alors combinées à ces techniques de détection **[I41]**. Les applications potentielles de ces techniques sont axées sur la détection de changement (analyse de la cohérence en fréquence, en angulaire), la détection de cibles mobiles dans les images SAR (analyse de la détection de changement dans les images provenant d'une décomposition en sous-ouvertures par exemple).

3.2.4 Détection de Changement dans les Images SAR Multi-Voies

Ces techniques analysent les changement ayant lieu dans des images prises à des instants différents. Les applications de ces techniques sont diverses (analyse des mouvement des glaciers, des dégâts provoqués par un séisme, analyse des changements de culture, analyse d'un champ de bataille pour des déplacements de véhicules, ...). Les signaux collectés étant multi-voies, il est possible d'étendre toutes les techniques connues (test de Box, test d'égalité de matrices de covariance, ...) au cas hétérogène, non stationnaire, cohérent ou non cohérent. Il est ainsi possible d'étudier la détection de changement :

- dans des images interférométriques multi passes (Imagerie SAR Interférométrique Différentielle),
- dans des images polarimétriques multi passes (Imagerie SAR Polarimétrique Différentielle),
- dans des images polarimétriques et interférométriques multi passes (Imagerie POLINSAR Différentielle).

La technique précédente des sous bandes ou sous ouvertures peut être ici utilisée pour encore augmenter les degrés de liberté.

3.2.5 Imagerie Hyperspectrale

L'apparition de nouvelles sources d'information (notamment optique et radar à haute résolution spatiale, polarimétrie, interférométrie, et l'imagerie hyperspectrale) permet effectivement d'augmenter le nombre de degrés de liberté de l'image. Cette quantité d'informations provenant de sources différentes peut bien entendu se combiner et se modéliser par une loi multivariée. L'aspect de l'hétérogénéité de cette information peut alors être prise en charge par la modélisation non gaussienne de type SIRV. Ceci m'a conduit à proposer un sujet de thèse 2011 conjointement avec le Département d'Optique Théorique et Appliqué (DOTA) de l'ONERA et avec le DSO à Singapour sur les applications potentielles des techniques développées dans ce document pour les applications d'imagerie hyperspectrale : *Robust Detection and Classification Using the SIRV Background Modelling for Hyperspectral Imaging* décrit ci-dessous :

Many military and civilian applications involve the detection of an object or activity such as a military vehicle or vehicle tracks. Hyperspectral imaging sensors provide image data containing both spatial and spectral information, and this information can be used to address such detection tasks. The basic idea for hyperspectral imaging stems from the fact that, for any given material, the amount of radiation that is reflected, absorbed, or emitted (the radiance) varies with wavelength. Hyperspectral imaging sensors measure the radiance of the materials within each pixel area at a very large number of contiguous spectral wavelength bands. In theory, the design and evaluation of detection algorithms is facilitated by assuming some meaningful probability distributions for the target and non-target spectra. The key features of the adopted probabilistic signal models and the used criterion of optimality determine the nature of the obtained detectors and their performance. The mathematical framework for the design and evaluation

of detection algorithms is provided by the area of statistics known as binary hypothesis testing. The detection problem is typically formulated as a binary hypothesis test with two competing hypotheses : background only or target and background. In practice the background statistics are unknown and have to be estimated from the data. If the detectors operate in a sparse target environment, the statistical parameters (covariance matrix, ...) of the background can be estimated by using all pixels within an area of interest. The size of the area has to be chosen large enough to assure the invertibility of the covariance matrix and small enough to assure both spectral homogeneity (stationarity) and spatial homogeneity. Since the two hypotheses contain unknown parameters (for example, the covariance matrix of the background) that have to be estimated from the data, the detector has to be adaptive, and it is usually designed by using the generalized-likelihood-ratio test (GLRT) approach. The problem of target detection in hyperspectral imaging can usually be split into two parts :

- For full-pixel targets, there is no significant interaction between target and background other than secondary illumination, shading, and other subtle illumination factors. Hence the spectrum observed by the sensor is produced by either the target spectrum or the background spectrum. In both cases, the observed spectrum is corrupted by additive sensor noise. However, it can be assumed the sensor noise is insignificant or can be accounted for by the target and background distributions. In this case detection performance is mainly determined by the variability of target and background spectra,
- For subpixel targets, the spectrum observed by the sensor results from a linear or nonlinear mixing of the target and background spectra. The most important consideration is that the background spectrum adds an interference component that makes detection more difficult. The problem of sensor noise is always present as well, and can be incorporated in the target and background signals or modeled as a separate source.

In these two cases, it is possible to derive optimal statistical tests depending or not on the knowledge of the target spectral signature (anomaly detection, matched filter, subspace based detection). All these techniques are well known but perform well only in some restrictive hypotheses : to date, analytic derivation of hyperspectral target detectors has been based on signal models involving multivariate normal (Gaussian) distributions. In many practical cases, however, the actual response of a detector to the background pixels differs from the theoretically predicted distribution for Gaussian backgrounds. In fact, the empirical distribution usually has heavier tails compared to the theoretical distribution, and these tails strongly influence the observed false-alarm rate of the detector [4]. Clearly, the detection problem cannot be at all addressed in an optimum manner until the departures of the background distribution from normality are understood.

The aim of this Ph.D. proposal is to introduce a non-Gaussian model for the background statistics (SIRV models) already used in radar signal processing and discuss the implications for detector design. The goal is here clearly to extend, in the SIRV context, all the previous already proposed detectors in an heterogeneous and/or non Gaussian background. This can lead to significantly enhance the performance of classical detectors based on the conventional Gaussian hypothesis. These new detectors are shown to be invariant with the spatial heterogeneity of the background and even robust to presence of some targets embedded in the reference background. Moreover, they can perfectly regulate the false alarm rate in non Gaussian and heterogeneous environment. Hence, they can be efficiently used for a greater variety of target and background classes. Finally, this new methodology can lead to define new classification techniques.

3.3 Méthodologie et Mise en Œuvre

La théorie de l'estimation vise à déterminer les paramètres physiques d'un système à partir d'observations. Parmi les méthodes existantes, la méthode du Maximum de Vraisemblance est la plus large-

4. D. Manolakis, D. Marden, and G.A. Shaw, Hyperspectral Image Processing for Automatic Target Detection Applications, Lincoln Laboratory Journal, Vol.14, No.1, 2003.

ment utilisée pour ses performances et ses bonnes propriétés statistiques. Cette méthode s'appuie sur la connaissance a priori de la famille de densités de probabilité décrivant les observations. Dans ce contexte, la famille des densités de probabilité gaussiennes est universellement utilisée en traitement de signal : on peut citer, comme exemple d'applications, l'estimation des caractéristiques du bruit additif (bruit thermique, fouillis) dans les détecteurs radar, le traitement d'antenne. La famille gaussienne rend bien compte de la distribution des données dans de nombreux cas et permet souvent de développer des estimateurs faciles à mettre en œuvre (cas de la SCM, Sample Covariance Matrix, covariance empirique des données). Malheureusement, elle ne permet pas la modélisation de bruits complexes à queue lourde et ne rend pas compte des variations extrêmes des mesures que l'on observe alors. Ce défaut peut conduire à de très mauvais estimateurs dans le cas de données impulsives modélisées à tort par une loi normale. Il est donc primordial de développer de nouveaux estimateurs dans le cadre de familles de densités de probabilité élargies conciliant la modélisation des données gaussiennes et des données impulsives. Une famille de densités de probabilité est connue par les statisticiens pour satisfaire à ces deux exigences : les densités de probabilité à contours elliptiques plus communément appelées SIRV ou encore processus gaussiens composés. Ces processus SIRV peuvent être mis sous la forme du produit d'un vecteur aléatoire gaussien (appelé speckle) centré, circulaire et caractérisée par sa matrice de covariance et d'une variable aléatoire scalaire et positive (appelée texture) caractérisée par sa densité de probabilité régissant la fluctuation de la puissance du vecteur de mesure. Ces processus possèdent des propriétés très intéressantes pour le traitement des signaux et des images : modélisation très générale de processus non gaussiens (loi K, de Weibull, du chi2, Cauchy, alpha stable, etc.) incluant également le processus gaussien, fonction caractéristique purement radiale, propriété d'invariance de la loi de la texture par filtrage linéaire, ...

Dans le contexte des SIRV, une procédure d'estimation du Maximum de Vraisemblance de la matrice de covariance a été définie comme étant l'unique solution d'une équation implicite. Hormis la propriété d'invariance à la texture, cet estimateur a des propriétés très intéressantes dans de nombreuses applications : détecteurs radar optimaux en environnement hétérogène et/ou non gaussien. Son intérêt pratique, sa mise en œuvre, ses propriétés (ex : robustesse) ainsi que sa suprématie par rapport à l'estimateur conventionnel SCM dans les applications liées à la détection radar STAP en environnement non-gaussien et à la classification polarimétrique dans les images SAR hautement texturées, peuvent être démontrées.

Ces estimateurs sont connus dans la littérature comme faisant partie des M-estimateurs [Maro76], [Maro06], beaucoup plus généraux qu'il est nécessaire d'étudier. La thèse de Mélanie Mahot **[T8]** a pour but d'étudier ces M-estimateurs et d'analyser leur robustesse dans différentes thématiques liées à la théorie de la détection et de l'estimation.

3.4 Déroulement de la Recherche

Les objectifs à atteindre à l'issue de cette recherche sont de :

- Robustifier à l'aide d'estimateurs de qualité, les détecteurs radar (radar de veille, radar aéroporté de type STAP) confrontés à la non-gaussianité, l'hétérogénéité et la non-stationnarité du fouillis,
- Étendre les outils de classifications polarimétriques SAR basés sur des estimateurs gaussiens au cas de fouillis non-gaussien, hétérogène et non stationnaire,
- Combiner les techniques d'estimateurs et de détection et l'analyse Temps-Fréquence (sous bandes, sous ouvertures) sur des données SAR mono-canal ou muti-canal pour augmenter le degré de liberté de l'information disponible,
- Montrer l'intérêt et la potentialité de ces nouvelles méthodes sur données réelles provenant de différentes expérimentations (ex : imagerie hyperspectrale),
- Contribuer à la diffusion de ces techniques, publier les résultats dans des journaux internationaux

à comité de lecture.

3.5 Les Moyens à Disposition pour la Réalisation du Projet

Pour mener à bien cette thématique, je vais bien évidemment m'appuyer essentiellement sur mes différents organismes d'accueil et partenaires à travers d'échanges d'étudiants, de publications et de collaborations dans différents projets ayant trait à cette thématique.

3.5.1 Liens avec l'ONERA

Mon activité principale à l'ONERA consiste principalement à participer voire améliorer les techniques existantes pour la détection, l'imagerie, que ce soit au travers d'activités contractuelles, d'études internes typiquement axées sur la recherche, d'encadrements de thèse ou de nouvelles propositions d'étude. J'ai ainsi encadré ou encadre à ce jour à l'ONERA 8 doctorants, 2 post-doc et quelques étudiants de Master depuis les années 2000 sur les thématiques présentées dans ce document. Je compte, bien évidemment, sur le soutien de mon organisme d'appartenance et de la DGA pour me permettre de mener à bien les projets futurs sur lesquels je compte m'investir.

3.5.2 Liens avec le Laboratoire SONDRA

Dans le cadre de mes activités de recherche à l'ONERA, je suis officiellement détaché depuis janvier 2008 à mi-temps dans le laboratoire franco-singapourien SONDRA de Supélec. Mon travail est de proposer et de superviser les activités liées au traitement du signal de ce laboratoire. Le financement de ce laboratoire est en grande partie effectué par l'agence de défense Singapourienne (DSTA). En lien avec cette dernière, le laboratoire définit chaque année des propositions de financement pour les chercheurs permanents de ce laboratoire mais également des propositions de thèse. J'ai ainsi monté et obtenu en 2009 deux bourses de thèse sous financement DGA (Méthodes robustes pour le STAP, classification dans les images SAR hautement texturées) correspondant à mes thématiques de recherche (thèses de Mélanie Mahot **[T8]** et Pierre Formont **[T9]**). Je participe également à l'encadrement de la thèse de Chin Yuan Chong (doctorante singapourienne en poste à SONDRA, **[T6]**) soutenance prévue en 2011) sur l'extension de ces nouvelles techniques au radar MIMO (voir Annexe L).

Depuis janvier 2010, j'ai obtenu un financement DGA pour un séjour de recherche d'un an au DSO (Defence Science Organization) National Laboratories à Singapour qui a manifesté un réel intérêt et a soutenu ces mêmes thématiques. Hormis le fait de continuer mon propre travail de recherche avec d'autres chercheurs (applications des techniques développées dans le cadre du SAR, de l'imagerie hyperspectrale, du STAP en sonar, ...), ce séjour permet de renforcer les liens existants (échange de doctorants ou de chercheurs confirmés) entre les deux laboratoires et de définir de futurs projets communs. La figure (Fig. 3.1) montre les axes de coopération initiés durant l'année 2010. Une thématique de détection de changement dans les images SAR multi-passes est en passe d'être élaborée pour proposer un sujet de thèse encadré conjointement par SONDRA et le DSO. Des travaux de collaboration sont également engagés avec l'équipe Radar de NTU (Nanyang Technology University) de Singapour pour mettre en œuvre ces techniques de détection au problème du radar HFSW (High Frequency Surface Wave). Dès mon retour de Singapour, mon projet est de poursuivre l'effort du travail de recherche déjà engagé avec l'équipe de recherche de SONDRA.

3.5.3 CLUB STAP

En janvier 2007, s'est créé, à l'initiative de l'ONERA et de la DGA un groupe scientifique de laboratoires universitaires (SONDRA, SATIE, ISAE, IETR, LSS, ...), d'entités étatiques (ONERA, CELAR, DGA) et d'industriels (THALES) ayant pour but de fédérer les recherches ayant trait aux traitements STAP et dont je suis le leader actuel. Cette fédération s'effectue par des présentations régulières de sujets

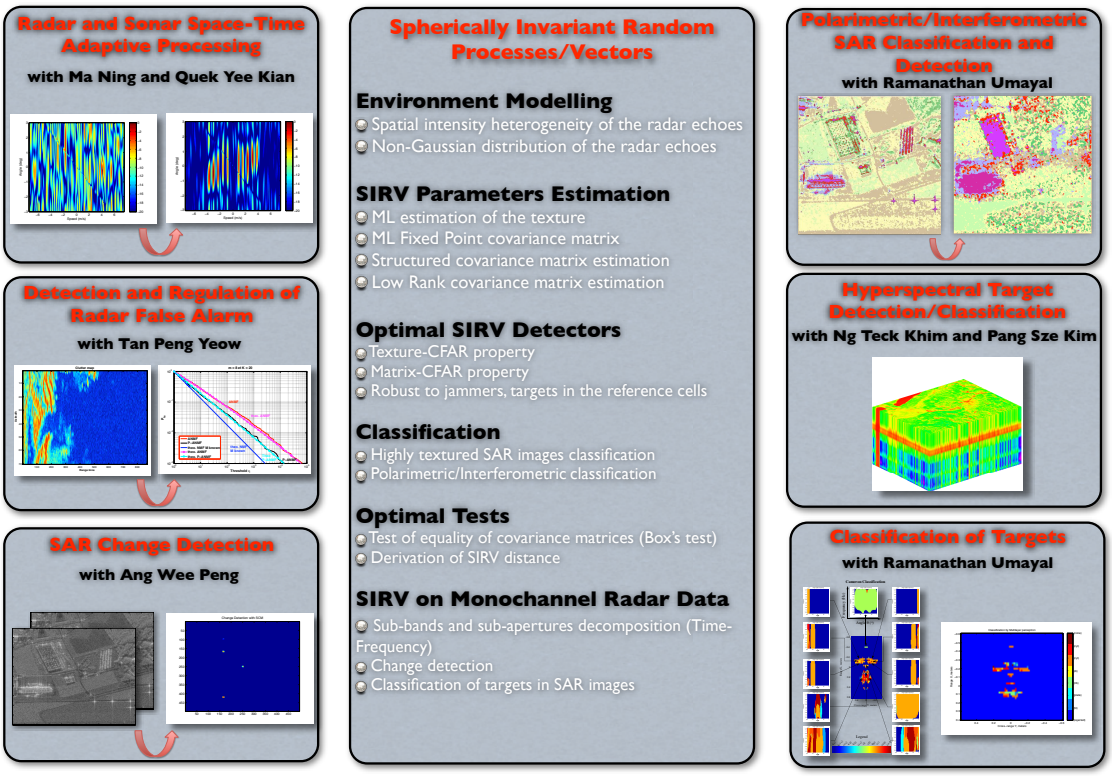


FIGURE 3.1 – Aperçu des axes de coopération développés durant mon année 2010 au DSO National Laboratories

de recherche par des chercheurs confirmés mais également par les doctorants intéressés pas le sujet. Une base de données de signaux expérimentaux permet de confronter et de comparer les idées et les résultats. Récemment étendu aux partenaires européens, ce réseau est un formidable vivier d'échanges d'idées et de futures collaborations. L'ONERA a ainsi proposé à l'EDA (European Defence Agency), une proposition de thématique ("*call*") sur le STAP et a fait, suite à cet appel à projet, acte de candidature en partenariat avec THALES et quelques laboratoires universitaires.

3.5.4 Projet ANR : Outils de Surveillance par Télédétection à partir de Radars Satellitaires

Élaboré par le GIPSA Lab (notamment Gabriel Vasile et Michel Gay), ce projet s'inscrit dans les thématiques de "Sécurité des ouvrages" et "Gestion de crise". Il concerne le site du barrage de Grand'Maison en Isère. Les parties pour le moment qui seraient impliquées sont le CEMAGREF, l'EDF, le GIPSA, l'ONERA et SONDRA. Plus précisément, la participation de SONDRA consisterait à l'analyse de la mesure de déplacement (glissement de terrain) par interférométrie et sur la prévision d'avalanche (analyse de la neige) par modélisation électromagnétique et par traitement polarimétrique. L'ONERA apporterait ici son soutien et son expertise avec le moyen SAR SETHI. La thématique commune avec mes activités de recherche est ici principalement la détection de changement.

3.5.5 Liens avec les Laboratoires Partenaires

Je voudrais ici citer des partenaires universitaires avec qui j'ai initié un certain nombre de travaux et avec lesquels je souhaiterais continuer de collaborer dans le futur. Je citerai le SATIE de l'ENS Cachan avec Philippe Forster sur toute la théorie des SIRV et des détecteurs optimaux, l'IETR avec Éric Pottier et Laurent Ferro-Famil en ce qui concerne l'imagerie SAR par méthodes temps-fréquence ou la polarimétrie,

le GIPSA-Lab avec Gabriel Vasile pour ce qui est de la classification polarimétrique en imagerie SAR, l'imagerie hyperspectrale et la détection de changement ainsi que le LSS de Supélec avec Sylvie Marcos et Alexandre Renaux pour les recherches ayant trait au STAP.

Chapitre 4

Conclusion

Au terme de la rédaction de ce mémoire d'HDR, un bilan synthétique de mes activités de recherche menées à l'ONERA depuis plus de vingt ans peut être établi.

Mon axe de recherche initial, dérivé de mes travaux de thèse sur l'Analyse Temps-Fréquence des signaux unidimensionnels large bande s'est étendu progressivement à l'analyse des signaux bidimensionnels SAR mono-voie puis à l'analyse des signaux SAR multi-voies (polarimétrie, interférométrie, ...). Ces travaux ont été menés principalement dans l'encadrement des thèses de M. Tria **[T2]** et M. Duquenoy **[T4]**. Alliées aux méthodes de classification, ces analyses, même si elles peuvent être jugées gourmandes en ressources, permettent d'exhiber de nombreux paramètres physiques caractéristiques des cibles et des fonds nécessaires à la compréhension de notre environnement. Les techniques d'extraction d'informations de ces hyperimages, qui ressemblent fortement à la thématique actuelle de recherche sur l'extraction d'information dans le plan temps-fréquence, est un nouvel axe potentiel de recherche. Les techniques Temps-Fréquence peuvent être aussi assez facilement appliquées sur toutes sortes de données multi-images, comme le multi-temporel, multi-passes, multi-bandes (hyperspectral). Cette possibilité ouvre, en outre, des pistes de recherches intéressantes pour la détection car elles permettent de dégager des degrés de liberté nécessaires, par exemple, à l'analyse statistique multivariée et à l'utilisation du modèle SIRV.

Parallèlement à ces méthodes temps-fréquence, je me suis intéressé au problème de la détection et de l'estimation radar en environnement hétérogène et non gaussien. Cette thématique s'est réellement développée avec les travaux de thèses menés par E. Jay **[T1]** et F. Pascal **[T3]**. Ils ont établi de nouveaux types de détecteur alliés à des estimateurs très puissants, comme l'estimateur de la matrice de covariance du point fixe qui ont prouvé leur efficacité sur signaux réels. Ces techniques d'estimation ont été étendues au STAP (thèse de Guilhem Pailloux **[T5]** en exploitant la propriété de persymétrie de la matrice de covariance du fouillis, au MIMO (thèse de C.Y. Chong à SONDRA). Elles ont ensuite permis d'améliorer la classification polarimétrique des images SAR hétérogènes (post-doc et travaux actuels de G. Vasile **[PD1]**, travaux actuels du post-doc de L. Bombrun **[PD2]**). Hormis l'application de régulation de fausse alarme dans les détecteurs radar ou SAR, l'estimateur du point fixe a la remarquable propriété d'être robuste. Il peut donc être très utile lorsque les données secondaires, même gaussiennes, sont polluées par des interférences, la présence d'autres cibles, la présence de réflecteurs ponctuels dans les images SAR qui biaisent la technique d'estimation usuelle de type SCM. Deux thèses sur l'analyse de la robustesse de ces techniques (M. Mahot, **[T8]**) et la classification des images SAR (thèse de P. Formont, **[T9]**) ont débutées en octobre 2009. De nouvelles pistes de recherche alliant conjointement les techniques Temps-Fréquence et ces détecteurs sont actuellement en cours. Elles concernent notamment la détection de changement, la détection de cibles mobiles dans les images SAR mono ou multi voies (polarimétriques, interférométriques, multi passes, ...).

Ces deux thématiques, couplées ou non, ouvrent ainsi un champ de recherche assez vaste, l'émergence des radars ou satellites d'observation de la terre contribuant et encourageant fortement aussi l'exploitation, l'amélioration ou l'émergence de ce genre de techniques d'analyse.

Annexe A

Bornes de Cramer Rao pour l'estimation du facteur d'échelle Doppler

A.1 La Transformation de Mellin

A.1.1 Définition

La transformation de Mellin [T0] est définie sur le signal analytique $Z(f)$ en fréquence par la relation :

$$M^{\xi}[Z](\beta) = \int_{0}^{+\infty} Z(f)\, e^{2i\pi\xi f}\, f^{2i\pi\beta+r}\, df \tag{A.1}$$

ainsi que par sa forme réciproque :

$$Z(f) = \int_{-\infty}^{+\infty} M^{\xi}[Z](\beta)\, e^{-2i\pi\xi f}\, f^{-2i\pi\beta-r-1}\, d\beta \tag{A.2}$$

Cette transformation est unitaire, c'est à dire

$$(Z_1, Z_2) = \int_{-\infty}^{+\infty} M^{\xi}[Z_1](\beta)\, M^{\xi *}[Z_2](\beta)\, d\beta \tag{A.3}$$

pour le produit scalaire invariant donné par :

$$(Z_1, Z_2) = \int_{0}^{+\infty} Z_1(f)\, Z_2^{*}(f)\, f^{2r+1}\, df \tag{A.4}$$

La transformation peut être interprétée comme le coefficient de la décomposition du signal $Z(f)$ sur une base de signaux hyperboliques $Z^{\xi}(f,\beta)$ définis par

$$Z^{\xi}(f,\beta) = e^{-2i\pi\xi f}\, f^{-2i\pi\beta-r-1} \tag{A.5}$$

dont la loi de retard de groupe est caractérisée par l'équation $t = \xi + \beta/f$.

Le paramètre réel r peut être choisi arbitrairement mais peut corriger le changement de dimension d'une quantité physique effectué lors d'une transformation affine sur le temps. On choisira par simplification, le paramètre temporel ξ égal à zéro, ce qui ne revient à considérer que le cas particulier des signaux centrés autour de l'époque moyenne nulle. On notera alors par la suite $M[Z] = M^{\xi=0}[Z]$.

La principale propriété de la transformation de Mellin est la propriété d'invariance aux changements d'échelle :

$$\begin{array}{ccc} Z(f) & \xrightarrow{U_\xi} & U_\xi Z(f) = a^{r+1}\, e^{-2i\pi\xi(1-a)f}\, Z(af) \\ \downarrow & & \downarrow \\ M^\xi[Z](\beta) & \longrightarrow & M^\xi[U_\xi Z](\beta) = a^{-2i\pi\beta}\, M^\xi[Z](\beta) \end{array} \tag{A.6}$$

Cette propriété et la propriété d'unitarité (Eq. A.3) permettront par la suite de réécrire dans l'espace de Mellin les différentes formes large bande de manière plus intéressante. Une autre propriété de la transformation de Mellin, très importante, est la diagonalisation de l'opérateur B défini comme :

$$BZ(f) = -\frac{1}{2i\pi}\left(f\frac{d}{df} + r + 1 + 2i\pi\xi f\right) Z(f) \tag{A.7}$$

qui se transforme par transformée de Mellin comme $M^\xi[BZ](\beta) = \beta M^\xi[Z](\beta)$.

Citons enfin la transformation de Mellin d'un produit invariant défini par :

$$(Z_1 \circ Z_2)(f) = f^{r+1}\, e^{2i\pi\xi f}\, Z_1(f)\, Z_2(f) \tag{A.8}$$

qui n'est que le produit de convolution des transformées :

$$M^\xi[Z_1 \circ Z_2](\beta) = \left(M^\xi[Z_1] * M^\xi[Z_2]\right)(\beta) \tag{A.9}$$

et le produit de convolution multiplicatif :

$$(Z_1 ** Z_2)(f) = \int_0^\infty Z_1\left(\frac{f}{f'}\right) Z_2(f')\, e^{2i\pi\xi\left(\frac{f}{f'}+f'-f\right)}\, \frac{df'}{f'} \tag{A.10}$$

qui se transforme en produit des deux transformées :

$$M^\xi[Z_1 ** Z_2](\beta) = M^\xi[Z_1](\beta)\, M^\xi[Z_2](\beta) \tag{A.11}$$

A.1.2 La transformation de Mellin Discrète

Toutes les propriétés importantes qui ont permis de construire la transformation discrète et de donner une interprétation de la variable β (paramètre d'hyperbole $t = \xi + \beta/f$ dans le plan temps-fréquence) peuvent être consultées dans **[T0]**, **[O2]**, **[C1]**. Cette transformée s'écrit :

$$M^\xi[Z]\left(\frac{p}{N\ln q}\right) = \ln q \sum_{k=L}^{L+N-1} q^{k(r+1)}\, e^{2i\pi\xi q^k}\, Z(q^k)\, e^{2i\pi\frac{kp}{N}} \tag{A.12}$$

où $0 \leq p \leq N-1$ et L vérifie $q^L = f_1$ avec f_1 fréquence min de Z.

La transformée de Mellin est ainsi calculée sur les échantillons géométriques du signal spectral par algorithme rapide (Transformée de Fourier Rapide). La raison géométrique q d'échantillonnage du signal Z doit vérifier une condition de non recouvrement :

$$\frac{1}{\ln q} \geq \beta_2 - \beta_1 \tag{A.13}$$

où $[\beta_1, \beta_2]$ est le support de $M^\xi[Z](\beta)$. Ce support peut à première vue ne pas être facile à déterminer. En fait, l'étude des distributions temps-fréquence affines et en particulier la construction tomographique de la distribution affine unitaire de J. et P. Bertrand [Bert91], notée $P_0(t,f)$ et d'expression

$$P_0(t,f) = f\int_{-\infty}^{+\infty} (\lambda(u)\lambda(-u))^{r+1}\, Z(f\lambda(u))\, Z^*(\lambda(-u))\, e^{-2i\pi ftu}\, du \tag{A.14}$$

avec

$$\lambda(u) = \frac{ue^{-u/2}}{2\sinh u/2} \tag{A.15}$$

va pouvoir apporter, de manière très élégante, une solution très simple à ce problème et conduire ainsi à l'élaboration du théorème d'échantillonnage de la transformation de Mellin.

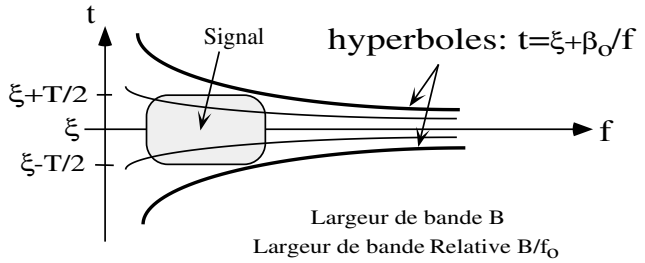


FIGURE A.1 – Localisation d'un signal entre les hyperboles d'équation $t = \xi + \beta_0/f$ et $t = \xi - \beta_0/f$

A.1.3 Théorème d'Échantillonnage

Grâce à l'étude générale des représentations temps-fréquence affines des signaux [Bert91] et à la formule de Moyal, la transformation de Mellin est reliée à la distribution temps-fréquence affine unitaire $P_0(t, f)$ par l'équation :

$$\int_{-\infty}^{+\infty} dt \int_{0}^{+\infty} P_0(t, f)\, \delta(t - \xi - \beta/f)\, f^{2r}\, df = \left| M^{\xi}[Z](\beta) \right|^2 \tag{A.16}$$

Le carré du module de la transformée de Mellin du signal Z est égal à l'intégrale de sa représentation temps-fréquence affine $P_0(t, f)$ sur un réseau d'hyperboles indexées par β dans le plan temps-fréquence (transformation de Radon généralisée). Ainsi, lorsque le signal est localisé en temps et en fréquence, il existe un certain paramètre β_0 à partir duquel l'intégrale de $P_0(t, f)$ devient nulle. La transformée de Mellin d'un signal localisé en temps et en fréquence est donc elle-même localisée.

Si le signal est localisé dans le plan temps-fréquence (largeur de bande B autour de la fréquence centrale f_0, durée T), il est possible de déterminer grossièrement ce paramètre β_0 par

$$\beta_0 = (f_0 + B/2)T/2 \tag{A.17}$$

en ne considérant que les hyperboles qui délimitent au mieux ce signal dans le plan temps-fréquence, comme le montre la figure A.1.

La condition de non recouvrement (Eq. A.13) s'écrit alors :

$$\frac{1}{\ln q} \geq \beta_2 - \beta_1 = 2\beta_0 \tag{A.18}$$

avec β_0 défini par la relation (Eq. A.17). Il vient :

$$\frac{1}{\ln q} \geq BT(1 + 2/R)/2 \tag{A.19}$$

avec BT définissant le produit bande durée du signal et R la largeur de bande relative B/f_0.

Si le support spectral de Z est $[f_1, f_2]$, la raison q doit être choisie de manière à ce que la restriction de la forme dilatocyclée $\overline{Z}$ de Z (périodisation de Z à toutes les échelles sur le demi-axe des fréquences positives) sur $[f_1, f_2]$

$$\overline{Z}(f) = \sum_{n=-\infty}^{+\infty} Q^{n(r+1)}\, Z(Q^n f)\, e^{2i\pi\xi f(1-Q^n)} \tag{A.20}$$

soit égale à Z, ce qui implique :

$$Q = q^N \geq \frac{f_2}{f_1} \tag{A.21}$$

Le nombre de points N qui caractérisent parfaitement $M^{\xi}[Z]$ est déduit de (Eq. A.19) et (Eq. A.21) :

$$N \geq BT \left(\frac{1}{2} + \frac{1}{R} \right) \ln \frac{1 + R/2}{1 - R/2} \tag{A.22}$$

Si le signal est à bande étroite, on peut remarquer que toutes les dilatations deviennent des translations, que la transformée de Mellin devient transformée de Fourier, et que le nombre de points N devient le BT du critère de Nyquist.

A.2 La Transformation de Mellin et l'Analyse Radar

A.2.1 Introduction

Considérons l'émission d'un signal $z(t)$ (analytique) sur une cible en mouvement. Le signal reçu par le radar peut être modélisé comme :

$$x(t, \underline{\alpha}_0) = A_0 \, T_{\underline{\theta}_0} z(t) \, e^{i\phi_0} + b(t) \tag{A.23}$$

où $\underline{\alpha}_0 = (A_0, \phi_0, \underline{\theta}_0)$ et où $T_{\underline{\theta}_0}$ est une sous transformation agissant sur le signal $z(t)$ avec un jeu de paramètres inconnus $\underline{\theta}_0$ (retard, décalage de fréquence ou compression Doppler). Le paramètre A_0 caractérise l'amplitude du signal écho, ϕ_0 un changement de phase et $b(t)$ un bruit blanc gaussien de moyenne nulle et de variance σ^2. Lorsque les densités de probabilité des paramètres A_0 et ϕ_0 sont inconnues, ce qui est souvent le cas, la statistique Λ à maximiser, d'après la théorie de l'estimation au sens du Maximum de Vraisemblance Généralisée, est donnée par le carré du module du produit scalaire entre le signal reçu $x(t, \underline{\alpha}_0)$ et une certaine réplique $T_{\underline{\theta}} z(t)$ du signal émis adaptée à un jeu de paramètres particuliers $\underline{\theta}$. On peut montrer que la théorie du filtre optimal, qui consiste à déterminer le filtre qui maximise le rapport signal à bruit, conduit de même, en présence de bruit blanc, au calcul de cette vraisemblance généralisée, appelée sortie du filtre adapté.

$$\Lambda(\underline{\alpha}_0, \underline{\theta}) = \frac{1}{2\sigma^2} \frac{\left| \int_{-\infty}^{+\infty} x(t, \underline{\alpha}_0) \, T^*_{\underline{\theta}} z(t) \, dt \right|^2}{\int_{-\infty}^{+\infty} \left| T_{\underline{\theta}} z(t) \right|^2 \, dt} \tag{A.24}$$

Lorsque le rapport signal à bruit est élevé ou qu'il n'y a pas de bruit additif, ce produit scalaire, plus connu sous le nom de fonction d'ambiguïté du signal, est maximal pour un jeu de paramètres $\underline{\theta}$ égal à celui de la cible $\underline{\theta}_0$. Il suffit alors de déterminer le maximum de cette fonction pour accéder directement aux paramètres cherchés. Il est clair que la qualité de ce genre d'estimation n'est pas parfaite et il convient alors de pouvoir quantifier l'erreur d'estimation commise sur les paramètres. La qualité d'un estimateur $\hat{\theta}_i$ est généralement mesurée par sa variance $Var(\theta_i - \hat{\theta}_i)$. Pour un estimateur non biaisé ($E(\hat{\theta}_i) = \theta_i$), cette variance possède une borne minimale donnée par les bornes de Cramer-Rao [Van71]. Les bornes de Cramer-Rao sont explicitées en inversant une matrice dite *Matrice d'Information de Fisher* définie par :

$$J_{i,j} = \left(-E \left[\frac{\partial^2 \Lambda}{\partial \theta_i \partial \theta_j} \right] \right)_{i,j} \tag{A.25}$$

où θ_i représente chaque composantes du vecteur $\underline{\theta}$.

A.2.2 La Fonction d'Ambiguïté

La fonction d'ambiguïté est donc une fonction de discrimination entre un signal et sa transformée par un opérateur $T_{\underline{\theta}}$ de paramètres inconnus $\underline{\theta}$ (translation de temps, de fréquence, compression, etc...).

A.2.2.1 La Fonction d'Ambiguïté Bande Étroite

La fonction d'ambiguïté bande étroite, telle qu'on la rencontre en théorie du radar ou du sonar, est construite sur le signal analytique $z(t)$ ou sur sa transformée de Fourier $Z(f)$. Supposons qu'une cible située à une distance R_0 et animée d'une vitesse radiale v_0 soit illuminée par un signal $z(t)$ caractérisé par une largeur de bande B, une fréquence porteuse f_0 et une durée T. Sous les hypothèses simplificatrices de Woodward [Wood53] énoncées ci-dessous,

- vitesse relative des cibles v_0 négligeable devant la célérité c dans le milieu de propagation
- signal bande étroite $B \ll f_0$
- $BT \ll \dfrac{c}{2v_0}$

l'effet Doppler peut être approximé par un simple décalage de fréquence sur le signal $z(t)$. Ainsi, le signal écho $x(t, \underline{\alpha}_0)$ peut être mis sous la forme :

$$x(t, \underline{\alpha}_0) = T_{\underline{\theta}} z(t) + b(t) = A_0\, z(t-\tau)\, e^{2i\pi\nu t}\, e^{i\phi_0} + b(t) \tag{A.26}$$

où $\nu = 2v_0 f_0/c$ est le décalage Doppler et τ le retard de propagation (position radiale $R_0 = c\tau/2$) et la fonction d'ambiguïté prend alors la forme suivante :

$$X(\tau, \nu) = \int_{-\infty}^{+\infty} z(t)\, z^*(t-\tau)\, e^{-2i\pi\nu t}\, dt \tag{A.27}$$

où $\nu = 2(v - v_0) f_0/c$ est la différence entre la fréquence Doppler du signal reçu et la fréquence Doppler du signal réplique et $\tau = 2(R - R_0)/c$ est la différence des retards entre le signal reçu et la réplique.

La Matrice d'Information de Fisher (Eq. A.25) est aisément calculable [Van71] et conduit à

$$J = \frac{4\pi^2 A_0^2}{\sigma^2} \begin{pmatrix} \sigma_f^2 & f_0\, t_0 - m \\ f_0\, t_0 - m & \sigma_t^2 \end{pmatrix} \tag{A.28}$$

où les moments du premier ordre f_0 et t_0 représentent la fréquence centrale et l'époque moyenne et où ceux du second ordre, σ_f et σ_t, représentent la largeur de bande et la durée du signal.

$$f_0 = \int_{-\infty}^{+\infty} f\, |Z(f)|^2\, df \qquad \sigma_f^2 = \int_{-\infty}^{+\infty} (f - f_0)^2\, |Z(f)|^2\, df \tag{A.29}$$

$$t_0 = \int_{-\infty}^{+\infty} t\, |z(t)|^2\, dt \qquad \sigma_t^2 = \int_{-\infty}^{+\infty} (t - t_0)^2\, |z(t)|^2\, dt \tag{A.30}$$

Le paramètre noté m définit l'indice de modulation du signal

$$m = -\frac{1}{2\pi}\, Im \int_{-\infty}^{+\infty} t\, z(t)\, \frac{dz^*}{dt}\, dt \tag{A.31}$$

Chaque valeur des bornes minimales de la variance des estimateurs est obtenue en inversant la matrice (Eq. A.28), ce qui conduit à :

$$E\left[(\nu - \hat{\nu})^2\right] = \frac{\sigma^2}{4\pi^2 A_0^2}\, \frac{\sigma_f^2}{\sigma_t^2 \sigma_f^2 - (m - f_0 t_0)^2} \geq \frac{\sigma^2}{4\pi^2 A_0^2}\, \frac{1}{\sigma_t^2} \tag{A.32}$$

$$E\left[(\tau - \hat{\tau})^2\right] = \frac{\sigma^2}{4\pi^2 A_0^2}\, \frac{\sigma_t^2}{\sigma_t^2 \sigma_f^2 - (m - f_0 t_0)^2} \geq \frac{\sigma^2}{4\pi^2 A_0^2}\, \frac{1}{\sigma_f^2} \tag{A.33}$$

Ces résultats bien connus prouvent qu'en théorie du radar, les signaux optimaux sont donnés par des codes qui réalisent à la fois un bon comportement en résolution radiale (grande largeur de bande σ_f) ainsi qu'en vitesse (grande durée σ_t). Ce sont les signaux à grand produit bande durée BT.

A.2.2.2 La Fonction d'Ambiguïté Large Bande

Lorsque l'une des hypothèses de Woodward [Wood53] n'est pas respectée, il est nécessaire de considérer la fonction d'ambiguïté dite large bande [Kelly65] ou encore généralisée, sous peine de dégrader les performances du système (forte atténuation du maximum de la sortie de filtre adapté, dégradation de la largeur des pics principaux, etc...). Dans ce cas, le signal écho $x(t)$ peut alors se mettre sous la forme :

$$x(t) = \frac{A_0}{\sqrt{a_0}}\, z\left(a_0^{-1}t - b_0\right)\, e^{i\phi_0} + b(t) \tag{A.34}$$

Les deux formes équivalentes de la fonction d'ambiguïté large bande sont alors données par :

$$X(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} z(t)\, z^*\left(\frac{t}{a} - b\right) dt \tag{A.35}$$

$$= \sqrt{a} \int_{0}^{+\infty} Z(f)\, Z^*(af)\, e^{2i\pi abf}\, df \tag{A.36}$$

où $a = (c - v_0)/(c + v_0)$ est le facteur de compression Doppler et $b = 2R_0/(c - v_0)$ est le retard.

La transformation de Mellin permet de calculer exactement cette quantité. En utilisant la propriété d'unitarité de la transformation de Mellin (Eq. A.3) avec $r = -1/2$, la fonction d'ambiguïté large bande (Eq. A.36) $X(a,b)$ devient :

$$X(a,b) = \int_{-\infty}^{+\infty} M[Z](\beta)\, M^*[U_0 Z](\beta)\, d\beta \tag{A.37}$$

avec $U_0 Z$ la transformation définie par

$$Z \longrightarrow U_0 Z(f) = \sqrt{a}\, e^{-2i\pi abf}\, Z(af) \tag{A.38}$$

En posant $Z_b(f) = Z(f)\, e^{-2i\pi bf}$, l'invariance d'échelle (Eq. A.6) de la transformée de Mellin permet d'écrire (Eq. A.37) comme :

$$X(a,b) = \int_{-\infty}^{+\infty} M[Z](\beta)\, M^*[Z_b](\beta)\, a^{2i\pi\beta}\, d\beta \tag{A.39}$$

Le facteur de compression a n'intervient plus dans le calcul des transformations de Mellin. Pour un facteur de translation temporelle donné b, il suffit d'effectuer la transformée de Fourier inverse sur β du produit des deux transformées de Mellin pour obtenir les échantillons a.

Dans le cas large bande, estimer la vitesse ne consiste pas en un simple problème d'estimation de décalage de fréquence Doppler mais en un problème d'estimation d'un facteur de compression. La statistique à maximiser est donnée par le carré du module de la fonction d'inter-ambiguïté large bande (reécrite dans le domaine spectral) :

$$\Lambda = \frac{a}{2\sigma^2} \left| \int_{0}^{+\infty} X(f)\, Z^*(af)\, e^{2i\pi abf}\, df \right|^2 \tag{A.40}$$

où les paramètres $a = (c + v)/(c - v)$ et b représentent les paramètres facteur de compression Doppler et retard à estimer. Un calcul direct de la matrice de Fisher par des méthodes classiques n'est, premièrement, pas facile à effectuer et, deuxièmement, ne conduit à aucune interprétation physique de ses coefficients, comme dans le cas bande étroite (Eq. A.28). En utilisant la Transformation de Mellin, le calcul des coefficients de la matrice de Fisher est, d'une part, très facilité et conduit, d'autre part, à une interprétation physique de chacun de ses coefficients.

De la même manière que pour l'équation (Eq. A.39), l'équation (Eq. A.40) se réécrit :

$$\Lambda = \frac{1}{2\sigma^2} \left| \int_{-\infty}^{+\infty} M[X](\beta)\, M^*[Z_b](\beta)\, a^{2i\pi\beta}\, d\beta \right|^2 \tag{A.41}$$

On peut alors montrer que la Matrice d'Information de Fisher prend la forme :

$$J = \frac{4\pi^2 A_0^2}{\sigma^2} \begin{pmatrix} \sigma_\beta^2 & f_0\beta_0 - M \\ f_0\beta_0 - M & \sigma_f^2 \end{pmatrix} \tag{A.42}$$

où les paramètres σ_f et f_0 définissent respectivement largeur de bande et fréquence centrale du signal et où les paramètres β_0, σ_β sont donnés par :

$$\beta_0 = \int_{-\infty}^{+\infty} \beta \left|M[Z](\beta)\right|^2 d\beta \quad \sigma_\beta^2 = \int_{-\infty}^{+\infty} (\beta - \beta_0)^2 \left|M[Z](\beta)\right|^2 d\beta \tag{A.43}$$

Les moments du premier et du second ordre sur le carré du module de la transformée de Mellin peuvent être interprétés respectivement comme le β moyen et l'étendue du signal Z dans l'espace de Mellin. L'indice de modulation large bande noté M et défini par :

$$M = -\frac{1}{2\pi} Im \int_0^{+\infty} f^2 \frac{dZ}{df} Z^*(f)\, df \tag{A.44}$$

joue ici le même rôle pour les signaux hyperboliques que l'indice de modulation bande étroite m pour les signaux 'chirps' (signaux à modulation linéaire de fréquence). Enfin, la quantité A_0/σ est le rapport signal à bruit.

Afin d'estimer la qualité des paramètres retard et compression, la Matrice d'Information de Fisher doit être inversée. Chaque terme de la matrice inverse J^{-1} donne la valeur minimale de la variance de chaque estimateur. Comme les estimateurs sont non biaisés et efficaces (fort rapport signal à bruit, nombre de mesures important), les bornes de Cramer-Rao sont atteintes et on obtient les résultats importants suivants :

- La variance de l'estimateur retard $\hat{b}$ est donnée par :

$$E\left[(b - \hat{b})^2\right] = \frac{\sigma^2}{4\pi^2 A_0^2} \frac{\sigma_\beta^2}{\sigma_f^2\sigma_\beta^2 - (M - \beta_0 f_0)^2} \geq \frac{\sigma^2}{4\pi^2 A_0^2} \frac{1}{\sigma_f^2} \tag{A.45}$$

Ce premier résultat (Eq. A.45) montre que la précision retard (ou distance) est, comme dans le cas bande étroite, proportionnelle à l'inverse de la largeur de bande du signal.

- La variance de l'estimateur facteur de compression $\hat{a}$ est donnée par :

$$E\left[(a - \hat{a})^2\right] = \frac{\sigma^2}{4\pi^2 A_0^2} \frac{\sigma_f^2}{\sigma_f^2\sigma_\beta^2 - (M - \beta_0 f_0)^2} \geq \frac{\sigma^2}{4\pi^2 A_0^2} \frac{1}{\sigma_\beta^2} \tag{A.46}$$

- La variance de l'estimateur vitesse $\hat{v}$ est donnée par :

$$E\left[(v - \hat{v})^2\right] = \frac{c^2}{4} E\left[(a - \hat{a})^2\right] \tag{A.47}$$

- La covariance des estimateurs croisés compression-retard est donnée par :

$$E\left[(a - \hat{a})(b - \hat{b})\right] = \frac{\sigma^2}{4\pi^2 A_0^2} \frac{M - \beta_0 f_0}{\sigma_f^2\sigma_\beta^2 - (M - \beta_0 f_0)^2} \tag{A.48}$$

Sous les hypothèses de Woodward, les hyperboles qui délimitent le signal dans le demi plan temps-fréquence peuvent être assimilées à des droites parallèles à l'axe des fréquences. De ce fait, les grandeurs β, β_0, σ_β, M et $a = (c + v)/(c - v)$ peuvent être respectivement approximées par $f_0 t$, $f_0 t_0$, $f_0\sigma_t$, $f_0 m$ et $1 + 2v/c$. En substituant ces approximations dans (Eq. A.46) et (Eq. A.47), on retrouve les résultats classiques du cas bande étroite :

$$E\left[(v - \hat{v})^2\right] = \frac{c^2\sigma^2}{16\pi^2 A_0^2 f_0^2} \frac{\sigma_f^2}{\sigma_t^2\sigma_f^2 - (m - f_0 t_0)^2} \tag{A.49}$$

Les deux résultats (Eq. A.46) et (Eq. A.47) sont très importants car ils montrent que la précision Doppler dépend maintenant, non plus seulement de la durée du signal comme dans le cas bande étroite, mais de l'étendue du signal dans l'espace de Mellin.

A titre d'exemple, si l'on considère un signal hyperbolique, c'est à dire à temps de propagation de groupe hyperbolique :

$$Z(f) = f^{2i\pi\beta_0 - \frac{1}{2}} \tag{A.50}$$

il est clair que son étendue temporelle σ_t n'est pas nulle. De ce fait, la variance bande étroite (Eq. A.32) de l'estimateur de vitesse qui est donnée, à un coefficient près, par l'inverse de son étendue temporelle, peut être considérée comme non-infinie, ce qui donne au signal un 'faux' certain pouvoir de précision vitesse. Calculons maintenant la variance large bande (Eq. A.46) en vitesse de ce signal : $\sigma_\beta = 0$ ce qui implique une variance infinie (pas de précision en vitesse). Le signal hyperbolique, aussi long soit-il, est donc un signal qui ne donne aucun pouvoir de séparation en vitesse.

Annexe B

Discriminating Real Objects in Radar Imaging by Exploiting the Squared Modulus of the CWT

Discriminating real objects in radar imaging by exploiting the squared modulus of the continuous wavelet transform

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Abstract: New technique based on continuous wavelet transform (CWT) for classifying objects in synthetic aperture radar (SAR) imaging is presented. The CWT allows to analyse two-dimensional SAR images to highlight the frequency and angular behaviour of the scatterers ref. 10, 11. This technique allows to build a SAR hyperimage, that is, a four-dimensional data cube which represents for each spatial location (x, y) of the scatterer in the image, its frequency and angular energy behaviour. When analysing different targets, objects or areas in SAR images, it has been recently observed that some scatterers belonging to a same class of objects could have similar frequency and angular energy responses. The previous observations have motivated the determination to exploit these energy responses to discriminate these objects. This discrimination is performed by frequency and angular correlations between the response of a particular scatterer (measured) and those of all the scatterers in the SAR image. Some examples of discrimination from real SAR data are presented and show an interest of the method for target classification and recognition for SAR imaging.

1 Introduction

The synthetic aperture radar (SAR) imaging process [1, 2] consists in the formation of high-resolution images. To do so, a moving radar emits pulses and collects the signals reflected by scatterers (see Fig. 1). The imaging plane is labelled using an x–y Cartesian coordinate system with origin at a reference point $\mathcal{O}$. We assume that the radar moves in a straight line and in a direction parallel to the cross-range direction. The radar position is described by azimuth angle θ defined counterclockwise from the y-direction. Moreover, we suppose far field conditions, and therefore we obtain plane-wave incidence on objects. In addition, we assume high-frequency radar measurements and consequently the scattering response of a man-made target is well approximated as a sum of responses from individual scatterers. In traditional SAR imaging, we consider a target as a collection of ideal scatterers, each with a complex amplitude σ_i and position $\boldsymbol{r}_i$. Hence, the scattering model for a given polarisation is [3–5]

$$H(\boldsymbol{k}) = \sum_{i=1}^{N} \sigma_i \exp(-j2\pi\, \boldsymbol{k} \cdot \boldsymbol{r}_i) \tag{1}$$

where $\boldsymbol{k}$ is the wavenumber vector in the direction of the scattered field. An example of monostatic SAR system is illustrated in Fig. 1 where the radar is carried by a moving aircraft. The position vector is $\boldsymbol{r} = (x, y)^{\mathrm{T}}$ where x and y represent, respectively, the slant-range and cross-range locations and the wavenumber vector $\boldsymbol{k}$ is

$$\boldsymbol{k} = \begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{pmatrix} k\cos\theta \\ k\sin\theta \end{pmatrix}$$

where $k = 2f/c$ is the wave number with f the frequency and c the speed of light, θ corresponds to the observation aspect.

Considering the scattering model in (1), the complex-valued image $I(\boldsymbol{r})$ is obtained by applying the 2D-Fourier transform on $H(\boldsymbol{k})$

$$\begin{aligned} I(\boldsymbol{r}) &= \mathcal{F}^{-1}_{\boldsymbol{k}\to\boldsymbol{r}} H(\boldsymbol{k}) \\ &= \sum_{i=1}^{N} \sigma_i \delta(\boldsymbol{r} - \boldsymbol{r}_i) \end{aligned} \tag{2}$$

However, the model proposed in (1) is less physically accurate for high relative bandwidth signals or when the SAR data are collected over a broad angular aperture. Indeed, if the radar measurements are carried out over a broad bandwidth and wide angular window, we have to suggest a model that takes the scattering phenomenology into account. This model is expressed as a summation of point scatterers multiplied by their respective frequency- and aspect-dependent amplitude function

$$H(\boldsymbol{k}) = \sum_{i=1}^{N} \sigma_i(f, \theta) \exp(-j2\pi\, \boldsymbol{k} \cdot \boldsymbol{r}_i) \tag{3}$$

where $\sigma_i(f, \theta)$ is a 2D-function determined by the geometry, composition and orientation of the scattering mechanism. First, the geometrical theory of diffraction (GTD) [6, 7] predicts that the frequency-dependent amplitude is of the form $(jf)^{\alpha}$ where α is related to the scatterer shape. For example, the scatterer can be a corner or a flat plate. Second, the

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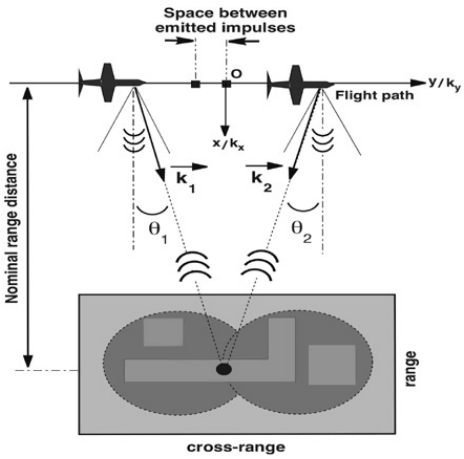

Fig. 1 *Illustration of a scatterer irradiated at two different aspect angles in non-squintmode SAR imaging*

amplitude dependence on aspect for localised scatterers such as trihedrals and corners is a slowly varying function modelled with a damped exponential function [8]. Third, the angle-dependent amplitude of distributed reflectors such as flat plates and dihedrals is dominated by a sinclike term as predicted by the physical optics far-field scattering theory [8]. By combining these dependences, the complex amplitude $\alpha_i(f, \theta)$ can be written in a single expression as follows

$$\sigma_i(f, \theta) = A_i\left(\frac{jf}{f_c}\right)^{\alpha_i} \sin c(kL_i \sin(\theta - \theta_i)) \times \exp(-2\pi f \gamma_i \sin(\theta)) \tag{4}$$

where A_i is a complex scalar, L_i and θ_i the length and the orientation of the ith scatterer, respectively, and γ_i a parameter with no physical interpretation. The parameters L_i and γ_i are zero for localised and distributed scatterers, respectively. The model in (3) combined with (4) provides a concise physically relevant description of measured scattering for use in target recognition, identification and discrimination.

Referring to the model in (3), we suggest the formation of an image that depends on the frequency and the aspect, that is, one that depends on the wave vector $\boldsymbol{k}$. This image will be written $I(\boldsymbol{r}, \boldsymbol{k})$ and called an hyperimage because its dimension is the product of the dimension of the vector $\boldsymbol{r}$ by the dimension of the vector $\boldsymbol{k}$. This hyperimage which provides a rich description of target scattering is not accessible from conventional Fourier-based imaging techniques and the construction needs a more advanced and appropriated analysis such as time–frequency methods. Among these methods, we propose to apply the multidimensional continuous wavelet transform (MCWT) in SAR imaging to construct such a hyperimage.

2 Construction of hyperimages using the MCWT

Before introducing the MCWT, we present the general construction of hyperimages from the time–frequency analysis. To do so, let us recall that the radar cross-section (RCS) of a target is defined as the squared modulus of the backscattering signal $H(\boldsymbol{k})$

$$\sigma(\boldsymbol{k}) = |H(\boldsymbol{k})|^2 \tag{5}$$

and is expressed in square meters [9]. The RCS is a function of the wavenumber $\boldsymbol{k}$, that is, the RCS depends on the variables (f, θ) and characterises the quantity of energy diffused by the target at the frequency f and aspect angle θ. Thus, the RCS provides in the f–θ plane, the distribution of the energy diffused by the scatterers. Unfortunately, the RCS provides no information concerning the position of these scatterers. To overcome this limitation, the time–frequency analysis has been introduced in SAR imaging to access to the energy distribution of the scatterers and also to their position. These two pieces of information can be obtained by constructing hyperimages $I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k})$ as hermitian and bilinear forms of the backscattering signal $H(\boldsymbol{k})$ [10]

$$I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k}) = \iint K(\boldsymbol{k}_1, \boldsymbol{k}_2; \boldsymbol{r}, \boldsymbol{k})\, H(\boldsymbol{k}_1)\, H^*(\boldsymbol{k}_2)\, \mathrm{d}\boldsymbol{k}_1\, \mathrm{d}\boldsymbol{k}_2 \tag{6}$$

The term on the right of (6) is, in fact, a generalisation of the RCS formulation in (5) and $I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k})$ is an energy distribution in the hyperplane $(\boldsymbol{r}, \boldsymbol{k})$. The kernel K is supposed to be hermitian and it can be determined with some physical constraints made on the distribution $I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k})$ [11, 12]; for example, one can impose that $I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k})$ should be positive in order that its integral on some surface $\mathcal{D}$ can be interpreted as the RCS contribution $\sigma_{\mathcal{D}}(\boldsymbol{k})$ of all the reflectors contained in $\mathcal{D}$

$$\sigma_{\mathcal{D}}(\boldsymbol{k}) = \int_{\mathcal{D}} I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k})\, \mathrm{d}\boldsymbol{r} \tag{7}$$

2.1 The MCWT, a particular method for constructing hyperimages

Let $\phi(k, \theta)$ be a mother wavelet supposed to represent the backscattering signal of a reference target localised around the position $\boldsymbol{r} = \boldsymbol{0}$. This target is supposed to diffuse mainly in the direction $\theta = 0$ and at the frequency f so that $k = 2f/c = 1$. In other words, $\phi(k, \theta)$ is a 2D-function localised around $(k, \theta) = (1, 0)$.

A family of wavelets $\Psi_{\boldsymbol{r}_0, \boldsymbol{k}_0}(\boldsymbol{k})$ can be generated from the mother wavelet $\phi(k, \theta)$ by rotation $\mathcal{R}_{\theta_0}$, translation $\boldsymbol{r}_0$ and contraction with the scale factor $1/k_0$ according to [9, 11]

$$\begin{aligned}\Psi_{\boldsymbol{r}_0, \boldsymbol{k}_0}(\boldsymbol{k}) &= \frac{1}{k_0} \mathrm{e}^{-2i\pi\, \boldsymbol{k}\cdot\boldsymbol{r}_0} \phi\left(\frac{1}{k_0} \mathcal{R}_{\theta_0}^{-1} \boldsymbol{k}\right) \\ &= \frac{1}{k_0} \mathrm{e}^{-2i\pi \boldsymbol{k}\cdot\boldsymbol{r}_0} \phi\left(\frac{k}{k_0}, \theta - \theta_0\right)\end{aligned} \tag{8}$$

The wavelet coefficients $C_{\mathrm{H}}(\boldsymbol{r}_0, \boldsymbol{k}_0)$ are introduced as the scalar product between the backscattering signal H and each element $\Psi_{\boldsymbol{r}_0, \boldsymbol{k}_0}$ of the wavelet basis

$$\begin{aligned}C_{\mathrm{H}}(\vec{\boldsymbol{r}}_0, \vec{\boldsymbol{k}}_0) &= \int H(\boldsymbol{k}) \Psi^*_{\boldsymbol{r}_0, \boldsymbol{k}_0}(\boldsymbol{k})\, \mathrm{d}\boldsymbol{k} \\ &= \int_0^2 \mathrm{d}\theta \int_0^{+\infty} k\, H(k, \theta) \\ &\quad \times \frac{1}{k_0} \mathrm{e}^{j2\pi\, \boldsymbol{k}\cdot\boldsymbol{r}_0} \phi^*\left(\frac{k}{k_0}, \theta - \theta_0\right) \mathrm{d}k\, \mathrm{d}\theta\end{aligned}$$

From the general form of hyperimages defined in (6), a particular solution can be proposed on the basis of wavelet analysis. This solution is expressed as the squared modulus of the wavelet coefficients divided by the

admissibility wavelet coefficient A_ϕ [11]

$$I_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k}) = \frac{1}{A_\phi} |C_{\mathrm{H}}(\boldsymbol{r}, \boldsymbol{k})|^2 \tag{9}$$

where A_ϕ is defined as

$$A_\phi = \int \frac{|\phi(\boldsymbol{k})|^2}{k^2} \mathrm{d}\boldsymbol{k} < \infty \tag{10}$$

The kernel K introduced in (6) which corresponds to the solution proposed in (9) is

$$K(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{r}, \boldsymbol{k}) = \frac{1}{A_\phi} \frac{k_1 k_2}{k^2} \mathrm{e}^{j2\pi(\boldsymbol{k}_1 - \boldsymbol{k}_2)\cdot \boldsymbol{r}} \times \phi^*\left(\frac{k_1}{k}, \theta_1 - \theta\right) \phi\left(\frac{k_2}{k}, \theta_2 - \theta\right)$$

The reconstruction property allows to recover the signal with the knowledge of its wavelet coefficients

$$H(\boldsymbol{k}) = \frac{1}{A_\phi} \int \mathrm{d}\boldsymbol{r}_0 \int C_{\mathrm{H}}(\boldsymbol{r}_0, \boldsymbol{k}_0) \Psi_{\boldsymbol{r}_0, \boldsymbol{k}_0}(\boldsymbol{k}) \, \mathrm{d}\boldsymbol{k}_0$$

2.2 Interpretation of the hyperimage $I(\boldsymbol{r}, \boldsymbol{k})$

Let us rewrite $I(\boldsymbol{r}, \boldsymbol{k}) \equiv I(x, y; f, \theta)$. For each reflector located at $\boldsymbol{r}_0 = (x_0, y_0)$, we can extract its energetic feature $I(x_0, y_0; f, \theta)$ as a function of frequency f and aspect angle θ [12].

To analyse this 4D structure, a visual display interface called i4d [13] has been developed and allows to carry out an interactive and dynamic analysis. Figs. 3*a* and *b* describes the interface i4d that consists in selecting a pixel located at (x_0, y_0) in the plane (x, y), and visualising its corresponding energy response $I(x_0, y_0; f, \theta)$. In Fig. 3*b*, the initial SAR image $I(x, y)$ is displayed inside the x–y plane (left plot), in order that the user be sure that the pixel he selects corresponds to a reflector belonging to the object to be analysed. On two sides of the image plane, there are sliders that allow to select a pixel in the x–y plane. The above and below right figures represent the energy response of the selected pixel, in 2D and 3D, respectively.

2.3 Wavelet parameters and resolutions

In this paper, we do not tackle the question of the mother wavelet choice. Actually, any mother wavelet (Klauder, Mexican hat, ...) that is well localised at $(k, \theta) = (1, 0)$ is suitable for analysing the SAR images. One of the more famous wavelets is the gaussian's wavelet. For our analysis, we have chosen the bi-dimensional gaussian, defined as

$$\phi(k, \theta) = Y(k) \mathrm{e}^{-(k-1)^2/\sigma_k^2} \mathrm{e}^{-\theta^2/\sigma_\theta^2}$$

where $Y(k)$ is the Heaviside distribution defined as

$$Y(k) = \begin{cases} 0 & \text{if } k \le 0 \\ 1 & \text{else} \end{cases}$$

Parameters $(\sigma_k, \sigma_\theta)$ characterise the frequency and angular resolutions, respectively: indeed, they determine the frequency and angular bandwidths δk and $\delta\theta$ of mother wavelet $\phi(k, \theta)$ and so control the frequency and angular spread of the wavelets $\Psi_{\boldsymbol{r}_0, \boldsymbol{k}_0}(\boldsymbol{k})$, respectively. Bandwidths δk and $\delta\theta$ are defined as the '3 dB pass-band' (also called 'mean-square bandwidth') [14]. This 3 dB pass-band is defined as follows:

- By denoting $k_1 = 1$ the reference wavenumber, 3 dB pass-band δk is defined according to

$$\frac{|\phi(k_1 \pm (\delta k/2), \theta)|^2}{|\phi(k_1, \theta)|^2} = \frac{1}{2}$$

- Angular bandwidth $\delta\theta$ is defined the same way with the reference angle $\theta_1 = 0$.

In our applications, $\delta\theta$ is equal to 15% of the full angular aperture $\Delta\theta$. Bandwidth δk has been determined so that the wavelets localised at $k_c = 2f_c/c$ (f_c is the central frequency) have a spread equal to 15% of $\Delta k = 2B/c$ (B is the frequency bandwidth).

2.4 Algorithm for computing the MCWT

To describe the algorithm which allows to extend the MCWT in SAR imaging, let us recall the mathematical expression of the wavelet transform in (9) by considering a real mother wavelet ϕ

$$\begin{aligned} C(\boldsymbol{r}_0, \boldsymbol{k}_0) &= \int_0^{2\pi} \int_0^{+\infty} k\, H(k, \theta) \Psi_{\boldsymbol{r}_0, \boldsymbol{k}_0}(k, \theta) \, \mathrm{d}\boldsymbol{k} \, \mathrm{d}\theta \\ &= \int_0^{2\pi} \int_0^{+\infty} k\, H(k, \theta) \frac{1}{k_0} \phi\left(\frac{k}{k_0}, \theta - \theta_0\right) \\ &\quad \times \exp(j2\pi[kx_0 \cos\theta + k y_0 \sin\theta]) \mathrm{d}\boldsymbol{k} \, \mathrm{d}\theta \end{aligned}$$

With a change of variables $(k_x, k_y) = (k \cos\theta, k \sin\theta)$, the second member of the previous equation can be expressed as

$$\begin{aligned} &C(\boldsymbol{r}_0, \boldsymbol{k}_0) \\ &= \int_{\mathbb{R}^2} H(k_x, k_y) \frac{1}{k_0} \phi\left(\frac{\sqrt{k_x^2 + k_y^2}}{k_0}, \arctan\left(\frac{k_y}{k_x}\right) - \theta_0\right) \\ &\quad \times \exp(j2\pi[k_x x_0 + k_y y_0]) \, \mathrm{d}k_x \, \mathrm{d}k_y \\ &= \mathcal{F}^{-1}_{\substack{k_x \to x_0 \\ k_y \to y_0}} [H(k_x, k_y) \Phi_{k_0, \theta_0}(k_x, k_y)] \end{aligned} \tag{11}$$

where

$$\Phi_{k_0, \theta_0}(k_x, k_y) = \frac{1}{k_0} \phi\left(\frac{\sqrt{k_x^2 + k_y^2}}{k_0}, \arctan\left(\frac{k_y}{k_x}\right) - \theta_0\right)$$

Hence, referring to the relation (11), the wavelet coefficients can be calculated by using the bi-dimensional Fourier transform. The steps of the algorithm are developed in the following paragraph.

2.4.1 Steps of the algorithm:

First, let us construct the spectrum $H(k_x, k_y)$ by performing the bi-dimensional Fourier transform of the complex-valued image $I(x, y)$

$$H(k_x, k_y) = \mathcal{F}_{\substack{x \to k_x \\ y \to k_y}} [I(x, y)] \tag{12}$$

then, for each couple of wavenumber–angle values (k_i, θ_j),

(1) Construct the analysis window localised around (k_i, θ_j), $\phi_{k_i, \theta_j}(k, \theta) = (1/k_i)\phi(k/k_i, \theta - \theta_j)$ using the

variables (k_x, k_y)

$$\Phi_{k_i,\theta_j}(k_x, k_y) = \frac{1}{k_i}\phi\left(\frac{\sqrt{k_x^2 + k_y^2}}{k_i}, \arctan\left(\frac{k_y}{k_x}\right) - \theta_j\right)$$

(2) Make the product: $\Pi_{k_i,\theta_j}(k_x, k_y) = \boldsymbol{H}(k_x, k_y)\,\Phi_{k_i,\theta_j}(k_x, k_y)$
(3) The wavelet coefficient C_{H} $(\boldsymbol{r}_0, \boldsymbol{k}_{ij})$ is obtained by bi-dimensional inverse fourier transform on the 2D-function Π_{k_i,θ_j} (k_x, k_y). To compute them efficiently, we use the fast Fourier transform (FFT)

$$C_{\mathrm{H}}(\boldsymbol{r}_0, \boldsymbol{k}_{ij}) = \mathrm{FFT}_{\substack{k_x \to x_0 \\ k_y \to y_0}}[\Pi_{k_i,\theta_j}(k_x, k_y)] \tag{13}$$

Once the wavelet coefficients are calculated for all the couples of wavenumber–angle values (k_i, θ_j), the hyperimage I_{H} $(\boldsymbol{r}_0, \boldsymbol{k}_0)$ is defined by the squared modulus of the wavelet coefficients divided by the admissibility coefficient A_ϕ

$$I_{\mathrm{H}}(\boldsymbol{r}_0, \boldsymbol{k}_0) = \frac{1}{A_\phi}\left|C_H(\boldsymbol{r}_0, \boldsymbol{k}_0)\right|^2$$

2.4.2 Complexity of the algorithm: The approximate complexity of the algorithm can be expressed in terms of the number of FFT performed in step 3, (13). Assuming a complex-valued image $I(x, y)$ of $(N_x \times N_y)$ samples and referring to the Fourier relation in (12), the spectrum $H(k_x, k_y)$ will be on a grid of $(N_{k_x} \times N_{k_y})$ samples with $N_{k_x} = N_x$ and $N_{k_y} = N_y$. We denote (N_{k_0}, N_{θ_0}) the numbers of values $k_0 = 2f_0/c$ (where f_0 represents a frequency value) and aspects θ_0.

Consequently, by referring to the relation in (13), we have to deal with

$$N_{k_0}N_{\theta_0}(N_{k_x}\mathrm{FFT}\,[\text{of } N_{k_y} \text{ samples}] + N_{k_y}\mathrm{FFT}\,[\text{of } N_{k_x} \text{ samples}])$$

In our SAR applications, the values used are typically

$$N_x = N_{k_x} = 512, \quad N_y = N_{k_y} = 512, \quad N_{k_0} = N_{\theta_0} = 40$$

The following section is devoted to the use of the hyperimage I_{H} for target discrimination in SAR imaging. Indeed, by fixing the first and the second dimension of the hyperimage, we access to I_{H} $(x_i, y_i; f, \theta)$ which is the energy distribution of the scatterer localised at (x_i, y_i). Referring to the scattering model given by (3) and (4) which takes into account the scatterers' geometry, our aim is to distinguish in a SAR image, one object from other ones by exploiting the energy distributions of all scatterers contained in this image.

3 Discriminating different kinds of object using the wavelet analysis

The ONERA airborne [15] SAR system, called RAMSES, allows to process real data collected in different frequency ranges (X-band, Ku-band, P-band, ...) for the formation of high-resolution SAR images. Therefore, the target discrimination method proposed in this paper can be potentially tested on many targets. In this paper, we analyse four SAR images and we test the target discrimination technique on different objects included in these images.

3.1 Highlighting the reflectors' energy distributions

In this paragraph, we highlight the frequency and aspect angle dependencies of real scatterers by analysing four SAR images. The analysis of the first two images is also described [12, 16, 17]. These images come from the same measurement campaign with a fixed objects and targets orientation on the ground. So it has not been possible to test the robustness of this technique for different object orientations. In the examples presented below, the central frequency is $f_c = 14.2$ GHz (Ku-band), the bandwidth is $B = 900$ MHz (that corresponds to a relative bandwidth $B_r = B/f_c = 6\%$) and the full angular aperture is $\Delta_\theta = 3°$. Hence, the analysed target or object is seen only for a subset of wavenumbers $\boldsymbol{k}$. So, it implies that the wavelet analysis can give a full description of the scatterers RCS only in this range of measure. This fact limits the possibility of discriminating the target for any orientation on the ground. The following is the description of the four analysed SAR images:

- The first image is composed of a pair of tanks, stairs (on the left) and a footbridge laid along the vertical (see area (a) in Fig. 2). Thanks to the interface i4d described in the Fig. 3, we have selected different pixels (in the x–y plane) localised on the tanks, the footbridge and the stairs, and we have visualised the corresponding energy response in the f–θ plane: we have carried the responses of these pixels forward to the Fig. 4. Observing Fig. 4, we can notice that the tanks' reflectors have similar responses. Also, on the footbridge and the stairs, the analysed scatterers' responses are almost identical: the aspect behaviours seem to be dominated by a gaussian or a sinclike as predicted by the model in (4) for distributed reflectors. However, these reflectors have energetic characteristics which differ from the ones of the reflectors on the pair of tanks. Considering the previous experimental observations, our aim has been to separate the pair of tanks from the combination of the footbridge and the stairs by exploiting the energetic characteristics of all the reflectors in the image.

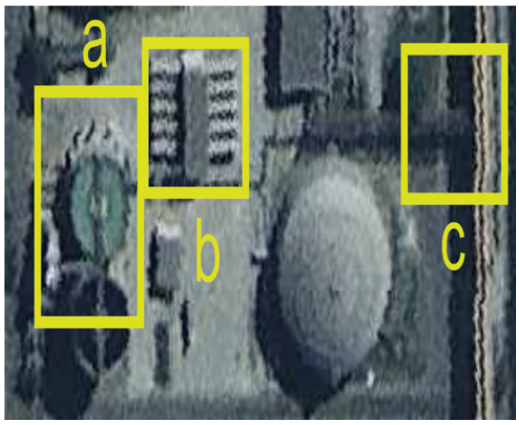

Fig. 2 *Aerial photography containing three of the four images analysed*

a Couple of tanks with a footbridge (along the vertical) and stairs (on the left)
b Lateral buildings attached to a big one
c Building (along the horizontal) and pipe (along the vertical)

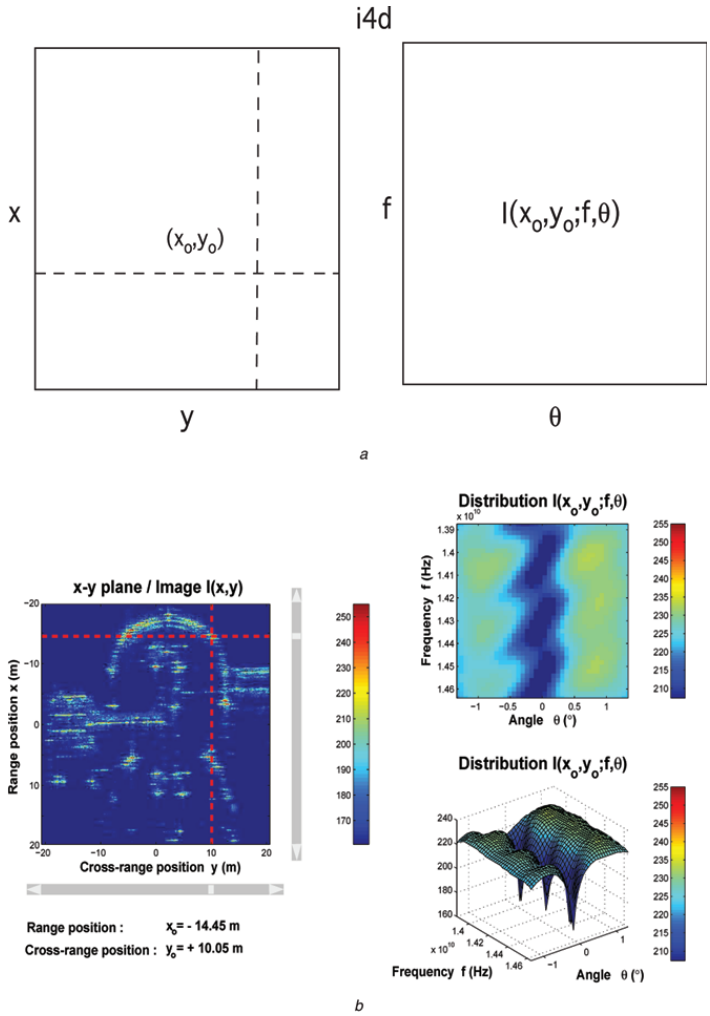

Fig. 3 *Visual interface i4d to visualise the energy response of a given reflector*
a Presentation of i4d
b Visualisation of the response of a reflector belonging to a tank

• The second study concerns a warehouse: lateral buildings are attached to a big one on both sides (see area (b) in Fig. 2). Again, with the help of the interface i4d, we have selected different pixels (in the x–y plane) localised on the lateral buildings' roofs and on the central building's ridge, and we have visualised the corresponding energy response in the f–θ plane: we have carried the responses of these pixels forward to Fig. 5. On the lateral buildings' roofs, the responses of the analysed reflectors are almost identical and again the aspect-varying responses seem to follow the prediction of the model in (4) in the case of distributed reflectors. However, the responses of these reflectors differ clearly from the responses of the scatterers on the central building's ridge that seem more random. At the sight of the results presented in the Fig. 5, the work has consisted in distinguishing the roof of the lateral buildings from the ridge of the central one thanks to the knowledge of the energetic characteristics of these two types of object. Following the above step, we have visualised the response of different scatterers in the following two SAR images, and the observations that we have made in terms of scatterers' energy response, have suggested us to discriminate some target or other.

• Area (c) in Fig. 2 represents a building laid along the horizontal, and a pipe along the vertical: our aim has been to extract the building and subtract the pipe.

• The last image analysed is a building with objects that we are not able to identify, on the roof (see Fig. 9): the purpose has been to extract one ridge of the building and subtract the objects on the roof.

The observations presented in the Figs. 4 and 5 reveal different classes of reflectors at the sight of their frequency and behaviours. Each class of reflectors corresponds to a specific type of object. These observations lead us to propose an algorithm whose purpose is to discriminate a

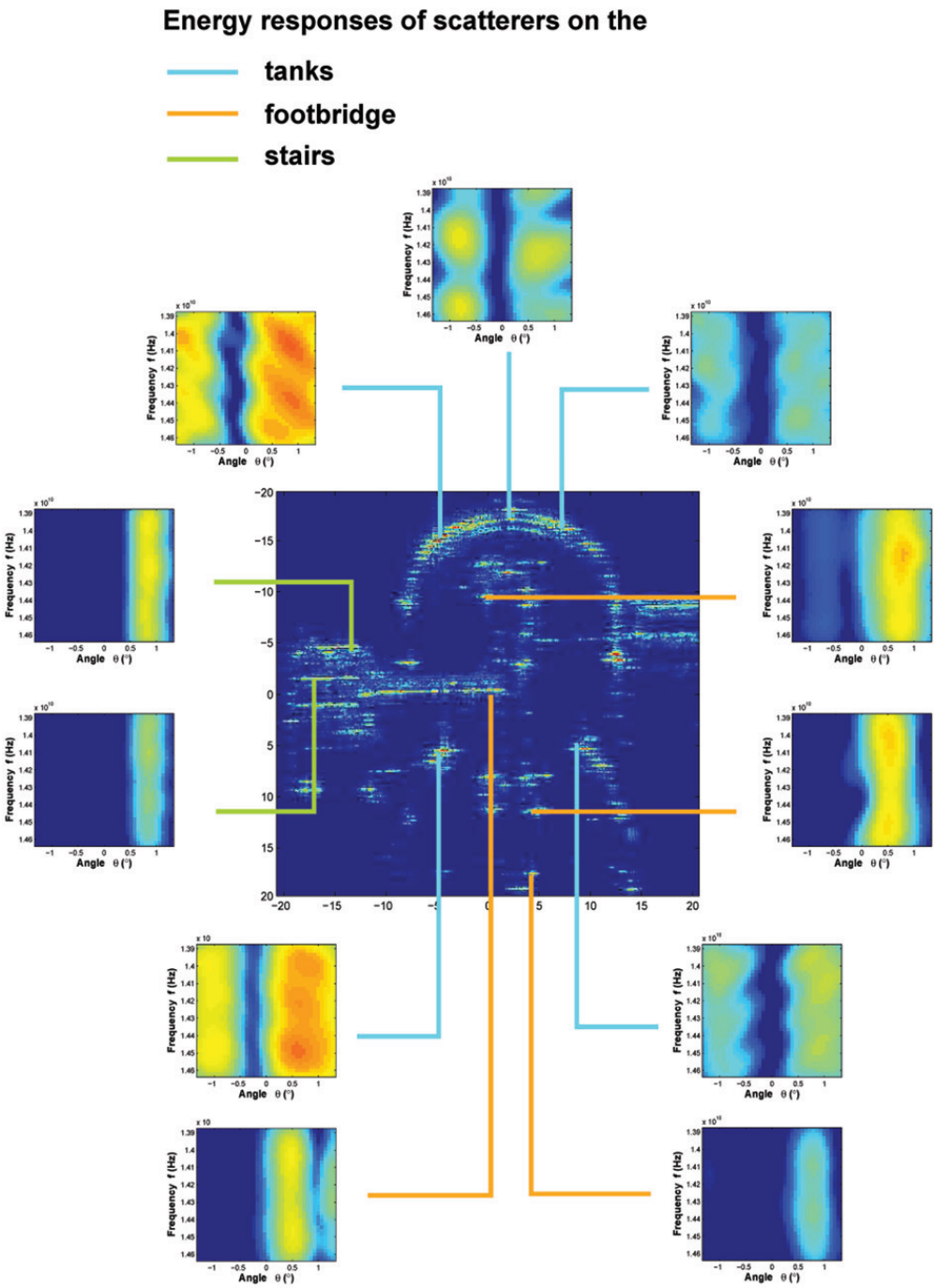

Fig. 4 *Energetic characteristics of some scatterers localised on the tanks and the combination of the footbridge and the stairs*

type of object in a SAR image by exploiting the energy distributions of all the scatterers.

3.2 Development of an algorithm for discriminating different types of object

The object type discrimination algorithm (OTDA) that we propose in this section makes two hypotheses:

- One scatterer which belongs to an object is supposed to have a frequency and aspect behaviour similar to a scatterer of another reflector localised in this object.
- The energy distribution of one reflector belonging to a first object is assumed to differ from the distribution of another reflector localised on a second object with different geometry and/or orientation relative to the former object.

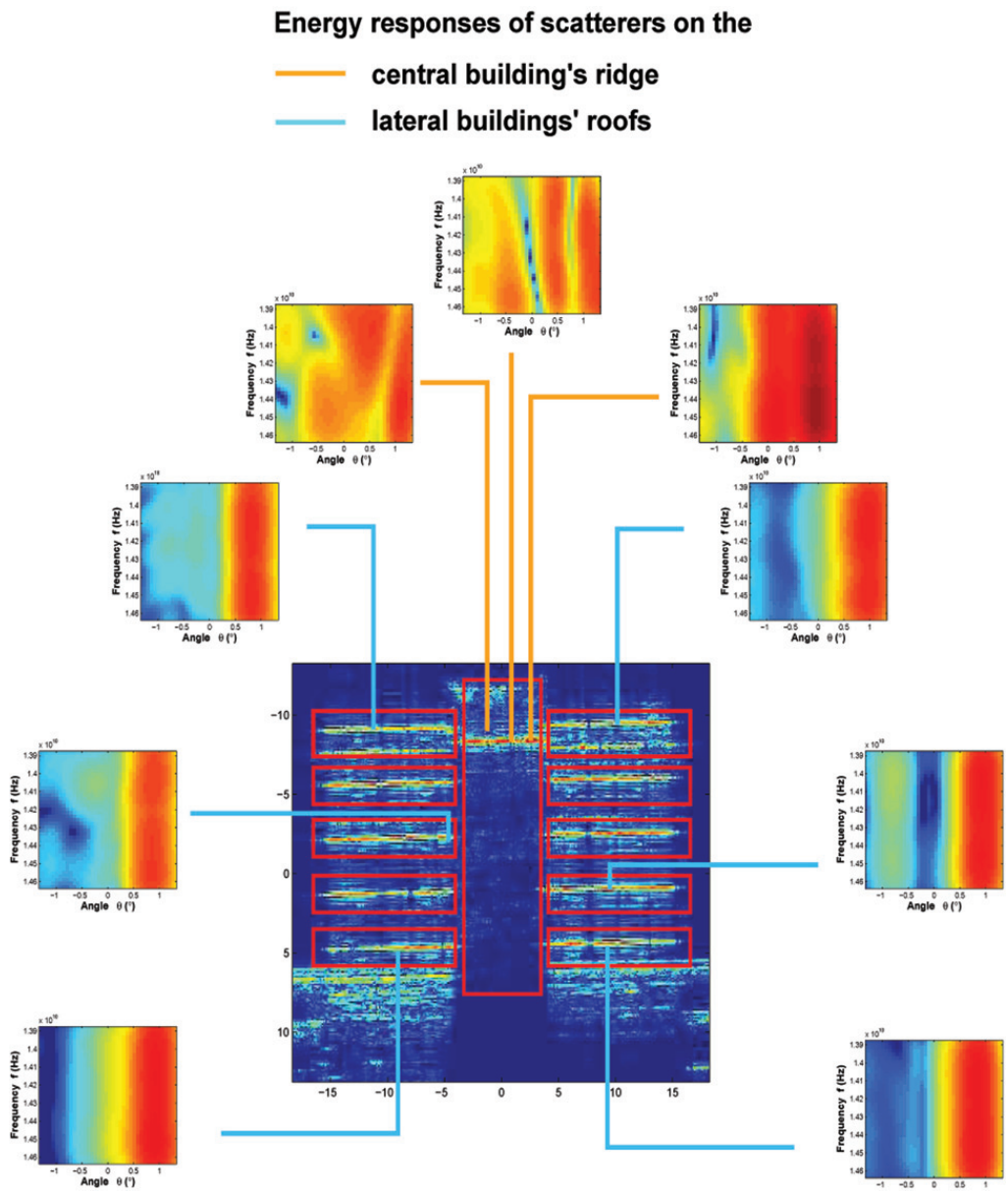

Fig. 5 *Energetic characteristics of some scatterers localised on the lateral buildings' roofs and on the central building's ridge*

The basic idea we propose is to select among all the scatterers' energy distributions, one energy distribution susceptible to be characteristic of the type of object to be discriminated: this distribution becomes a reference. Then, the purpose is to identify the scatterers in the image which have similar distributions to the reference.

Before introducing the OTDA, a threshold is applied on all the energy distributions since we suppose that the energetic characteristics of a scatterer remain within a confined dynamic of its energy distribution. Consequently, we choose a dynamic $-\Delta_{\mathrm{dB}}$ identical for all the distributions: the threshold, for each distribution, is placed at $-\Delta_{\mathrm{dB}}$ from the maximum of this distribution. To do so, we sweep across the x–y plane and for each pixel localised in (x_i, y_j), we determine the maximum value of the corresponding distribution $I(x_i, y_j; f, \theta)$: $I_{i,j}^{\max} = \max_{f,\theta} [I(x_i, y_j; f, \theta)]$ then we calculate the corresponding threshold value: $T_{i,j} = I_{i,j}^{\max} 10^{-(\Delta_{\mathrm{dB}}/20)}$, finally we set to zero all the energy values that are smaller than the threshold $T_{i,j}$

$$\text{if } I(x_i, x_j; f, \theta) \leq T_{i,j}, \text{ then } I(x_i, x_j; f, \theta) = 0$$

Once the threshold process is done, we execute the OTDA. The OTDA consists in selecting arbitrarily a scatterer on the target to be extracted, whose energy response seems to be characteristic of this target. The process for selecting the reflector is the following: by using the interface i4d (see Fig. 3*b*), we visualise the energy response of different

pixels localised on the object, and we select a reflector whose energy response is similar to another responses that we have visualised and corresponding to reflectors on the target. Then, we correlate the high-energy response of the reflector selected, with the high-energy response of all the pixels in the image plane. The more the high-energy response of a pixel similar to the reference high-energy response, the larger the corresponding correlation coefficient value. The objective of the above method is that the large correlation coefficient values highlight the scatterers that compose the target to be discriminated. The steps of the OTDA are described as follows:

Object type discrimination algorithm

- Select a pixel on the object to be distinguished. By denoting (x_0, y_0) the coordinates of the selected pixel, the energy distribution of this pixel $I_0(f, \theta) \equiv I(x_0, y_0; f, \theta)$ is considered as a reference function.
- Calculate the following 2D-correlation function by introducing the reference distribution and the distribution of the other pixels in the image $\{I(x_i, x_j; f, \theta); (i, j) \in [1, N_x] \times [1, N_y]\}$

$$C_0(x_i, y_j) = \frac{1}{\sqrt{E_0}\sqrt{E_{i,j}}} \int I_0(f, \theta) I_{i,j}(f, \theta)\, \mathrm{d}f\, \mathrm{d}\theta$$

where (N_x, N_y) are the numbers of samples in slant-range and cross-range, respectively, we adopt the simplified notation $I_{i,j}(f, \theta) \equiv I(x_i, x_j; f, \theta)$ and $E_{i,j}$ is defined as

$$E_{i,j} = \int |I(x_i, y_j; f, \theta)|^2\, \mathrm{d}f\, \mathrm{d}\theta$$

Remark 1: This algorithm does not exploit the phase of the wavelet coefficients which contains information. Thus, it can be improved by introducing the complex wavelet coefficients in the correlation function [21] instead of the energy distribution: the 2D correlation function becomes

$$C_{\mathrm{ref}}(i, j) = \frac{1}{\sqrt{E_{\mathrm{ref}}}\sqrt{E_{i,j}}} \times \int W(x_{\mathrm{ref}}, y_{\mathrm{ref}}; f, \theta) W^*(x_i, y_j; f, \theta)\, \mathrm{d}f\, \mathrm{d}\theta$$

where

$$E_{\mathrm{ref}} = \int |W(x_{\mathrm{ref}}, y_{\mathrm{ref}}; f, \theta)|^2\, \mathrm{d}f\, \mathrm{d}\theta$$

$$E_{i,j} = \int |W(x_i, y_i; f, \theta)^2\, \mathrm{d}f\, \mathrm{d}\theta$$

This substitution of the distributions by the wavelet coefficients is justified by the fact that the wavelet coefficient contains a phase information that is susceptible to discriminate the target. This idea is strengthened by the fact that the use of the complex wavelet coefficients in polarimetry and interferometry has allowed to improve the distinction of natural targets from artificial ones and the estimation of the target height, respectively [18–20]. We have noticed that the target discrimination method (involving the squared modulus of the wavelet coefficients) presented in this paper, which depends on the choice of the reference pixel, is not reliable in some cases. We have shown that exploiting not only the modulus but also the argument of the wavelet coefficients, can improve the detection of a certain class of reflectors. A first result stemming from the full exploitation of the wavelet coefficients can be found in the work of Tria *et al.* [21].

4 Results in object type discrimination

In this section, the purpose is to separate the tanks and the combination of the footbridge and the stairs in the first SAR image (see area (a) in the Fig. 2), distinguish the ridge of the central building from the roofs of the lateral buildings in the second image (see area (b) in Fig. 2), extract one ridge of the building and subtract the pipe in the third image (see area (c) in Fig. 2) and extract a ridge of the building in the fourth image (see Fig. 9).

4.1 Separation of the tanks and the combination of the footbridge and the stairs

To extract the tanks, we apply the algorithm by selecting in the image, a pixel whose energy distribution is susceptible to be characteristic of the tanks: this distribution is considered as the reference function. Then, we calculate the 2D-correlation function introduced in step 2 of the OTDA. Fig. 6*a* represents this bi-dimensional function in

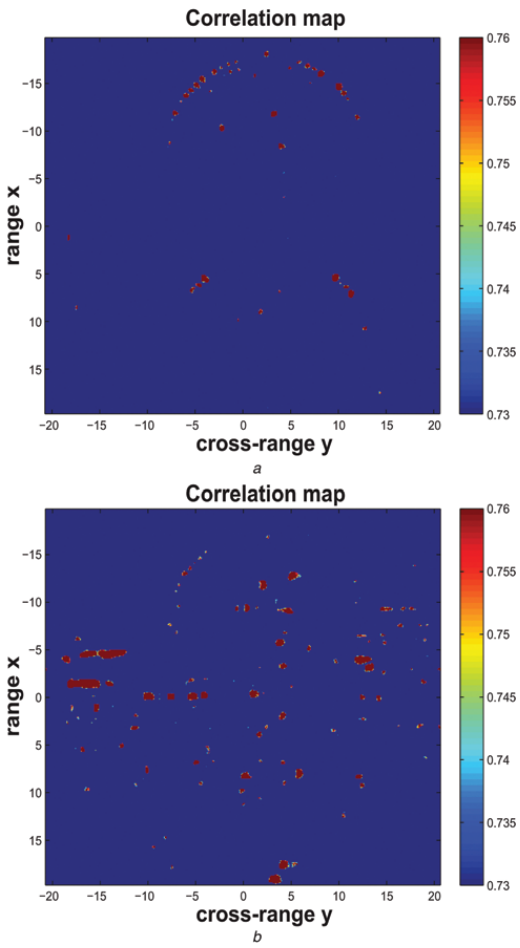

Fig. 6 *Separation of the pair of tanks and the set composed of the footbridge (on the vertical) and the stairs (on the left)*

a Extraction of the tanks using the ODTA
b Extraction of the set footbridge–stairs using the ODTA

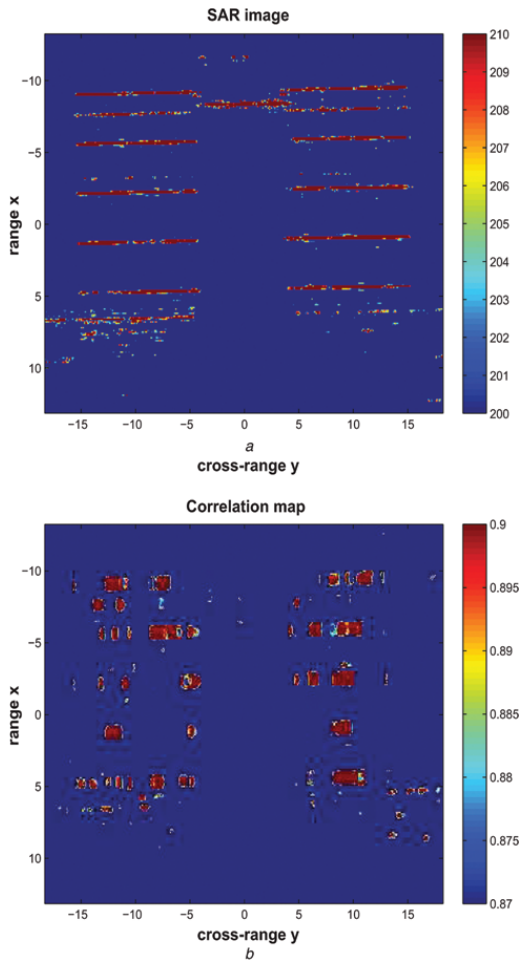

Fig. 7 *Subtraction of the central building's ridge, whereas the lateral buildings' roof are extracted*

a Initial SAR image composed of lateral buildings attached to a big one
b Extraction of the lateral buildings using the ODTA

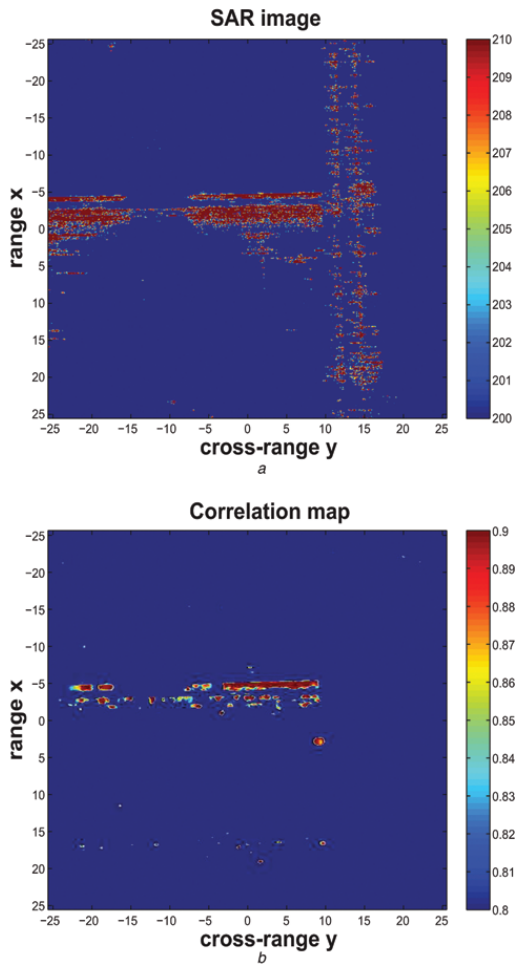

Fig. 8 *Subtraction of the pipe, whereas a portion of the building is distinguished*

a Initial SAR image composed of a building (along the horizontal) and a pipe (along the vertical)
b Extraction of a portion of the building, whereas the pipe is substracted, using the ODTA

the $x-y$ plane (we call this representation 'correlation map': high- and low-correlation coefficients are represented by the light gray and dark gray colours respectively): we can notice that the high-correlation values correspond to scatterers belonging to the tanks. To discriminate the footbridge and the stairs, we proceed the same way by considering as the reference function, the energy distribution of one pixel situated on the footbridge. The correlation map is illustrated in Fig. 6*b*: the footbridge and the stairs are discriminated.

4.2 Subtraction of a building's ridge

In this second study, our objective is to extract the roof of the lateral buildings and distinguish them from the ridge of the central one. More precisely, we want to subtract the second object. To do so, we execute the OTDA by considering the energy distribution of one pixel localised on the roof of one lateral building, as the reference function. In Fig. 7*b*, the correlation map is illustrated and compared with the initial SAR image, in Fig. 7*a*: the roof of the lateral buildings are discerned, whereas the ridge of the central building is not visible.

4.3 Extraction of a portion of a building

Fig. 8*a* represents the initial SAR image composed of a building and a pipe. Our aim has been to extract a building's ridge and subtract the pipe, by correlating the high-energy response of all the pixels in the $x-y$ plane, with the high energy of one scatterer belonging to the building's ridge. The correlation map is illustrated in the Fig. 8*b*: by comparing with Fig. 8*a*, we can see that the ridge of the building is clearly discriminated.

4.4 Extraction of a building's ridge

The last study concerns the building represented in the Fig. 9. The purpose has been to extract a ridge of the building and subtract the components on the roof. To do so, we

Fig. 9 *Aerial photography corresponding to the fourth image analysed: a building seen from the air*

have used the interface i4d to detect the energy characteristic response of the ridge localised around the range $x = -15$ m (see Fig. 10*a*), and we have applied the ODTA in order that the high-correlation values correspond to scatterers belonging to the ridge to be distinguished. The correlation map is illustrated in Fig. 10*b* and compared with the initial SAR image in the Fig. 10*a*: we can observe that the ridge is clearly extracted around the range position $x = -15$ m, the second ridge around the range $x = 17$ m is weakly discriminated and two other filiform components around $x = -17$ m and $x = -12$ m are also extracted. The component around the range $x = -17$ m is actually the intersection between the floor and the building (see Fig. 9); the component around the range $x = -12$ m is an edge of the straight object, laid along the horizontal (see Fig. 9). It is to notice the existence of a straight object laid along the horizontal around the range $x = 2$ m (see Fig. 10*a*), which is not present in the aerial photography representing the building (see Fig. 9).

5 Conclusion and perspectives

The following conclusions can be drawn from the experimental results stemming from the wavelet analysis and the OTDA. First, some reflectors analysed have an aspect-dependent response, which follows the prediction of the physical optics for distributed scatterers. Concerning the object discrimination, the results are encouraging since different objects have been extracted from the four images analysed. However, we can propose some modifications in the OTDA to improve the results. Instead of choosing a reference distribution in the analysed image, it should be more convenient to establish a bank of theoretical frequency and aspect varying responses corresponding to canonical objects (e.g. trihedral, dihedral, sphere, cylinder, flat plate and so on), each of which can be considered as a reference response.

In addition, we can propose to introduce in the 2D correlation function, a distribution resulting from the contribution of a neighbourhood $V_{i,j}$ around the pixel (x_i, y_j)

$$I(V_{i,j}; f, \theta) = \int_{V_{i,j}} I(x, y; f, \theta)\, \mathrm{d}x\, \mathrm{d}y$$

instead of the distribution of one pixel localised at (x_i, y_j), since the slant-range and cross-range resolutions degrade (in virtue of the Heisenberg's principle) by introducing

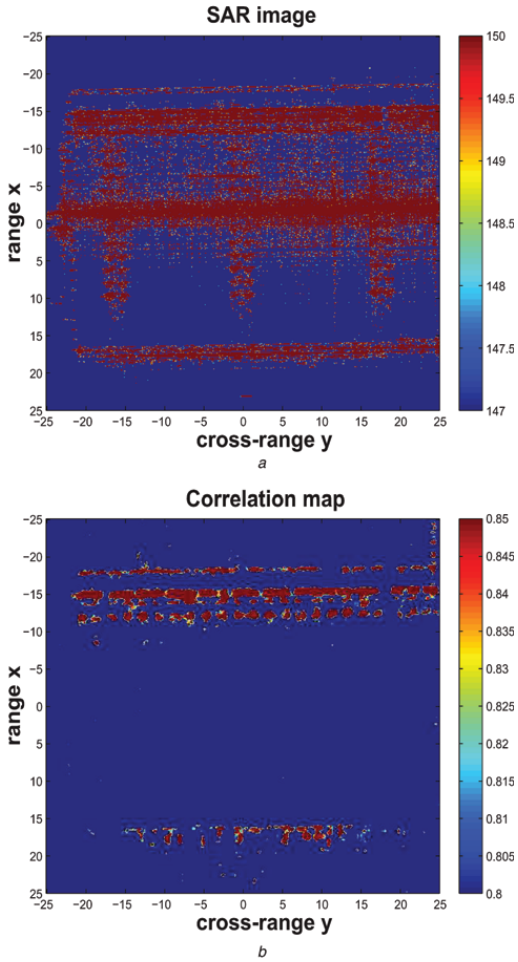

Fig. 10 *Building's ridges, the intersection between the floor and the building, and an edge of an object on the roof are extracted, whereas the other components on the roof are subtracted*

a Initial SAR image of a building with longitudinal objects on the roof
b Extraction of the building's ridges, the intersection between the floor and the building, and an edge of an object on the roof

the wavelet analysis and consequently the reflector's position to be analysed is not known accurately. Here, the choice of the size of the window around the pixel localised at (x_i, y_j), has to be discussed. Moreover, the wavelet analysis has to be extended to SAR images formed over wide angular apertures. For example, in X-band and in P-band, this aperture can reach $\Delta\theta = 20^\circ$ and $\Delta\theta = 120^\circ$, respectively, although the SAR images analysed in this paper have been formed over a small angular aperture $\Delta\theta = 3^\circ$, which limits the target discrimination. This extension to wide apertures can be interesting to classify the objects according to their orientation. As well, it will be important to analyse images built with broad bandwidths in order to check whether the frequency-dependent behaviours of real scatterers follows the GTD prediction: for example, with the P-band, we have the possibility to work with relative bandwidth $B_r = 16\%$, whereas our analysis have been made in Ku-band with a relative bandwidth $B_r = B/f_c = 6\%$, which limits the target discrimination again.

Finally, the aim of separating different types of object can be connected to a problem of sources classification: the scatterers (on the different types of object) can be identified as sources since they emit a signal whose amplitude is a function of frequency and aspect angle. The signal received by the radar is a mix of the signals emitted by these sources. Consequently, this problem of separating objects can be studied using the sources classification methods.

Presently, our priority has been turned to the full exploitation of the wavelet coefficients, in order to improve the detection of a specific class of reflectors, and more globally in order to obtain a better discrimination of targets.

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Annexe C

Scatterers Characterisation in Radar Imaging Using Joint Time–Frequency Analysis and Polarimetric Coherent Decompositions

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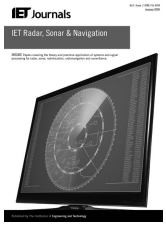

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Scatterers characterisation in radar imaging using joint time–frequency analysis and polarimetric coherent decompositions

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Abstract: Classical radar imaging generally considers targets as set of isotropic independent sources with a constant response in the measured frequency band. Nevertheless, new radar capabilities, in terms of signal bandwidth and angular excursion, may challenge this bright point model. Studies based on multidimensional time–frequency (TF) analysis, describing the angular and frequency behaviour of a scene's reflectivity, showed that some scatterers may have anisotropic and dispersive responses. Polarisation diversity is an interesting additional source of information in radar imaging, and provides indicators closely linked to some geometric and electromagnetic properties of the observed objects. In this study, a fully polarimetric TF analysis is proposed for radar imaging (SAR, ISAR) that characterises the anisotropic and dispersive behaviour of the polarimetric response of deterministic targets. This method is based on the hyper-image concept, which describes the response of scatterers as a function of the observation angle, the emitted frequency and polarimetric canonical behaviours. Polarimetric hyper-images point out that non-stationary behaviours can be related to physical properties of the target (geometrical shape, relative orientation) and allow a better understanding of the scattering mechanisms. This polarimetric hyper-image representation is then used to detect non-stationary scatterers and to classify their behaviour.

1 Introduction

Radar is an instrument traditionally used to pinpoint the position and velocity of a target from its backscattered microwave energy. Synthetic aperture radar (SAR) images represent maps of the spatial distribution of the reflectivity of a scene. High-range resolution may be obtained in the range direction by emitting waveforms with a large bandwidth of the transmitted waveform, whereas high cross-range resolution is achieved by coherently processing returned signals from correlated sequences of small apertures at different aspect angles of the radar in order to synthesise a large aperture [1].

Conventional radar imaging techniques consider targets as a set of bright points. Indeed, scatterers are considered to have an isotropic response, constant over the frequency band described by the signal waveform [2, 3]. Recent studies, based on time–frequency (TF) analysis, demonstrated how to estimate the spatial distribution of the angular and frequency behaviours of a scene imaged by a SAR [4–6]. These representations, called hyper-images, showed that some scatterers were neither isotropic nor white in the frequency domain. Such non-stationary behaviours may be particularly frequent over images acquired by modern high-resolution SAR sensors using large frequency band width and azimuth beam width. The

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varying response of scatterers may be due to material dispersion or dispersive or anisotropic effects induced by their geometry or their orientation.

Polarimetry is another information source about the geometry and the orientation of scatterers in radar imaging. We are specially interested in its application on man-made targets, because they present a dispersive and anisotropic behaviour. We use polarimetric coherent decompositions [7], which express the scattering or Sinclair matrix as a combination of the scattering responses of simpler objects, as a primary tool to compare the geometry of scatterers to those of canonical objects, and to get notions on their orientation.

The goal is to combine TF analysis and polarimetry in order to obtain a better understanding of the backscattering mechanisms and to explain the non-stationary behaviour of scatterers. In this context, two approaches are possible. The first consists in using TF analysis and polarimetry separately, and to merge their data in a second time. The problem raised by this first approach is that data can be redundant. The second proposes to use TF analysis and polarimetry jointly. This technique consists in decomposing processed polarimetric radar images into frequency/angular/polarisation information of the image scatterers (called polarimetric hyper-image).

In this paper, a fully polarimetric TF analysis method is proposed to describe the polarimetric nature relative to the angle of illumination and the emitted frequency. The polarimetric hyper-images construction is explained from the classical radar imaging, the 2D TF analysis and the coherent decompositions. Then, polarimetric hyper-images are applied to full polarimetric anechoic chamber data. Finally, we extract statistics that characterise the dispersive behaviour, the anisotropic behaviour and the non-stationary polarimetric behaviour, and use them to process a classification.

2 Classical radar imaging

The backscattering coefficient $H(\boldsymbol{k})$ of an object illuminated by a radar is defined as

$$H(\boldsymbol{k}) = \lim_{R\to\infty} \sqrt{4\pi R^2}\,\frac{E_{\mathrm{r}}}{E_{\mathrm{i}}} \tag{1}$$

where E_{r} and E_{i} represent the complex amplitude of the incoming and reflected field, respectively, and R stands for the distance between the radar and the object.

The squared modulus of $H(\boldsymbol{k})$ is called the radar cross section (RCS) and is expressed in squared metres. The two-dimensional (2-D) wave vector, $\boldsymbol{k}$, is related to the frequency f and to the direction θ of illumination by $|\boldsymbol{k}| = k = 2\pi f/c$ and $\theta = \arg(\boldsymbol{k})$.

Under the bright point model assumption, commonly used in radar imaging [8], the object under analysis is considered as a set of ideal independent reflectors, that is independent point whose reflection properties remain constant over the measured frequency domain and for all directions of presentation. Let $S(\boldsymbol{r})$ be the complex amplitude of the bright point response located at $\boldsymbol{r} = (x, y)^{\mathrm{T}}$ in a set of Cartesian axes related to the object. Under far field conditions, the complex backscattering coefficient for the whole object is given by the in-phase summation of each reflector contribution

$$H(\boldsymbol{k}) = \int S(\boldsymbol{r})\,\mathrm{e}^{-2i\boldsymbol{k}.\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r} \tag{2}$$

After a Fourier transform of (2), one can obtain the spatial distribution $S(\boldsymbol{r})$ of the complex amplitude of the scatterers' response around the centre frequency and mean angle of presentation

$$S(\boldsymbol{r}) = \int H(\boldsymbol{k})\,\mathrm{e}^{2i\boldsymbol{k}.\boldsymbol{r}}\,\mathrm{d}\boldsymbol{k} \tag{3}$$

The spatial distribution of the backscattered energy is denoted by

$$I(\boldsymbol{r}) = |S(\boldsymbol{r})|^2 \tag{4}$$

The polarimetric generalisation of the scattering coefficient is called the scattering matrix $\boldsymbol{S}$ or Sinclair matrix, defined in the polarimetric horizontal–vertical basis, as

$$\boldsymbol{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \tag{5}$$

where S_{pq} represents the target response in the polarimetric axis p to an incoming wave polarised along the axis q.

One may realistically expect that the response of some scatterers, illuminated over a large frequency domain and/or a large angular extent, may vary with the acquisition parameters. The corresponding coherent (complex amplitude) or incoherent (energy) reflectivity spatial distributions may then depend on the considered wave vector $\boldsymbol{k}$ and will be denoted as $S(\boldsymbol{r}, \boldsymbol{k})$ and $I(\boldsymbol{r}, \boldsymbol{k})$, respectively, in the sequel.

3 Extended radar imaging

Extended radar images can be constructed using TF analysis and physical group theory [5]. The dimension of the resulting hyper-images is the product of the dimension of the spatial vector $\boldsymbol{r}$ by the dimension of the spectral vector $\boldsymbol{k}$.

The principle of extended radar imaging is based on a physical group of transformations, the similarity group $\mathcal{S}$,

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that acts on the physical variables $\boldsymbol{r}$ and $\boldsymbol{k}$ through rotation $[\boldsymbol{R}]_\alpha$, dilatation a and translation $\delta\boldsymbol{r}$ as

$$\begin{array}{ccc} \boldsymbol{r} & \longrightarrow & \boldsymbol{r}' = a\,[\boldsymbol{R}]_\alpha\,\boldsymbol{r} + \delta\boldsymbol{r} \\ \downarrow & & \downarrow \\ \boldsymbol{k} & \longrightarrow & \boldsymbol{k}' = a^{-1}\,[\boldsymbol{R}]_\alpha\,\boldsymbol{k} \end{array} \tag{6}$$

The transformation laws for the reflected signal $H(\boldsymbol{k})$ and its extended image $I(\boldsymbol{r}, \boldsymbol{k})$ are therefore given by

$$\begin{array}{ccc} H(\boldsymbol{k}) & \longrightarrow & H'(\boldsymbol{k}) = a\exp(-2i\boldsymbol{k}\cdot\delta\boldsymbol{r})\,H(a[\boldsymbol{R}]_\alpha^{-1}\,\boldsymbol{k}) \\ \downarrow & & \downarrow \\ I(\boldsymbol{r}, \boldsymbol{k}) & \longrightarrow & I'(\boldsymbol{r}, \boldsymbol{k}) = I(a^{-1}\,[\boldsymbol{R}]_\alpha^{-1}\,(\boldsymbol{r} - \delta\boldsymbol{r}),\, a\,[\boldsymbol{R}]_\alpha^{-1}\,\boldsymbol{k}) \end{array} \tag{7}$$

This covariance law depicts the invariance of the form of physical laws under arbitrary differentiable coordinate transformations.

In radar imaging, the most commonly used changes of reference coordinates are the shift of the reference origin (translation $\delta\boldsymbol{r}$), the modification of the axis orientation (rotation $[\boldsymbol{R}]_\alpha$), and the change of scale (dilatation a in space or time). Two different acquisition configurations, $\mathcal{A}$ and $\mathcal{B}$, characterised by their spatial coordinate system, $\boldsymbol{r}$ and $\boldsymbol{r}'$, respectively, have spectral coordinate systems, $\boldsymbol{k}$ and $\boldsymbol{k}'$, given by the transformation law in (6). The measured backscattering coefficients, $H(\boldsymbol{k})$ and $H'(\boldsymbol{k})$, and the corresponding extended images, $I(\boldsymbol{k})$ and $I'(\boldsymbol{k})$, are related by the covariance law given in (7).

3.1 General formulation of extended images

A first approach to derive the energy distribution $I(\boldsymbol{r}, \boldsymbol{k})$ consists in representing it as a hermitian and bi-linear form of the signal $H(\boldsymbol{k})$ reflected by the target

$$I(\boldsymbol{r}, \boldsymbol{k}) = \int\int K(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{r}, \boldsymbol{k})\,H(\boldsymbol{k}_1)\,H^*(\boldsymbol{k}_2)\,\mathrm{d}\boldsymbol{k}_1\,\mathrm{d}\boldsymbol{k}_2 \tag{8}$$

where the hermitian kernel $K(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{r}, \boldsymbol{k})$ can be chosen so as to satisfy physical constraints made on the distribution $I(\boldsymbol{r}, \boldsymbol{k})$:

- the distribution can satisfy the property of covariance by the similarity group $\mathcal{S}$,
- $I(\boldsymbol{r}, \boldsymbol{k})$ can be considered, in $\mathbb{R}^2$, as a spatial density of energy (for a given $\boldsymbol{k}$), implying a constraint of positiveness. Its integration over a spatial region $\mathcal{D}$ can, therefore, be interpreted as the RCS contribution $\sigma_{\mathcal{D}}(\boldsymbol{k})$ of all the reflectors contained in $\mathcal{D}$

$$\sigma_{\mathcal{D}}(\boldsymbol{k}) = \int_{\mathcal{D}} I(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\boldsymbol{r} \tag{9}$$

- if $\mathcal{D}$ represents the whole spatial domain, the distribution can respect the well known marginal property

$$\int I(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\boldsymbol{r} = |H(\boldsymbol{k})|^2 \tag{10}$$

- the conservation of energy between the distribution and the reflected signal spaces leads to an important relation (Moyal formula) that connects the inner product between two given reflected signals H_1 and H_2 and their associated distributions I_1 and I_2

$$\left|\int H_1(\boldsymbol{k})\,H_2^*(\boldsymbol{k})\,\mathrm{d}\boldsymbol{k}\right|^2 = \int\int I_1(\boldsymbol{r}, \boldsymbol{k})\,I_2^*(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{k} \tag{11}$$

Studies on TF analysis have shown that no distribution can satisfy simultaneously all these properties [9]. As an example, kernels satisfying property (11) do not always lead to positive distributions, and are inconsistent with the RCS nature of the distribution given by (9) or (10).

To overcome this drawback, it is possible to construct a regularised form by smoothing the general distribution given in (8). These regularised distributions verify the covariance property, the RCS property (9) and the Moyal formula (11) but not the marginalisation property (10). The construction of such regularised extended images using a wavelet transform is developed in the next section.

3.2 Construction of extended images by wavelet transform

Let $\phi(\boldsymbol{k}) = \phi(k, \theta)$ be a mother wavelet representing the signal reflected by a reference target. The associated distribution $I_\phi(\boldsymbol{r}, \boldsymbol{k})$ is supposed to be well located around the spatial origin $\boldsymbol{r} = 0$ and the spectral location $(k, \theta) = (1, 0)$. One can, for instance, use a (2-D) separate Gaussian function

$$\phi(k, \theta) = \exp\left(-\frac{(k-1)^2}{\sigma_k^2}\right)\exp\left(-\frac{\theta^2}{\sigma_\theta^2}\right) \tag{12}$$

where the two free parameters σ_k and σ_θ control the spread in frequency and in angular domain and relations between spatial and spectral resolutions.

Using the similarity group, $\mathcal{S}$, a family of wavelet bases $\Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}(\boldsymbol{k})$ can be generated from the mother wavelet $\phi(k, \theta)$ as

$$\begin{aligned} \Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}(\boldsymbol{k}) &= \frac{1}{k_0}\exp(-2i\boldsymbol{k}\boldsymbol{r}_0)\,\phi\left(\frac{1}{k_0}\,[\boldsymbol{R}]_{\theta_0}^{-1}\,\boldsymbol{k}\right) \\ &= \frac{1}{k_0}\exp(-2i\boldsymbol{k}\boldsymbol{r}_0)\,\phi\left(\frac{k}{k_0},\, \theta - \theta_0\right) \end{aligned} \tag{13}$$

A regularised distribution $I_{\mathrm{r}}(\boldsymbol{r}_0, \boldsymbol{k}_0)$ can be built by smoothing the general distribution $I(\boldsymbol{r}, \boldsymbol{k})$ given by (8). Using the Moyal formula (11), the covariance property

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with $H_1(\boldsymbol{k}) = H(\boldsymbol{k})$, $H_2(\boldsymbol{k}) = \Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}(\boldsymbol{k})$, $I_1 = I_H$ and $I_2 = I_\phi$, we obtain

$$I_{\mathrm{r}}(\boldsymbol{r}_0, \boldsymbol{k}_0) = \int\int I_H(\boldsymbol{r}, \boldsymbol{k})\, I_\phi^*\left(k_0[\boldsymbol{R}]_{\theta_0}^{-1}(\boldsymbol{r}-\boldsymbol{r}_0), \frac{1}{k_0}[\boldsymbol{R}]_{\theta_0}^{-1}\boldsymbol{k}\right)\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{k}$$
$$= \left|\int H(\boldsymbol{k})\frac{1}{k_0}\exp(2i\boldsymbol{k}\boldsymbol{r}_0)\,\phi^*\left(\frac{1}{k_0}[\boldsymbol{R}]_{\theta_0}^{-1}\boldsymbol{k}\right)\mathrm{d}\boldsymbol{k}\right|^2 \quad (14)$$

The right-hand side of (14) is the wavelet coefficient $F(\boldsymbol{r}_0, \boldsymbol{k}_0)$, defined as the scalar product between the reflected signal H and each element $\Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}$ of the wavelet basis

$$F(\boldsymbol{r}_0, \boldsymbol{k}_0) = \int H(\boldsymbol{k})\,\Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}^*(\boldsymbol{k})\,\mathrm{d}\boldsymbol{k}$$
$$= \int_0^{2\pi}\mathrm{d}\theta\int_0^{+\infty} kH(k, \theta)\,\Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}^*(\boldsymbol{k})\,\mathrm{d}k \quad (15)$$

One may note that the wavelet coefficient is invariant by any transformation of the $\mathcal{S}$ group.

The reconstruction property allows one to recover the signal from its wavelet coefficients as

$$H(\boldsymbol{k}) = \frac{1}{K(\phi)}\int \mathrm{d}\boldsymbol{r}_0\int F(\boldsymbol{r}_0, \boldsymbol{k}_0)\,\Psi_{\boldsymbol{r}_0,\boldsymbol{k}_0}(\boldsymbol{k})\,\mathrm{d}\boldsymbol{k}_0 \quad (16)$$

where $K(\phi)$ is the admissibility coefficient given by

$$K(\phi) = \int \frac{|\phi(\boldsymbol{k})|^2}{k^2}\,\mathrm{d}\boldsymbol{k} < +\infty \quad (17)$$

From (4), (14) and (15), one can conclude that the wavelet coefficient $F(\boldsymbol{r}_0, \boldsymbol{k}_0)$ is equal to the coherent complex image $S(\boldsymbol{r}_0, \boldsymbol{k}_0)$

$$F(\boldsymbol{r}_0, \boldsymbol{k}_0) = S(\boldsymbol{r}_0, \boldsymbol{k}_0) \quad (18)$$

3.3 Interpretation of the distribution $I(\boldsymbol{r}, \boldsymbol{k})$

Rewriting $I(\boldsymbol{r}, \boldsymbol{k})$ as $I(x, y, f, \theta)$ shows that for each frequency f_0 and each angle of radar illumination θ_0, $I(x, y, f_0, \theta_0)$ represents a spatial distribution of the backscattered energy for this frequency and angle.

Reciprocally, for a given spatial location, $\boldsymbol{r}_0 = (x_0, y_0)^{\mathrm{T}}$, $I(x_0, y_0, f, \theta)$ provides the 2-D spectral behaviour of the corresponding scatterer.

To analyse this 4D structure, a visual display interface called i4D has been developed and allows one to carry out an interactive and dynamic analysis.

3.4 Other multidimensional TF transforms

The hyper-image concept can be generalised to all TF transforms [10], like the smoothed pseudo Wigner–Ville transform used to detect targets in [11]. However, TF techniques dealing with polarimetric data need preserve the relative phase information between the polarimetric channels. This coherent constraint is only verified by atomic decompositions, like the short time Fourier transform (STFT) and the continuous wavelet transform. Some examples of TF processing of polarimetric SAR data using the STFT can be found in [12] and [13]. In this study, the continuous wavelet transform is preferred since its covariance property is better adapted to the analysis of anechoic chamber measurements [5].

Moreover, the proposed TF technique, as a particular case of atomic decompositions (wavelet and STFT), decomposes the UWB signal into 2-D sub-spectra that can be interpreted as frequency sub-bands and angular sub-sectors.

It is not the case of other TF approaches, like in particular quadratic distributions (Wigner–Ville, smoothed pseudo Wigner–Ville, the Choi–Williams distribution, the Born–Jordan, Page or Rihaczek distributions). Indeed, these distributions may achieve higher performance (resolution, ...) that can be used to detect and locate particular signals (chirps, ...) but as the expense of physical interpretation (they might be real-valued, non-positive, have pseudo energy, ...).

Anyway, as it uses atomic decompositions, the approach presented in this paper relies on strong physical considerations. The use of wavelets or STFT is simply a matter of representation and does not violate the physical considerations radar imaging is based on.

3.5 Polarimetric generalisation of the hyper-image concept

The wavelet transform defined in (15) is applied in a similar way to each of the four polarimetric channels mentioned in (5). The resulting scattering matrix is a multivariate function of the frequency and on the illumination angle and is called hyper-scattering matrix

$$\boldsymbol{S}(\boldsymbol{r}, \boldsymbol{k}) = \begin{bmatrix} S_{hh}(\boldsymbol{r}, \boldsymbol{k}) & S_{hv}(\boldsymbol{r}, \boldsymbol{k}) \\ S_{vh}(\boldsymbol{r}, \boldsymbol{k}) & S_{vv}(\boldsymbol{r}, \boldsymbol{k}) \end{bmatrix} \quad (19)$$

Due to the use of a phase preserving TF transform the hyper-scattering matrix given in (19) can be analysed and interpreted using traditional polarimetric approaches.

Similarly to the classical polarimetric span, the extended span is defined as the sum of the squared modulus of each

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element of the hyper-scattering matrix (19).

$$P(\boldsymbol{r},\boldsymbol{k}) = |S_{hh}(\boldsymbol{r},\boldsymbol{k})|^2 + |S_{hv}(\boldsymbol{r},\boldsymbol{k})|^2 + |S_{vh}(\boldsymbol{r},\boldsymbol{k})|^2 + |S_{vv}(\boldsymbol{r},\boldsymbol{k})|^2 \tag{20}$$

The extended span does not represent the whole extended polarimetric information, but is a basic representation of the energetic polarimetric behaviour in both space and spectral domains.

The proposed approach is applied to fully polarimetric measurements acquired in an anechoic chamber. The observed target is a 'Cyrano' weapon scaled model, made out of steel (length 1.2 m – width 0.60 m) and represented on Fig. 1. Backscattering coefficients are measured over a frequency band ranging from 12 up to 18 GHz, with a sampling rate of 0.75 MHz, and for an azimuthal orientation varying from −25 to +25° with 0.5° increments. The evolution of the extended span as a function of the emitted frequency and the observation angle is represented on Fig. 2. In the following, scatterers of interest, whose location is indicated on Fig. 2, are selected and their TF behaviour is described.

- *Head (P1):* The response of the weapon model's head is characterised by a non-dispersive and isotropic behaviour.

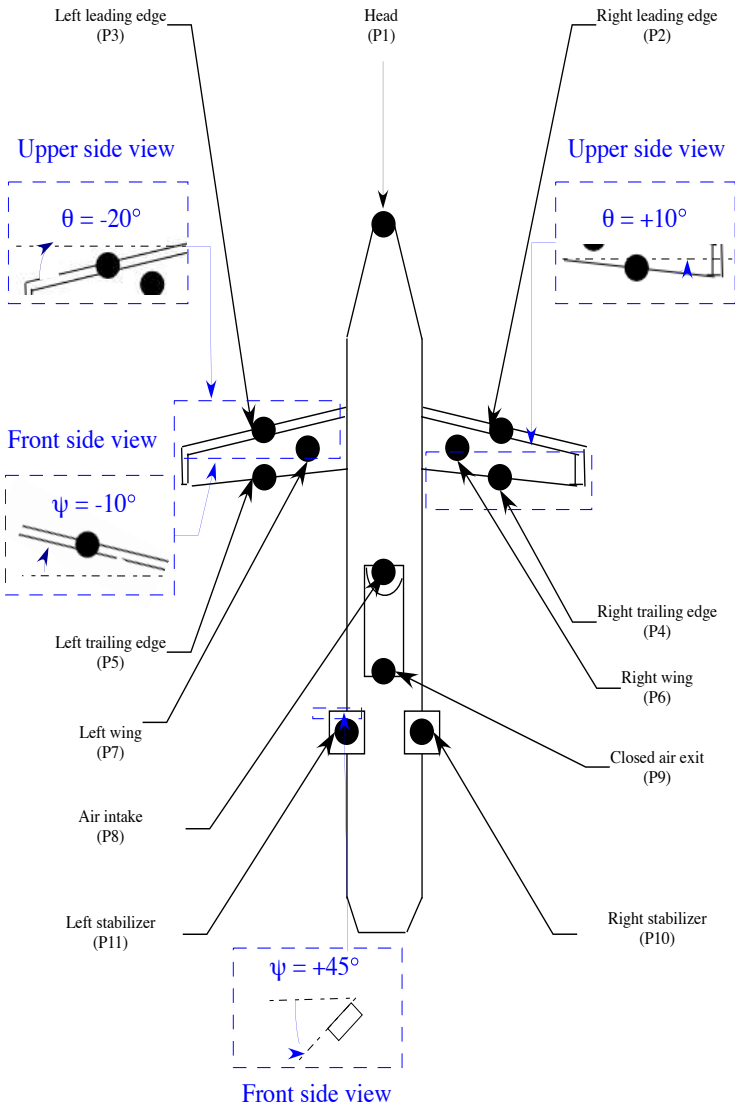

Figure 1 *Cyrano weapon model*

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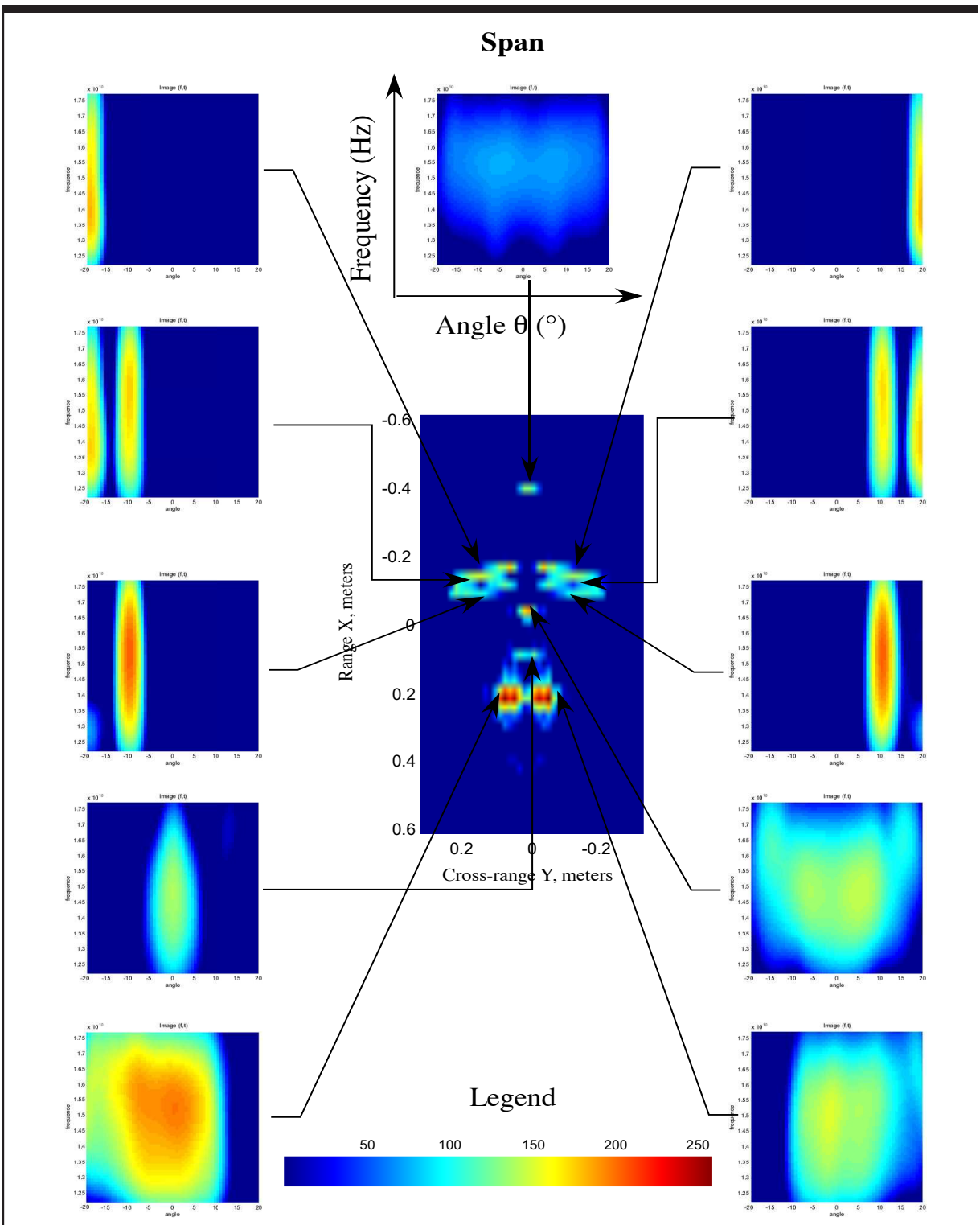

Figure 2 *Extended span over selected locations*

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This stationary behaviour is due to the particular geometry of the head, whose shape is a semi-sphere known to reflect electromagnetic waves in an isotropic and non-dispersive way over the angular-frequency domain used for this experiment.

• *Leading edges (P2, P3):* Leading edges are identified by a directional response in the angle–frequency domain. The aspect angle corresponding to the maximum of the backscattered energy, $\theta = \pm 20^\circ$, indicates the orientation of the wing edges with respect to the radar beam. As explained in [14] this directional behaviour expresses a diffraction phenomenon.

• *Trailing edges (P4, P5):* Similarly to leading edges, trailing edges have a directional behaviour in the angle–frequency plane, due to wave diffraction as well [14]. The backscattered energy reaches a maximum for an angle of view of $\theta = \pm 10^\circ$, which exactly corresponds to the orientation of this part of the illuminated object with respect to the horizontal to the radar beam.

• *Wings (P6, P7):* These particular points demonstrate a limitation of the proposed TF analysis approach, known as the Heisenberg uncertainty principle, inherent to the kind of TF transform used in this study. Indeed, the resolution of a joint TF analysis using continuous wavelets is limited by the Heisenberg inequality, which provides a physical lower bound for the product of the spatial and spectral resolutions. As a consequence, for a given spectral resolution, the TF responses of too closely spaced scatterers cannot be discriminated. This resolution trade-off phenomenon is well illustrated over the wings, whose angle–frequency response presents two directional features ($\theta = \pm 10^\circ$ and $\theta = \pm 20^\circ$), corresponding to artefacts generated by the leading and trailing edges as presented above.

This case highlights potential limitations, in terms of signal separation, of the proposed hyper-image approach.

• *Air intake (P8):* The air intake response appears as dispersive and anisotropic, indicating the presence of complex scattering mechanisms that are highly dependent on the conditions of acquisition.

• *Closed air exit (P9):* The 2-D characteristic of the close air exit is highly directive and is centred around $\theta = 0^\circ$. This behaviour is due to the specular reflection of the emitted signal over the closed air exit, which occurs as the radar faces the exit, that is when the air exit behaves as a waveguide.

• *Stabilisers (P10, P11):* These responses do not show any particular angular of frequency behaviours. One may note that the geometrical symmetry between the left and right stabilisers can be observed from their 2-D characteristics.

As shown by the results depicted in Fig. 2, the extended span permits one to highlight scatterers with an anisotropic or dispersive polarimetric energetic response and may be used to investigate local scattering phenomena, in terms of scattering mechanisms and orientation with respect to the radar beam. In order to characterise the degree of anisotropy and dispersion of the extended span, one can compute the marginal densities in the frequency and angular domains as

$$P_f(\boldsymbol{r}, f) = \frac{\int_\theta P(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\theta}{\int P(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\boldsymbol{k}} \tag{21}$$

and

$$P_\theta(\boldsymbol{r}, \theta) = \frac{\int_k k\,P(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}k}{\int P(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\boldsymbol{k}} \tag{22}$$

Frequency and angular marginal densities of the extended span, computed over particular locations of the Cyrano weapon, are represented in Figs. 3 and 4. The marginal densities in frequency indicate that all the selected points are affected by a similar dispersive behaviour, characterised by a Gaussian-like shape and centered around the mean frequency. It is likely that this pattern corresponds to the frequency response of the radar device itself and not to a frequency-sensitive target response. The marginal densities in angle relate very well the sensitivity of the response of some of the target elements to this parameter, as explained above.

The extended span and its marginal densities are basic tools that may be used to roughly characterise the polarimetric TF behaviour of an object. Indeed, the span is a global energetic indicator, but does not account for phase and amplitude imbalances between the polarimetric channels, and other polarimetric approaches, like the decomposition into scattering mechanisms, might be used to refine the analysis of the target polarimetric response. Among the wide variety of polarimetric decomposition approaches, the so-called coherent decompositions, applying directly onto scattering matrices, are particularly well adapted to the study of high-resolution anechoic chamber measurements of deterministic targets that are not affected by incoherent perturbations from natural environments. Due to the phase preservation property of the selected TF transform, coherent polarimetric decomposition techniques can be directly applied onto hyper-scattering matrices. The most popular coherent polarimetric decomposition schemes are presented in the following section.

4 Coherent polarimetric decompositions

4.1 Pauli decomposition

The Pauli decomposition expresses the measured scattering matrix $\boldsymbol{S}$ in a modified Pauli basis [15], given by the

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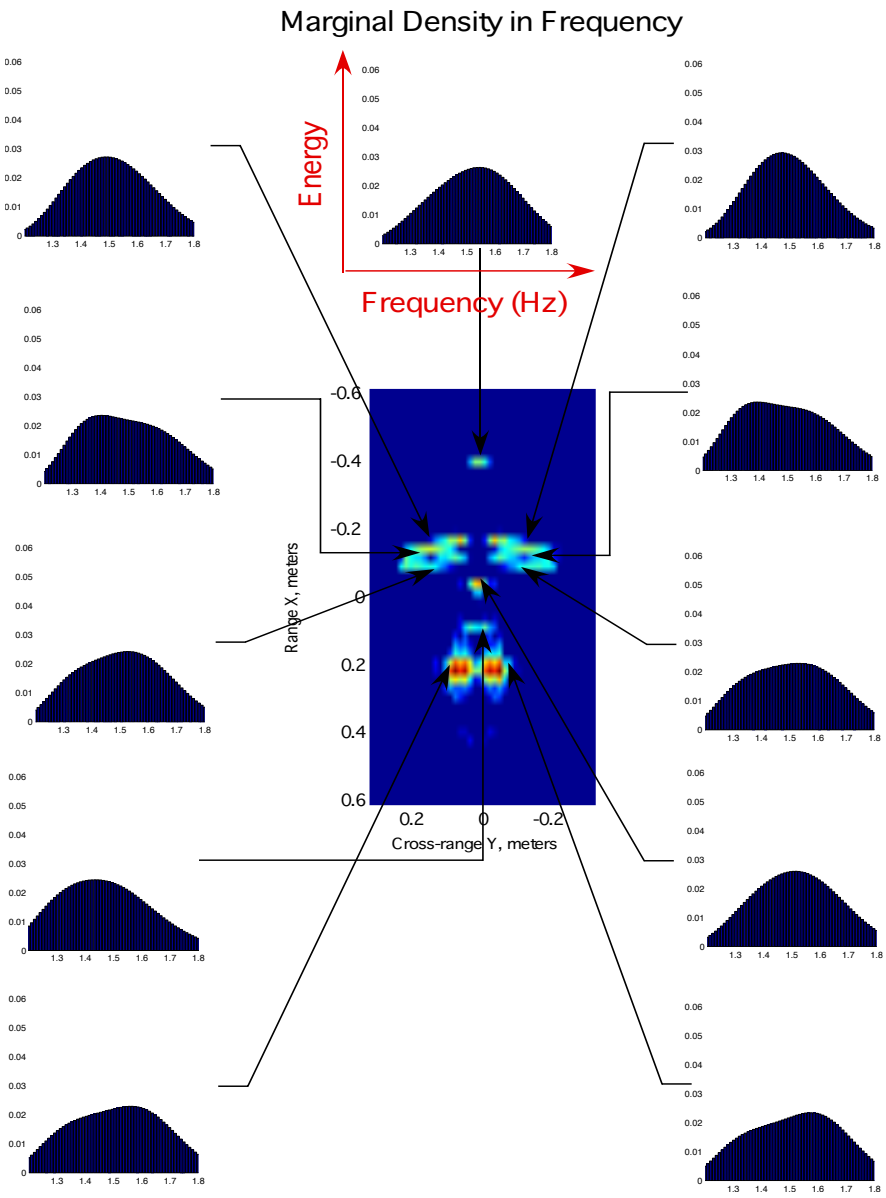

Figure 3 *Extended span marginal density in frequency*

following four matrices

$$\boldsymbol{S}_a = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

$$\boldsymbol{S}_b = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{24}$$

$$\boldsymbol{S}_c = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{25}$$

$$\boldsymbol{S}_d = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{26}$$

In a monostatic system configuration, the reciprocity principle $S_{hv} = S_{vh}$ applies, and the modified Pauli basis can be reduced to $\boldsymbol{S}_a$, $\boldsymbol{S}_b$ and $\boldsymbol{S}_c$. Consequently, a symmetric scattering matrix $\boldsymbol{S}$ can be expressed as

$$\boldsymbol{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{bmatrix} = \alpha\boldsymbol{S}_a + \beta\boldsymbol{S}_b + \gamma\boldsymbol{S}_c \tag{27}$$

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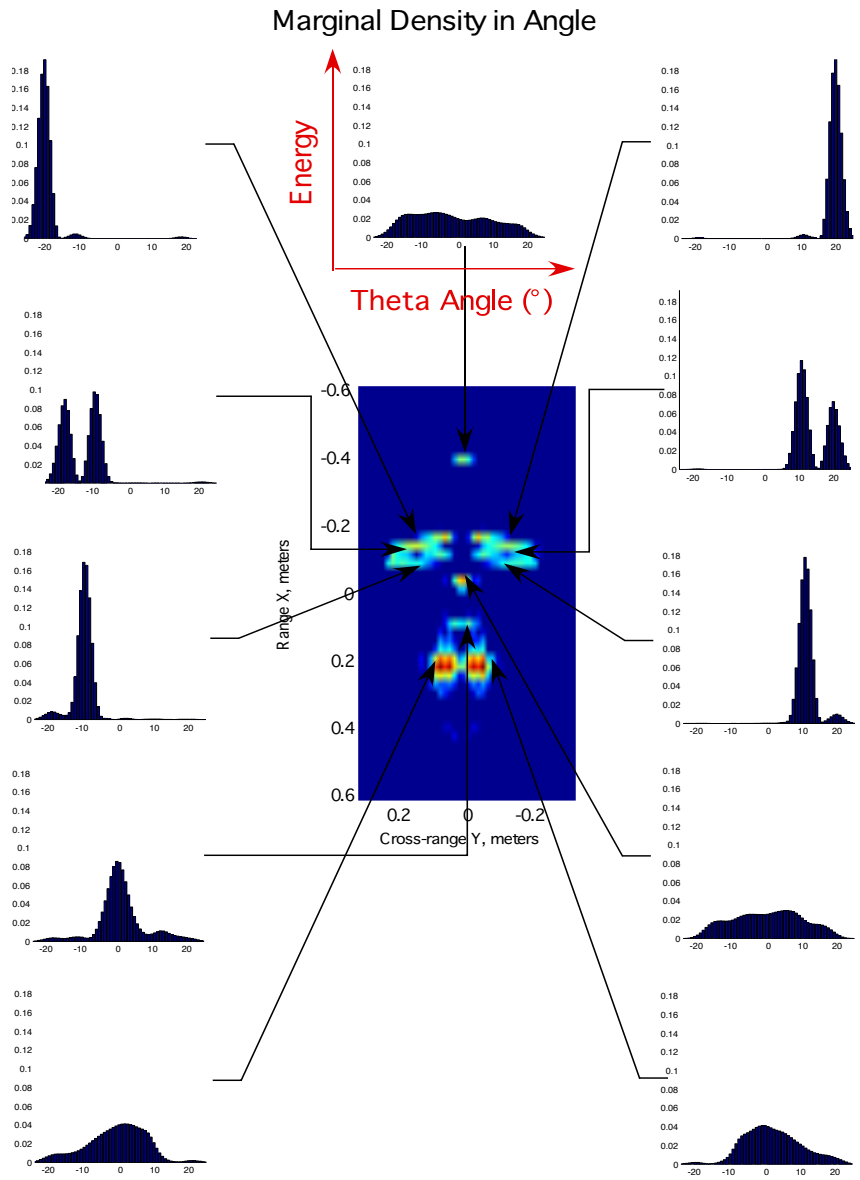

Figure 4 *Extended span marginal density in angle*

where the complex coefficients α, β, γ are given by

$$\alpha = \frac{S_{hh} + S_{vv}}{\sqrt{2}}, \quad \beta = \frac{S_{hh} - S_{vv}}{\sqrt{2}}, \quad \gamma = \sqrt{2} S_{hv} \tag{28}$$

The span can be obtained from the decomposition coefficients α, β, γ, as

$$P = |\alpha|^2 + |\beta|^2 + |\gamma|^2 \tag{29}$$

This polarimetric processing approach decomposes a scattering matrix into components associated to orthogonal canonical scattering mechanisms. The term $\boldsymbol{S}_a$ represents

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the response of a plate observed at normal incidence or a sphere, whereas $\boldsymbol{S}_b$ is characteristic of a horizontal metallic dihedral. The last component $\boldsymbol{S}_c$ represents the scattering matrix of a metallic dihedral oriented at 45° with respect to the radar line of sight.

The squared modulus of the decomposition coefficients $|\alpha|^2, |\beta|^2, |\gamma|^2$, represents the amount of the scattered energy that can be associated to each of the canonical scattering mechanism.

4.2 Krogager decomposition

A more refined alternative approach, proposed by E. Krogager [16–18] and considering a scattering matrix as the combination of the responses of a sphere, an oriented diplane and a helix, can be formulated as

$$\boldsymbol{S} = \mathrm{e}^{\mathrm{j}\varphi}\{\mathrm{e}^{\mathrm{j}\varphi_s} k_s \boldsymbol{S}_{\text{sphere}} + k_d \boldsymbol{S}_{\text{diplane}}(\vartheta) + k_h \boldsymbol{S}_{\text{helix}}(\vartheta)\} \quad (30)$$

with

$$\boldsymbol{S}_{\text{sphere}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \boldsymbol{S}_{\text{diplane}}(\vartheta) = \begin{bmatrix} \cos 2\vartheta & \sin 2\vartheta \\ \sin 2\vartheta & -\cos 2\vartheta \end{bmatrix}$$

$$\boldsymbol{S}_{\text{helix}}(\vartheta) = \mathrm{e}^{(\pm 2\mathrm{j}\vartheta)} \begin{bmatrix} 1 & \pm\mathrm{j} \\ \pm\mathrm{j} & -1 \end{bmatrix} \quad (31)$$

where the $\pm$ sign in the helix component varies its the handedness, left or right, and has to be fixed during the estimation of the decomposition components. The identification of the parameters introduced in (30) and (31) is generally performed in the right–left circular basis, where the expression of the scattering matrix is

$$\boldsymbol{S}_{(r,l)} = \begin{bmatrix} S_{rr} & S_{rl} \\ S_{rl} & S_{ll} \end{bmatrix} = \begin{bmatrix} |S_{rr}|\,\mathrm{e}^{\mathrm{j}\varphi_{rr}} & |S_{rl}|\,\mathrm{e}^{\mathrm{j}\varphi_{rl}} \\ |S_{rl}|\,\mathrm{e}^{\mathrm{j}\varphi_{rl}} & -|S_{ll}|\,\mathrm{e}^{\mathrm{j}(\varphi_{ll}+\pi)} \end{bmatrix} \quad (32)$$

with

$$S_{rr} = \mathrm{j}S_{hv} + \frac{1}{2}(S_{hh} - S_{vv}), \quad S_{ll} = \mathrm{j}S_{hv} - \frac{1}{2}(S_{hh} - S_{vv})$$

$$S_{rl} = \frac{\mathrm{j}}{2}(S_{hh} + S_{vv}) \quad (33)$$

The decomposition may then be formulated as

$$\boldsymbol{S}_{(r,l)} = \mathrm{e}^{\mathrm{j}\varphi}\left\{\mathrm{e}^{\mathrm{j}\varphi_s} k_s \begin{bmatrix} 0 & \mathrm{j} \\ \mathrm{j} & 0 \end{bmatrix} + k_d \begin{bmatrix} \mathrm{e}^{\mathrm{j}2\vartheta} & 0 \\ 0 & -\mathrm{e}^{-\mathrm{j}2\vartheta} \end{bmatrix} + k_h \begin{bmatrix} \mathrm{e}^{\mathrm{j}2\vartheta} & 0 \\ 0 & 0 \end{bmatrix}\right\} \quad (34)$$

from which parameters can be estimated as follows

$$\begin{cases} \varphi = \dfrac{1}{2}(\varphi_{rr} + \varphi_{ll} - \pi) \\ \vartheta = \dfrac{1}{4}(\varphi_{rr} - \varphi_{ll} + \pi) \\ \varphi_s = \varphi_{rl} - \dfrac{1}{2}(\varphi_{rr} + \varphi_{ll}) \end{cases} \quad (35)$$

$$\begin{cases} k_s = |S_{rl}|, \\ k_d = |S_{ll}|, \quad k_h = |S_{rr}| - |S_{ll}| \quad \text{if } |S_{rr}| > |S_{ll}| \\ k_d = |S_{rr}|, \quad k_h = |S_{ll}| - |S_{rr}| \quad \text{otherwise} \end{cases}$$

where the condition $|S_{rr}| > |S_{ll}|$ denotes the presence of a left-handed helix contribution. The absolute phase φ is fixed by the acquisition geometry and calibration and does not contain any relevant polarimetric information. The remaining five parameters, φ_s, k_s, k_d, k_h and ϑ, can be associated to physical properties of the target. The term φ_s is proportional to the path travelled by the radar signal between the phase centres of the sphere and diplane/helix contributions. The parameter ϑ stands for the orientation angle of the diplane and the helix scattering mechanisms around the radar line of sight. The coefficients k_s, k_d and k_h represent the amplitude of each canonical scattering mechanisms contributing to the originally measured scattering matrix $\boldsymbol{S}$.

4.3 Cameron decomposition

4.3.1 Principle of polarimetric symmetry: A scattering matrix may be represented under the form of a complex four-element scattering vector as

$$\boldsymbol{k}_S = [S_{hh}, S_{hv}, S_{vh}, S_{vv}]^{\mathrm{T}} \quad (36)$$

For reciprocal scattering responses, $S_{hv} = S_{vh}$, the decomposition of a scattering vector on the modified Pauli basis vector set may be written from (27) as

$$\boldsymbol{k}_S = \alpha \boldsymbol{k}_{S_a} + \beta \boldsymbol{k}_{S_b} + \gamma \boldsymbol{k}_{S_c} \quad (37)$$

where each coefficient of the decomposition is obtained by projecting the scattering vector $\boldsymbol{k}_S$ onto each element of the orthonormal Pauli vector set, for example $\alpha = \boldsymbol{k}_S^{\mathrm{T}} \boldsymbol{k}_{S_a}$.

A scattering mechanism is considered as symmetric if the projection of its scattering vector onto the cross-polarised component of the Pauli basis, $\boldsymbol{k}_{S_c}$, can be nullified by rotating the polarisation basis around the radar line of sight [19, 20]. Such a scattering vector may then be written as

$$\boldsymbol{k}_{S_{\text{sym}}} = a\,\mathrm{e}^{\mathrm{i}\rho} \boldsymbol{R}_\psi \boldsymbol{\lambda}(z), \text{ with } \boldsymbol{\lambda}(z) = \frac{1}{\sqrt{1+|z|^2}}[1, 0, 0, z]^{\mathrm{T}} \quad (38)$$

where $\boldsymbol{\lambda}(z)$ is a scattering vector satisfying $\boldsymbol{\lambda}^{\mathrm{T}}(z)\boldsymbol{k}_{S_c} = 0$, parametrised by the complex valued scalar z, a is the

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amplitude of the scattering matrix, ρ is the absolute phase, and $\boldsymbol{R}_\psi$ is a rotation matrix given by

$$\boldsymbol{R}_\psi = \boldsymbol{R}_2(\psi) \otimes \boldsymbol{R}_2(\psi), \text{ with } \boldsymbol{R}_2(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \tag{39}$$

where ψ represents the angle of rotation and $\otimes$ the Kronecker product.

A symmetric scattering mechanisms corresponds to a target having an axis of symmetry around the radar line of sight, whose orientation, often called the Huynen orientation, is given by ψ. Examples of normalised scattering vectors corresponding to symmetric canonical scattering mechanisms are given in Table 1.

4.3.2 Decomposition into canonical scattering mechanisms: A reciprocal scattering vector can be decomposed as a weighted sum of two symmetric orthogonal scattering components [19, 20], as

$$\boldsymbol{k}_S = \cos\tau\, \boldsymbol{k}_{S_{\max}^{\text{sym}}} + \sin\tau\, \boldsymbol{k}_{S_{\min}^{\text{sym}}} \tag{40}$$

The angle τ represents the degree to which the scattering vector deviates from being symmetric

$$\cos\tau = \frac{|\boldsymbol{k}_S^{\text{T}} \boldsymbol{k}_{S_{\max}^{\text{sym}}}|}{|\boldsymbol{k}_S|\,|\boldsymbol{k}_{S_{\max}^{\text{sym}}}|} \quad 0 \le \tau \le \frac{\pi}{4} \tag{41}$$

A scattering mechanism with $\tau = 0$ represents a fully symmetric scatterer like a trihedral or a dihedral, whereas a scattering matrix with $\tau = \pi/4$ represents a fully asymmetric scatterer, like a left or right-handed helix.

The symmetric scattering vectors $\boldsymbol{k}_{S_{\max}^{\text{sym}}}$ and $\boldsymbol{k}_{S_{\min}^{\text{sym}}}$ are given by

$$\boldsymbol{k}_{S_{\min}^{\text{sym}}} = \alpha\, \boldsymbol{k}_{S_a} + \varepsilon\, \boldsymbol{k}_{S_\beta}, \quad \boldsymbol{k}_{S_{\max}^{\text{sym}}}^{\text{T}} \boldsymbol{k}_{S_{\min}^{\text{sym}}} = 0 \tag{42}$$

Table 1 Examples of normalised vectors associated to canonical symmetric scatterers

Symmetric scatterer	Normalised vector
trihedral	$\boldsymbol{\lambda}(1)$
diplane	$\boldsymbol{\lambda}(-1)$
dipole	$\boldsymbol{\lambda}(0)$
cylinder	$\boldsymbol{\lambda}(1/2)$
narrow diplane	$\boldsymbol{\lambda}(-1/2)$
quarter wave device	$\boldsymbol{\lambda}(i)$

where ε is given by

$$\varepsilon = \beta\cos(\xi) + \gamma\sin(\xi), \quad \tan(2\xi) = \frac{\beta\gamma^* + \beta^*\gamma}{|\beta|^2 - |\gamma|^2} \tag{43}$$

The minimum and maximum attributes of the symmetric scattering vectors refer to the relative amplitude of their contribution to $\boldsymbol{k}_S$, steered by τ, which by construction verifies $\cos\tau \ge \sin\tau$. The orientation of the target is generally estimated from the representation of $\boldsymbol{k}_{S_{\min}^{\max}}$ under the form shown in (38).

Based on the decomposition of the measured scattering matrix $\boldsymbol{S}$ into symmetric components, Cameron proposed a classification scheme described on Fig. 5 [19, 20]. The initial test of reciprocity is verified in the monostatic acquisition configuration of this study. The input scattering matrix is then decomposed using (40), (42) and (43). The characteristic parameter τ is used to estimate the degree of symmetry of the input matrix: if $\tau \le \pi/8$ the scatterer is considered as mainly symmetric, whereas for $\tau > \pi/8$ it is associated to an non-symmetric object or a helix if $\tau \simeq \pi/4$. Scatterers labelled as symmetric may be further characterised using the decomposition of $\boldsymbol{k}_{S_{\max}^{\text{sym}}}$ shown in (38). The complex parameter z is compared to those of the reference canonical targets shown in Table 1, that is trihedral, dihedral, dipole, cylinder, narrow-diplane and quarter wave device. The scatterer under observation is assigned to the class of the reference canonical response

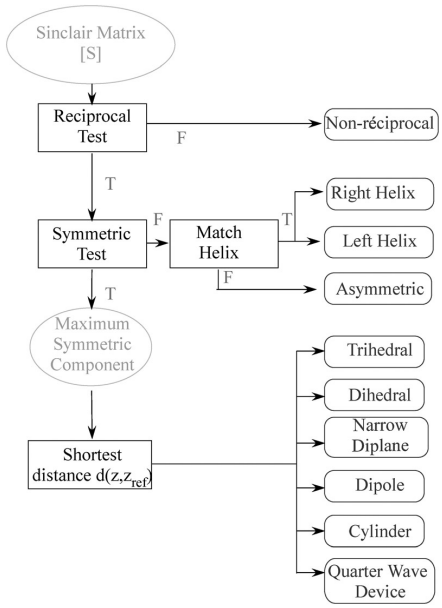

Figure 5 *Classification process of the Cameron decomposition*

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minimising the following metric

$$d(z, z_{\mathrm{ref}}) = \cos^{-1}\left(\frac{\max(|1 + z z_{\mathrm{ref}}^*|, |z + z_{\mathrm{ref}}^*|)}{\sqrt{1 + |z|^2}\sqrt{1 + |z_{\mathrm{ref}}|^2}}\right) \quad (44)$$

5 Polarimetric TF analysis using coherent decompositions

5.1 Polarimetric decomposition of hyper-images

As mentioned in Section 3.5, hyper-scattering matrices may be obtained by simultaneously applying a phase preserving TF transform over each polarisation channel. The resulting multidimensional information may then be analysed through polarimetric coherent decompositions in order to generate hyper-images of relevant parameters, that is images representing values taken by these parameters over the measured frequency and observation angle domains, as introduced in [21, 22].

The Krogager decomposition of a hyper-scattering matrix may be written as

$$\boldsymbol{S}(\boldsymbol{r}, \boldsymbol{k}) = \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r},\boldsymbol{k})}\{\mathrm{e}^{\mathrm{i}\varphi_s(\boldsymbol{r},\boldsymbol{k})} k_s(\boldsymbol{r}, \boldsymbol{k})\, \boldsymbol{S}_{\mathrm{sphere}} + k_d(\boldsymbol{r}, \boldsymbol{k})\, \boldsymbol{S}_{\mathrm{diplane}}(\vartheta(\boldsymbol{r}, \boldsymbol{k})) + k_h(\boldsymbol{r}, \boldsymbol{k})\, \boldsymbol{S}_{\mathrm{helix}}(\vartheta(\boldsymbol{r}, \boldsymbol{k}))\} \quad (45)$$

where the most significant hyper-images are given by $|k_s(\boldsymbol{r}, \boldsymbol{k})|^2$, $|k_d(\boldsymbol{r}, \boldsymbol{k})|^2$, $|k_h(\boldsymbol{r}, \boldsymbol{k})|^2$, the intensity associated to the sphere, diplane and helix components, respectively, and $\vartheta(\boldsymbol{r}, \boldsymbol{k})$, the orientation angle of the diplane and helix components.

The decomposition of a reciprocal hyper-scattering matrix using Cameron's approach leads to

$$\boldsymbol{S}(\boldsymbol{r}, \boldsymbol{k}) = \cos \tau(\boldsymbol{r}, \boldsymbol{k})\, \boldsymbol{S}_{\mathrm{sym}}^{\max}(\boldsymbol{r}, \boldsymbol{k}) + \sin \tau(\boldsymbol{r}, \boldsymbol{k})\, \boldsymbol{S}_{\mathrm{sym}}^{\min}(\boldsymbol{r}, \boldsymbol{k}) \quad (46)$$

The most significant parameters obtained from this decomposition approach are the classification into canonical scattering mechanisms, $W(\boldsymbol{r}, \boldsymbol{k})$, the Huynen orientation $\psi(\boldsymbol{r}, \boldsymbol{k})$, and the measure of symmetry $\tau(\boldsymbol{r}, \boldsymbol{k})$.

5.2 Experimental results

The proposed polarimetric hyper-images processing techniques are applied to the anechoic chamber measurements of the Cyrano weapon model described in Section 3.5. Results obtained using the Krogager decomposition are shown in Fig. 6 under the form of colour-coded images, computed from the intensity of the sphere, diplane and helix components. Hyper-images of the classification map and Huynen oriental angle, obtained using the Cameron decomposition are depicted in Figs. 7 and 8, respectively. In order to avoid meaningless classification results, the representation of the parameters derived from the Cameron decomposition is limited to frequency and angular domains corresponding to significant extended span values.

The analysis of TF parameters derived from both polarimetric decomposition approaches is discussed in the following over the particular locations indicated in Fig. 2.

- *Head (P1):* Both decompositions indicate a stationary TF polarimetric behaviour. The polarimetric behaviour of the weapon model's head is stationary. The dominant scattering behaviour is identified as a canonical sphere or trihedral reflector response, both characterised by a single bounce reflection of the electromagnetic wave by this part of the object. This constant polarimetric patterns observed at this spatial location is conferred by the geometrical shape of the head as explained during the analysis of the extended span.

- *Leading edges (P2, P3):* These parts of the object show a very directional response, mainly driven by the span, with a peak of the backscattered energy, centred on the horizontal orientation of the edges, that is $\theta = \pm 20^\circ$. The Krogager decomposition identifies the corresponding polarimetric response as a mixture of sphere and diplane contributions, whereas the Cameron approach more accurately classifies these narrow and anisotropic scatterers as dipoles. The Huynen orientation provides a precise estimate of the orientation of the wings with the radar line of sight, $\psi = \pm 10^\circ$. One may note the non-dispersive behaviour of the extended span and other extended polarimetric indicators. The edges clearly illustrate the potential of a polarimetric TF analysis to deeply characterise an object in terms of canonical scattering mechanism and relative orientation with respect to both horizontal and vertical planes.

- *Trailing edges (P4, P5):* A similar polarimetric behaviour is observed over the trailing edges. The horizontal orientation is well estimated with $\theta = \pm 10^\circ$. The poor estimation, $\psi = \pm 90^\circ$, of the vertical orientation is due to the fact that radar does not see the slope (upper view).

- *Wings (P6, P7):* The effects of the time and frequency-limited resolutions, linked to the Heisenberg uncertainty principle introduced earlier in this paper, are clearly visible on the TF polarimetric hyper-images, the TF behaviour of the wings being indeed a mixture of the polarimetric patterns of the leading and trailing wings.

- *Air intake (P8):* The polarimetric indicators confirm the dispersive and anisotropic behaviours observed during the analysis of the extended span, that are probably linked to the geometrical complexity of this scatterer.

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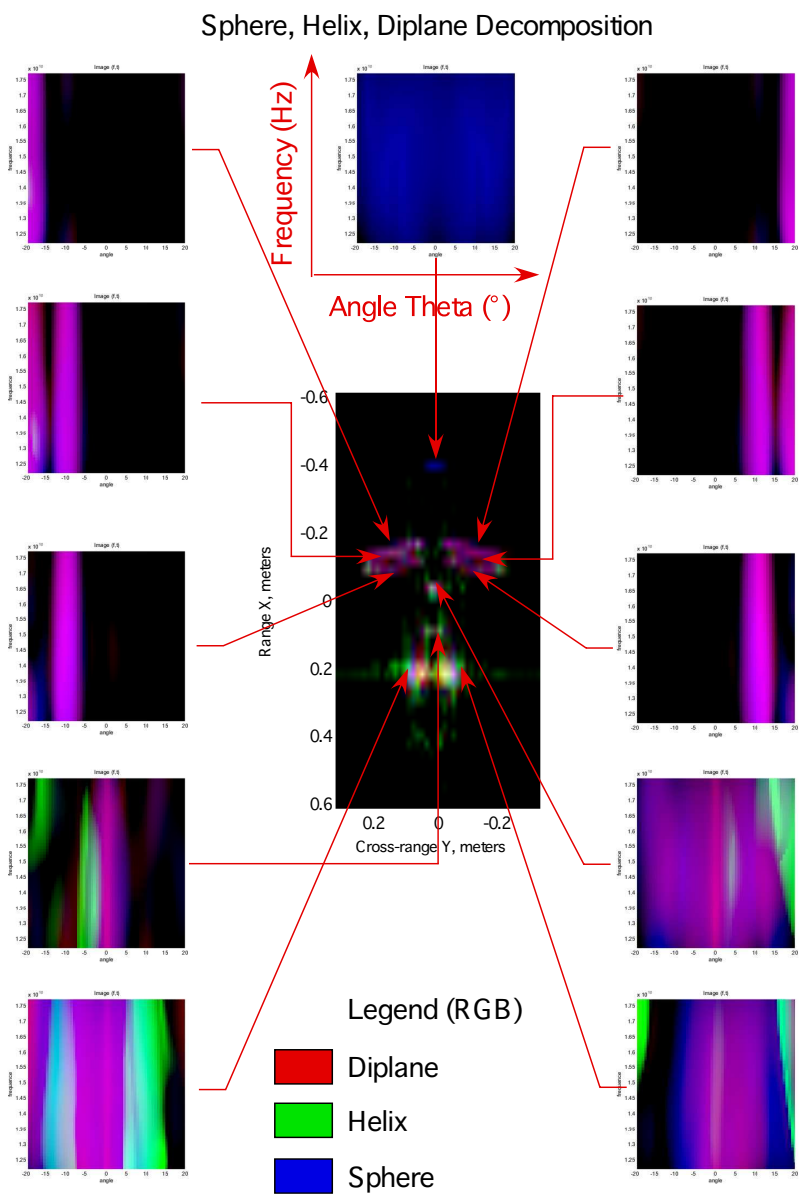

Figure 6 *Colour-coded hyper-images of the Krogager decomposition amplitude coefficients*

• *Closed air exit (P9):* The indicators derived from the Krogager and Cameron decompositions depict relate well the highly directive behaviour of the closed air exit centred on $\theta = 0^\circ$. The non-stationary aspect of the polarimetric features indicates that the scattering mechanism may not be a specular reflection over the closed exit, as previously presented during the analysis of the extended span, but a more complicated combination of scattering modes.

• *Stabilisers (P10, P11):* Similarly to the extended span, polarimetric decomposition parameters show a quasi-perfect symmetry between right and left wings. The hyper-images show that the TF scattering mechanism is clearly a function of the observation angle θ. This non-stationary behaviour is due to the strong variations of the slant range geometry as the aspect angle varies: the horizontal, $\theta \simeq 0^\circ$, dipole contribution with a Huynen orientation $\psi = \pm 45^\circ$ is caused by the edge of the stabiliser, whereas, as

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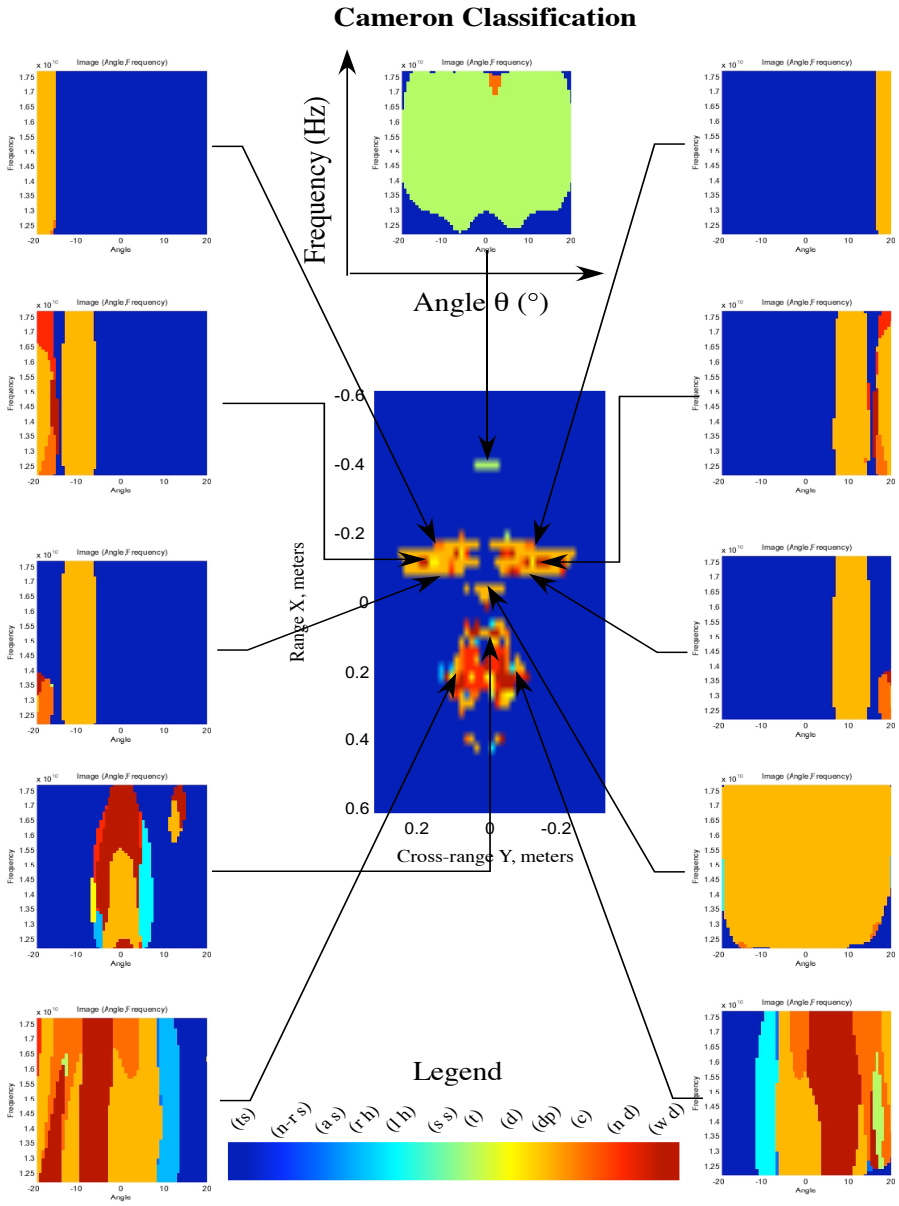

Figure 7 *Cameron classification of hyper-scattering matrices*

$\theta \simeq \pm 10°$, the observation of two perpendicular edges (two dipoles) in the same resolution cell induces a helix contribution. For $\theta \simeq \pm 20°$ the stabiliser is masked by the body of the Cyrano weapon and the backscattered energy reaches a minimum.

This analysis shows that the joint use 2D TF analysis and polarimetric decompositions, may provide efficient and highly descriptive features of scatterers, related to their geometry and orientation. Such an approach may be an efficient tool to improve targets recognition based on physical parameters.

6 Classification based on the polarimetric anisotropic and dispersive behaviour of scatterers

Numerous approaches to classify targets using frequency or angular scattering characteristics may be found in the

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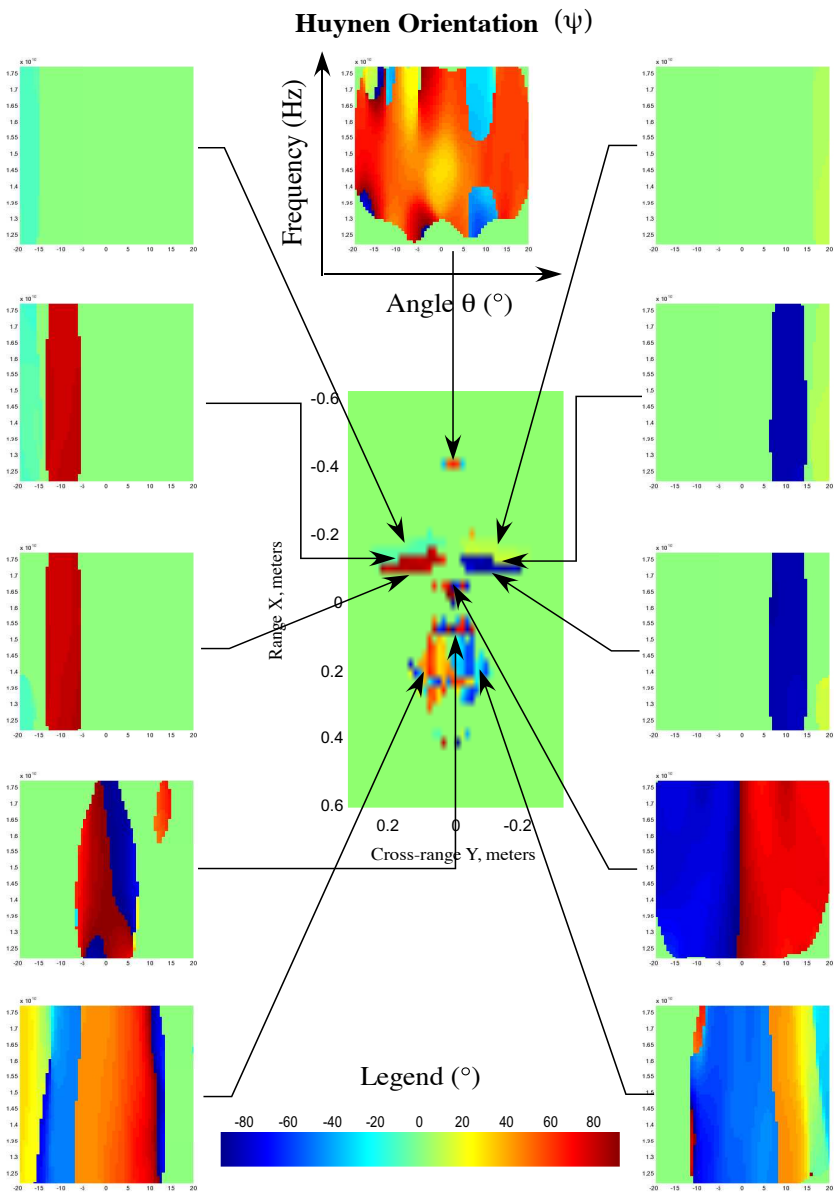

Figure 8 *Huynen orientation derived from the Cameron decomposition*

literature. Among them, one may note the method introduced by Aldhubaib and Shuley [23], based on the determination of an optimal bistatic angle, for which the scattered signal has a robust strength level, irrespective of the target aspect. This optimal bistatic angle concept may then be used to improve the scattered signal-to-noise ratio for targets of interest, prior to classification. Another technique, developed by F. Sadjadi [24], proposes to improve target classification performance using optimum polarimetric SAR signatures that minimise the distance between targets belonging to the same class, while maximising a between class metric.

The classification method we proposed aims to characterise the back-scattering behaviour of a target from characteristic features of its polarimetric hyper-image, that is according to the frequency, angular and polarimetric back-scattering properties of its most significant contributors.

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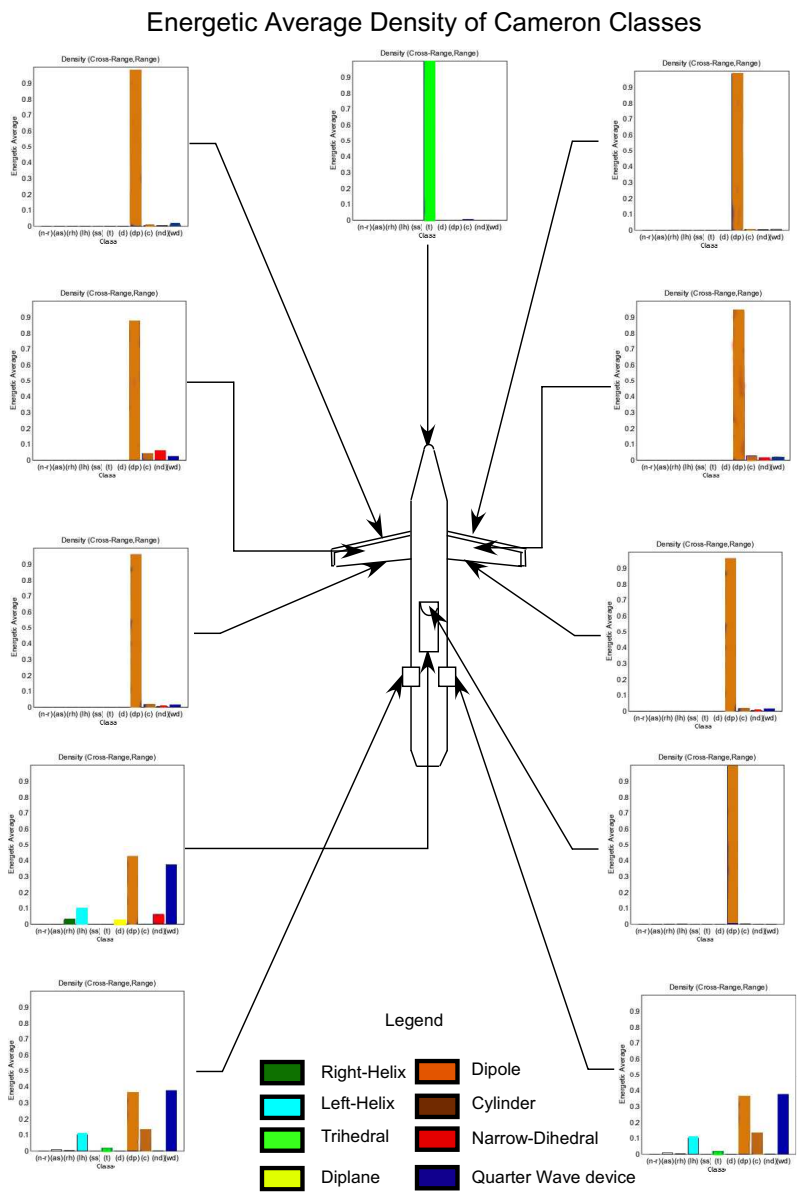

Figure 9 *Average density of polarimetric behaviours*

As revealed by the TF study of the Cyrano weapon model response presented in this paper, some typical properties, like frequency dispersion, angular anisotropy or polarimetric non-stationary polarimetric features, may be efficiently used to discriminate targets. Multidimensional representations, like polarimetric hyper-images, represent a consequent amount of information and may not be well adapted to fast and efficient target classification schemes. In the following, we propose to characterise a target by extracting relevant, orientation independent and highly descriptive indicators from its polarimetric hyper-image and to perform a hierarchical classification of its polarimetric TF behaviour.

6.1 Characteristic TF parameters

As can be seen in Figs. 2, 6 and 7, two kinds of variations of the scattering features in the frequency and angular domains may be considered:

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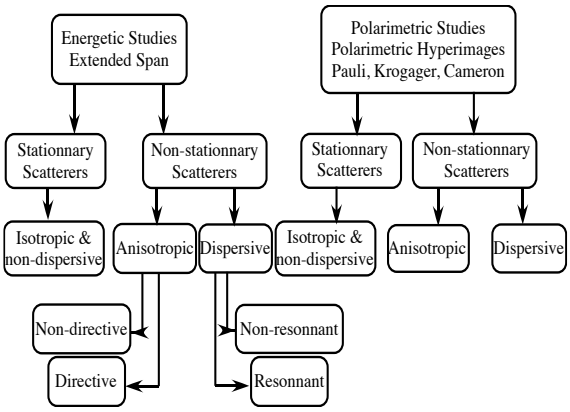

Figure 10 *Propose TF polarimetric classification scheme*

- Energetic variations that modulate the polarimetric signal level and can be characterised using the extended span information.

- Purely polarimetric variations that can be measured in a refined way using output parameters of the Cameron decomposition.

The level of energetic dispersion or anisotropy of a scatterer's response can be quantified by computing the standard deviation of the marginal density of the extended span as

$$\sigma_q(\boldsymbol{r}) = \left[\int_q (q-\mu_q)^2 P_q(\boldsymbol{r}, q)\,\mathrm{d}q\right]^{\frac{1}{2}} \quad \text{with} \qquad \mu_q(\boldsymbol{r}) = \int_q q P_q(\boldsymbol{r}, q)\,\mathrm{d}q,\ q = \theta, f \tag{47}$$

where the marginal densities, $P_f(\boldsymbol{r}, f)$ and $P_\theta(\boldsymbol{r}, \theta)$ are given in (21) and (22), respectively. The resulting indicators of frequency dispersion and angular anisotropy, $\sigma_f(\boldsymbol{r})$ and $\sigma_\theta(\boldsymbol{r})$, respectively, permit to detect efficiently varying energetic behaviours over each part of the measured object.

The amount of polarimetric variability over the frequency–angle domain is estimated from the distribution of the back-scattered energy over the different canonical targets of the Cameron decomposition of the polarimetric hyper image

$$\rho_i(\boldsymbol{r}) = \frac{\int P(\boldsymbol{r}, \boldsymbol{k})\,\delta(W(\boldsymbol{r}, \boldsymbol{k}) - C_i)\,\mathrm{d}\boldsymbol{k}}{\int P(\boldsymbol{r}, \boldsymbol{k})\,\mathrm{d}\boldsymbol{k}} \qquad i = 1, \ldots, 10 \tag{48}$$

where $W(\boldsymbol{r}, \boldsymbol{k})$ is hyper-image of the Cameron identification, C_i the ith canonical scattering mechanism and δ the Dirac function.

The characteristic vector, $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_{10}]$, is an histogram that characterises an average energetic polarimetric

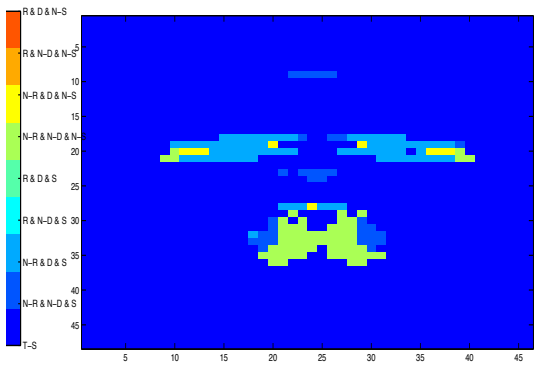

Figure 11 *TF polarimetric classification results*

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Table 2 Legend of TF polarimetric classification results

T-S	thresholded span
N-R	non-resonant
N-D	non-directional
S	polarimetric stationary
R	resonant
D	directional
N-S	polarimetric non-stationary

behaviour in the angular and frequency fields [25] and is illustrated on Fig. 9.

6.2 Classification

The polarimetric TF behaviour of an object may then be characterised using a hierarchical classification scheme, as proposed in [26], presented in Fig. 10. Thresholds for the dispersion and anisotropy indicators are fixed in an automatic way by considering that, due to the intrinsic variability of scattering patterns, the marginal densities of non-dispersive and isotropic scatterers follow a bounded Gaussian distribution $\mathcal{N}(\mu, \sigma)$ whose support is $\pm 3\,\sigma$. For the case under consideration, the angular support being $\Delta\theta = 50^\circ$, $\sigma_\theta(\boldsymbol{r})$ is compared to a threshold equal to 1/6 50°, whereas for a frequency range given by [12, 18] GHz, the threshold for $\sigma_\theta(\boldsymbol{r})$ is fixed at 1 GHz.

A scatterer is considered as having a stationary polarimetric response if one of the Cameron class dominates all the others, that is if there exists $\rho_i \geq 50\%$.

The classification results presented in Fig. 11 with legend given in Table 2 show that the head, the air intake, and the border of stabilisers are classified as non-resonant (N-R), non-directional (N-D), and polarimetrically stationary (S) scatterers. The wings and their edges, the closed air exit are classified as non-resonant, directional, and polarimetrically stationary scatterers. The stabilisers are classified as non-resonant, non-directional, and polarimetrically non-stationary scatterers.

7 Conclusion

In this paper we proposed a new polarimetric TF method that allows to describe the polarimetric behaviour of the scatterers in the frequency and angular domains and is adapted to polarimetric radar imaging. This method is based on the multidimensional wavelet radar imaging method [5] and has been extended to the polarimetric case using coherent polarimetric decomposition methods, like as the Pauli, Krogager and Cameron technique.

This method permits one to discriminate scatterers with a directive, resonant and polarimetrically non-stationary behaviour and allows one to provide physical polarimetric from the extracted scattering mechanisms.

A new classification procedure has been proposed, which allows one to characterise the global polarimetric behaviour of scatterers.

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Annexe D

Supervised Classification of Scatterers on SAR Imaging Based on Incoherent Polarimetric Time-Frequency Signatures

SUPERVISED CLASSIFICATION OF SCATTERERS ON SAR IMAGING BASED ON INCOHERENT POLARIMETRIC TIME-FREQUENCY SIGNATURES

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ABSTRACT

This paper deals with the analysis of the non-stationary behavior of scatterers in polarimetric SAR imaging . A method based on continuous wavelet and incoherent polarimetric decompositions is proposed to extract the polarimetric time-frequency signatures of scatterers. These signatures characterize scatterers according to their polarimetric /or energetic behavior versus the emitted frequency and the observation angle. Then, signatures from reference targets are used to train a multi-layer perceptron (MLP). All in all, SAR imaging data are classified by the MLP. The efficiency of this method is demonstrated, for the deterministic targets (man-made targets). It can be explained by the fact that the man-made targets present a strong non-stationary behavior. But for the vegetation and canopy the results are not convincing. It can be interpreted by the fact that the behavior of vegetation is stationary.

1. INTRODUCTION

This paper suggests a classification based on polarimetric time-frequency signatures for wideband and strong angular excursion SAR imaging. Indeed, in this case the model of bright point is not valid. Time-frequency analysis allows to build HyperImages [1], [2], [3] to correct this main drawback. Polarimetry is another information source to characterize scatterers. The aim of this paper is to use jointly time-frequency analysis and polarimetry incoherent decomposition to extract polarimetric time-frequency signatures and use them in a neural network to classify scatterers.

2. CONSTRUCTION OF THE HYPER-SCATTERING MATRIX BASED ON THE CONTINUOUS WAVELET

2.1 Classical radar imaging

The model usually used in radar imaging is the model of bright points [4]. The object under analysis can be seen as a set of bright points, i.e. a set of independent sources that reflect in the same way for all frequencies (white points) and all directions of presentation (isotropic points). Let $S(\mathbf{r})$ be the complex amplitude of the bright point response located at $\mathbf{r} = (x,y)^T$ in a set of cartesian axes related to the object. Under far field conditions (decomposition into planes waves), the complex backscattering coefficient for the whole object is then given by the in-phase summation of each reflector contribution:

$$H(\mathbf{k}) = \int S(\mathbf{r})\, e^{-2i\pi \mathbf{k}.\mathbf{r}}\, d\mathbf{r}. \tag{1}$$

After a Fourier Transform of (1), one can obtain the spatial distribution $I(\mathbf{r})$ of the reflectors complex amplitude for a mean frequency (the center frequency) and for a mean angle of presentation:

$$S(\mathbf{r}) = \int H(\mathbf{k})\, e^{2i\pi \mathbf{k}.\mathbf{r}}\, d\mathbf{k}. \tag{2}$$

The spatial distribution of the scatterers energy will be denoted in the following by:

$$\tilde{I}(\mathbf{r}) = |S(\mathbf{r})|^2 \tag{3}$$

A full polarimetric radar is generally designed to transmit and receive microwave radiations horizontally (h) or vertically (v) polarized. The polarimetric generalization of the scattering coefficient is called the scattering matrix $[\mathbf{S}]$ or Sinclair matrix:

$$[\mathbf{S}(\mathbf{r})] = \begin{bmatrix} S_{hh}(\mathbf{r}) & S_{hv}(\mathbf{r}) \\ S_{vh}(\mathbf{r}) & S_{vv}(\mathbf{r}) \end{bmatrix}. \tag{4}$$

When a target is illuminated by a broad-band signal and/or for a large angular extent, it is realistic to consider that the spatial distribution $\tilde{I}(\mathbf{r})$ of the reflectors energy (or the Sinclair complex image $S(\mathbf{r})$) depends on frequency f and on illumination angle θ. These two amplitude and energy distributions depending on the vector $\mathbf{k}$, they will be denoted respectively by $S(\mathbf{r},\mathbf{k})$ and $\tilde{I}(\mathbf{r},\mathbf{k})$ in the sequel.

2.2 Extended radar imaging

Let $\phi(\mathbf{k})$ be a mother wavelet supposed to represent the signal reflected by a reference target. This target is supposed

located around $\mathbf{r} = \vec{0}$ and backscatters the energy in the direction $\theta = 0$ and at the frequency f given by $k = \frac{2f}{c} = 1$. A family of function is built $\Psi_{\mathbf{r}_0,\mathbf{k}_0}$ from $\phi(\mathbf{k})$ by the similarity group S [1], [2]:

$$\Psi_{\mathbf{r}_o,\mathbf{k}_o}(\mathbf{k}) = \frac{1}{k_o}\, e^{-j2\pi\mathbf{k}.\mathbf{r}_o}\, \phi\left(\frac{1}{k_o}\mathscr{R}_{\theta_o}^{-1}\mathbf{k}\right) \quad (5)$$

$$= \frac{1}{k_o}\, e^{-j2\pi\mathbf{k}.\mathbf{r}_o}\, \phi\left(\frac{k}{k_o}, \theta - \theta_o\right). \quad (6)$$

The wavelet coefficient $S_{xx}(\mathbf{r}_o, \mathbf{k}_o)$ is defined as the scalar product between the complex backscattering coefficient H_{xx} and the wavelet $\Psi_{\mathbf{r}_0,\mathbf{k}_0}$:

$$S_{xx}(\mathbf{r}_o, \mathbf{k}_o) = < H_{xx}, \Psi_{\mathbf{r}_o,\mathbf{k}_o} > \quad (7)$$

The scalar product is defined following [5]:

$$S_{xx}(\mathbf{r}_o, \mathbf{k}_o) = \int_0^{2\pi} d\theta \int_0^{+\infty} k\, H_{xx}(k,\theta)\, \frac{1}{k_o}\, e^{+j2\pi\mathbf{k}.\mathbf{r}_o}\, \phi^*\left(\frac{k}{k_o}, \theta - \theta_o\right) dk \quad (8)$$

The scalogram which is the square modulus of the wavelet coefficients defines the hyperImage $\tilde{I}(\mathbf{r}, \mathbf{k})$.

2.3 Properties

The continuous wavelet transform has two interesting properties. The first is the reconstruction: it is possible to rebuild the complex backscattering coefficient $H_{xx}(\mathbf{k})$ from the wavelet coefficient $S_{xx}(\mathbf{r}_o, \mathbf{k}_o)$:

$$H_{xx}(\mathbf{k}) = \frac{1}{K_\phi} \int_S d\mathbf{r}_o \int S_{xx}(\mathbf{r}_o, \mathbf{k}_o)\, \Psi_{\mathbf{r}_o,\mathbf{k}_o}(\mathbf{k})\, d\mathbf{k}_o \quad (9)$$

with K_ϕ defined as the *admissibility coefficient* of the mother wavelet which must, to build $H_{xx}(\mathbf{k})$ from the wavelet coefficients, check:

$$K_\phi = \int |\phi(\mathbf{k})|^2\, \frac{d\mathbf{k}}{k^2} < +\infty \quad (10)$$

The second property is the isometry:

$$\frac{1}{K_\phi} \int_S d\mathbf{r}_o \int |S_{xx}(\mathbf{r}_o, \mathbf{k}_o)|^2\, d\mathbf{k}_o = \|H_{xx}\|^2 \quad (11)$$

2.4 Limitations

The continuous wavelet is limited by the Heisenberg principle. Indeed, this concept tells that we cannot obtain a spatial good resolution with a good resolution in the frequency domain and reciprocally. However, the continuous wavelet offers a resolution which changes with the frequency and the spatial domain. It allows multiresolution analysis [6].

2.5 Hyper-Scattering matrix definition and extended Span

The wavelet transform is applied on each of the four polarimetric channels. The resulting Sinclair scattering matrix now depends on the frequency and on the illumination angle and is called hyper-scattering matrix:

$$[\mathbf{S}](\mathbf{r}, \mathbf{k}) = \begin{bmatrix} S_{hh}(\mathbf{r}, \mathbf{k}) & S_{hv}(\mathbf{r}, \mathbf{k}) \\ S_{vh}(\mathbf{r}, \mathbf{k}) & S_{vv}(\mathbf{r}, \mathbf{k}) \end{bmatrix}. \quad (12)$$

The span is generally defined as the sum of the squared modulus of each element of the matrix (4). The extended span is now defined as the sum of the squared modulus of each element of the hyper-scattering matrix (12).

$$P(\mathbf{r}, \mathbf{k}) = |S_{hh}(\mathbf{r}, \mathbf{k})|^2 + |S_{hv}(\mathbf{r}, \mathbf{k})|^2 + |S_{vh}(\mathbf{r}, \mathbf{k})|^2 + |S_{vv}(\mathbf{r}, \mathbf{k})|^2. \quad (13)$$

The extended span provides a first polarimetric time-frequency signatures. Indeed, if one scatterer is selected at the position $\mathbf{r}_0$, $P(\mathbf{r}_0, \mathbf{k})$ describes the polarimetric energetic behavior of this scatterer versus the emitted frequency and the observation angle.

3. CONSTRUCTION OF THE INCOHERENT POLARIMETRIC TIME-FREQUENCY SIGNATURES

3.1 Definition of the covariance and coherency matrix

A scattering vector can be obtained by the projection of the Sinclair matrix on an orthogonal basis of special unitary group [7]. The two bases which are the most used, are the the lexicographic basis and the Pauli basis. In monostatic scenario, the reciprocity theorem holds and hence: $S_{hv} = S_{vh}$. So, by projecting the hyper-scattering matrix on the two bases, two hyper-scattering vectors which contain all polarimetric information can be obtained:

$$k_L(\mathbf{r}, \mathbf{k}) = [S_{hh}(\mathbf{r}, \mathbf{k}), \sqrt{2}S_{hv}(\mathbf{r}, \mathbf{k}), S_{vv}(\mathbf{r}, \mathbf{k})]^T \quad (14)$$

$$k_P(\mathbf{r}, \mathbf{k}) = \frac{1}{\sqrt{2}}[S_{hh}(\mathbf{r}, \mathbf{k}) + S_{vv}(\mathbf{r}, \mathbf{k}), S_{hh}(\mathbf{r}, \mathbf{k}) - S_{vv}(\mathbf{r}, \mathbf{k}), 2S_{hv}(\mathbf{r}, \mathbf{k})]^T \quad (15)$$

where ^{T} is the transpose operator.

From these targets vectors, the covariance hyper-matrix $[C(\mathbf{r}, \mathbf{k})]$ and the coherency hyper-matrix $[T(\mathbf{r}, \mathbf{k})]$ can be defined:

$$[C(\mathbf{r}, \mathbf{k})] = \langle k_L(\mathbf{r}, \mathbf{k}) k_L^{*T}(\mathbf{r}, \mathbf{k}) \rangle \quad (16)$$

$$[T(\mathbf{r}, \mathbf{k})] = \langle k_P(\mathbf{r}, \mathbf{k}) k_P^{*T}(\mathbf{r}, \mathbf{k}) \rangle \quad (17)$$

where * and $\langle\rangle$ are respectively the conjugate and the statistical mean operators.

The aim of this part is to use incoherent decompositions to obtain polarimetric time-frequency signatures. Hence, The objective of the incoherent decompositions is to separate the covariance or coherency matrices as the combination of second order descriptors corresponding to simpler or canonical objects, presenting an easier physical interpretation [8].

$$[C(\mathbf{r}, \mathbf{k})] = \sum_{i=1}^{k} p_i(\mathbf{r}, \mathbf{k})[C(\mathbf{r}, \mathbf{k})]_i \quad (18)$$

$$[T(\mathbf{r}, \mathbf{k})] = \sum_{i=1}^{k} q_i(\mathbf{r}, \mathbf{k})[T(\mathbf{r}, \mathbf{k})]_i \quad (19)$$

where the canonical responses are represented by $C[(\mathbf{r},\mathbf{k})]_i$ and $T[(\mathbf{r},\mathbf{k})]_i$, and $p_i(\mathbf{r},\mathbf{k})$ and $q_i(\mathbf{r},\mathbf{k})$ denote the coefficients of these components.

3.2 The Freeman-Durden Decomposition

3.2.1 Construction of the polarimetric time-frequency signatures

By applying the Freeman-Durden decomposition [9] on the covariance hyper-matrix, we obtain the three components scattering mechanism model:

$$\begin{aligned}[C(\mathbf{r},\mathbf{k})] &= f_s(\mathbf{r},\mathbf{k})[C(\mathbf{r},\mathbf{k})]_s + f_d(\mathbf{r},\mathbf{k})[C(\mathbf{r},\mathbf{k})]_d \\ &\quad + f_v(\mathbf{r},\mathbf{k})[C(\mathbf{r},\mathbf{k})]_v \end{aligned} \tag{20}$$

where $f_s(\mathbf{r},\mathbf{k})[C(\mathbf{r},\mathbf{k})]_s$ represents the single scattering, $f_d(\mathbf{r},\mathbf{k})[C(\mathbf{r},\mathbf{k})]_d$ is the double scattering and $f_v(\mathbf{r},\mathbf{k})[C(\mathbf{r},\mathbf{k})]_v$ the volume scattering. To calculate the different parameters, there are four observed equations for five unknown real coefficients:

$$\begin{aligned}
[C(\mathbf{r},\mathbf{k})]_{1,1} &= f_s(\mathbf{r},\mathbf{k})\beta(\mathbf{r},\mathbf{k})^2 + f_d(\mathbf{r},\mathbf{k})\alpha(\mathbf{r},\mathbf{k})^2 \\ &\quad + f_v(\mathbf{r},\mathbf{k}) \\
[C(\mathbf{r},\mathbf{k})]_{1,3} &= f_s(\mathbf{r},\mathbf{k})\beta(\mathbf{r},\mathbf{k}) - f_d(\mathbf{r},\mathbf{k})\alpha(\mathbf{r},\mathbf{k}) \\ &\quad + \frac{f_v(\mathbf{r},\mathbf{k})}{3} \\
[C(\mathbf{r},\mathbf{k})]_{2,2} &= \frac{2f_v(\mathbf{r},\mathbf{k})}{3} \\
[[C(\mathbf{r},\mathbf{k})]_{3,1} &= f_s(\mathbf{r},\mathbf{k})\beta(\mathbf{r},\mathbf{k}) - f_d(\mathbf{r},\mathbf{k})\alpha(\mathbf{r},\mathbf{k}) \\ &\quad + \frac{f_v(\mathbf{r},\mathbf{k})}{3} \\
[[C(\mathbf{r},\mathbf{k})]_{3,3} &= f_s(\mathbf{r},\mathbf{k}) + f_d(\mathbf{r},\mathbf{k}) + f_v(\mathbf{r},\mathbf{k})
\end{aligned} \tag{21}$$

So, an assumption is made [9]:

- if $\mathscr{R}e\{\langle S_{hh}(\mathbf{r},\mathbf{k})S(\mathbf{r},\mathbf{k})^*_{vv}\rangle - \frac{f_v(\mathbf{r},\mathbf{k})}{3}\} > 0 \rightarrow \alpha(\mathbf{r},\mathbf{k}) = 1$
- if $\mathscr{R}e\{\langle S_{hh}(\mathbf{r},\mathbf{k})S(\mathbf{r},\mathbf{k})^*_{vv}\rangle - \frac{f_v(\mathbf{r},\mathbf{k})}{3}\} < 0 \rightarrow \beta(\mathbf{r},\mathbf{k}) = 1$

Consequently the different parameters can be processed.

3.2.2 Interpretation of the polarimetric time-frequency signatures

The term $f_v(\mathbf{r},\mathbf{k})$ corresponds to the contribution of the volume scattering of the final hyper-covariance matrix. Hence, the scattered power by this component can be written as follows:

$$P_v(\mathbf{r},\mathbf{k}) = \frac{8f_v(\mathbf{r},\mathbf{k})}{3} \tag{22}$$

The power scattered by the double-bounce component of the hyper-covariance matrix has the expression:

$$P_d(\mathbf{r},\mathbf{k}) = f_d(\mathbf{r},\mathbf{k})(1+|\alpha(\mathbf{r},\mathbf{k})|^2) \tag{23}$$

Finally, the power scattered by the surface-like component is:

$$P_s(\mathbf{r},\mathbf{k}) = f_s(\mathbf{r},\mathbf{k})(1+|\beta(\mathbf{r},\mathbf{k})|^2) \tag{24}$$

For a scatterer located at $\mathbf{r_0}$, $P_v(\mathbf{r_0},\mathbf{k})$ (respectively, $P_d(\mathbf{r_0},\mathbf{k})$, and $P_s(\mathbf{r_0},\mathbf{k})$) represents the polarimetric behavior of volume scattering (respectively double scattering and simple scattering) versus the emitted frequency and the observation angle. These representations are called polarimetric time-frequency signatures.

3.3 The H/A/Alpha decomposition

3.3.1 Construction of the polarimetric time-frequency signatures

An Hermitian matrix 3×3 can be factorized according to [7], [10]:

$$T(\mathbf{r},\mathbf{k}) = P(\mathbf{r},\mathbf{k})D(\mathbf{r},\mathbf{k})P(\mathbf{r},\mathbf{k})^{-1} \tag{25}$$

The matrix $D(\mathbf{r},\mathbf{k})$ is diagonal with three real eigenvalues $\lambda_1(\mathbf{r},\mathbf{k}) > \lambda_2(\mathbf{r},\mathbf{k}) > \lambda_3(\mathbf{r},\mathbf{k})$ The unitary matrix is composed with eigenvectors whose the form is:

$$\begin{aligned}P_k(\mathbf{r},\mathbf{k}) = [&\cos(\alpha_k(\mathbf{r},\mathbf{k})), \sin(\alpha_k(\mathbf{r},\mathbf{k}))\cos(\beta_k(\mathbf{r},\mathbf{k})) \\ &\exp(j\delta_k(\mathbf{r},\mathbf{k})), \sin(\alpha_k(\mathbf{r},\mathbf{k}))\cos(\beta_k(\mathbf{r},\mathbf{k}))\exp(j\gamma_k(\mathbf{r},\mathbf{k}))]^T\end{aligned}$$

So, the decomposition can be written according to:

$$[T(\mathbf{r},\mathbf{k})] = \sum_{k=1}^{3} \lambda_k(\mathbf{r},\mathbf{k})P_k(\mathbf{r},\mathbf{k})P_k(\mathbf{r},\mathbf{k})^{*T} \tag{26}$$

From these eigenvectors and eigenvalues, secondary characteristics parameters can be extracted. Indeed, the eigenvalues show the power of each mechanism of the decomposition. The eigenvalues can be normalized following [8]:

$$p_k(\mathbf{r},\mathbf{k}) = \frac{\lambda_k(\mathbf{r},\mathbf{k})}{\sum_{k=1}^{3}\lambda_k(\mathbf{r},\mathbf{k})} \tag{27}$$

So, the entropy can be processed to determine the degree of randomness of the scattering process, which can be also interpreted as the degree of statistical disorder:

$$H(\mathbf{r},\mathbf{k}) = -\sum_{k=1}^{3} p_k(\mathbf{r},\mathbf{k})\log_3(p_k(\mathbf{r},\mathbf{k})) \tag{28}$$

The anisotropy can be defined to describe the secondary mechanisms:

$$A(\mathbf{r},\mathbf{k}) = \frac{p_2(\mathbf{r},\mathbf{k}) - p_3(\mathbf{r},\mathbf{k})}{p_2(\mathbf{r},\mathbf{k}) + p_3(\mathbf{r},\mathbf{k})} \tag{29}$$

The last parameter of the decomposition indicates the nature of the mechanism:

$$\alpha(\mathbf{r},\mathbf{k}) = \sum_{k=1}^{3} p_k(\mathbf{r},\mathbf{k})\alpha_k(\mathbf{r},\mathbf{k}) \tag{30}$$

3.3.2 Interpretation of the polarimetric time-frequency signatures

For a scatterer located at $\mathbf{r_0}$, $H(\mathbf{r_0},\mathbf{k})$ (respectively, $A(\mathbf{r_0},\mathbf{k})$, and $\alpha(\mathbf{r_0},\mathbf{k})$) represents the entropy (anisotropy and the α parameter) versus the emitted frequency and the observation angle. These representations are called polarimetric time-frequency signatures.

4. SUPERVISED CLASSIFICATION USING NEURAL NETWORKS

Neural networks are non-linear statistical data modeling tools. They can be used to find pattern data [11].

4.1 Architecture of the multi-layer perceptron (MLP)

A multi-layer perceptron is a feedforward artificial neural network model that maps sets of input data onto a set of appropriate output. The structure of our multi-layer perceptron is described figure (1). It is composed of nodes whose the processing is [12]:

$$a_j^{(1)} = \sum_{i=1}^{d} w_{ij}^{(1)} x_i + b_j^{(1)}, \tag{31}$$

where $a_j^{(1)}$ associated input with each hidden unit. Here $w_{ij}^{(1)}$ represents the elements of the first-layer weight matrix and b_j are the bias parameters associated with the hidden unit.

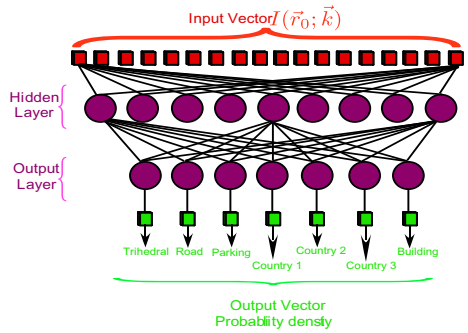

Figure 1: Architecture of the multi-layer perceptron

The variables $a_j^{(1)}$ are then transformed by the non-linear activation function of the hidden layer. The activation function is tanh(.). The outputs of the hidden units are given by:

$$z_j = \tanh(a_j^{(1)}) \tag{32}$$

which has the property that:

$$\frac{dz_j}{da_j^{(1)}} = 1 - z_j^2 \tag{33}$$

The z_j are then transformed by the second layer of weights and biases to give second-layer activation values $a_k^{(2)}$:

$$a_k^{(2)} = \sum_{j=1}^{M} w_{ij}^{(2)} z_j + b_k^{(2)} \tag{34}$$

Finally, these values are passed through the output-unit activation function to give output values y_k. For the more usual kind of classification problem in which we have of c mutually exclusive classes, we use the softmax activation function of the form [12]:

$$y_k = \frac{\exp(a_k^{(2)})}{\sum_{k'} a_{k'}^{(2)}} \tag{35}$$

Our multi-layer perceptron is a three layers whose the number of nodes of the input layer is equal to the number of input, the output layer is equal to the number of class to obtain a probability density whose the maximum defines the class which the scatterer is and the number of nodes of the hidden-layer is calculated following:

$$N_{Hidden-Layer} = \sqrt{N_{input} N_{output}} \tag{36}$$

4.2 Learning Basis

In supervised learning, a set of known signatures is given and the aim is to find a function in the allowed class of functions that matches the examples. The cost function is related to the mismatch between the mapping and the data and it implicitly contains prior knowledge about the pattern recognition problem.

The choice of the mother wavelet is moving toward a Gaussian shape. Indeed, a Gaussian have good properties and it has proved itself [13]. The spreading band of the Gaussian is chosen to $\frac{1}{6}$ band because it represents the best compromise of resolution between spatial and frequency domains. Indeed, we want a good resolution on frequency domain.

The polarimetric time-frequency signatures of manually selected scatterers are extracted as explained in the former part. On the image, the scatterers selected are the trihedral, the parking, the building, the road and three countries.

An example of Freeman-Durden learning basis is presented on the figure 2. The three contributions are coded respectively: P_d in red, P_v in green and P_s in blue. The image in the center is the image full resolution of the Freeman-Durden decomposition.

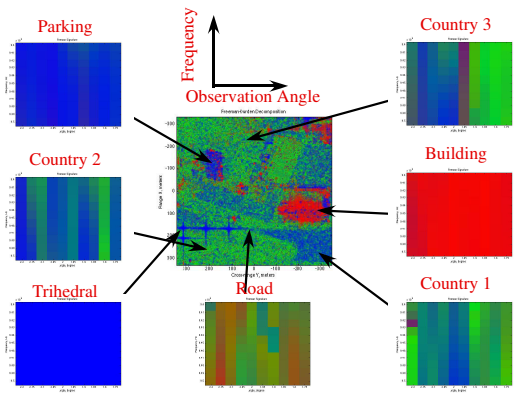

Figure 2: Learning basis obtained by the Freeman Durden polarimetric time-frequency signatures

5. RESULTS

The data under study is the full resolution image, (see Fig. 2). It is a X band image with an angular excursion of two degrees. The polarimetric time-frequency signatures are processed as explained in the former part. Then, these signatures are sent to the neural networks.

5.1 Freeman-Durden polarimetric time-frequency signatures

The results of the Freeman-Durden time-frequency signatures are represented on the figure 3. The trihedrals are classified as trihedral. The parking is identified by a melting pot of parking and trihedral contributions. It can be explained by their signatures. The three buildings are identified as a building. For the vegetation the results show that time-frequency analysis is not sufficient for these behaviors.

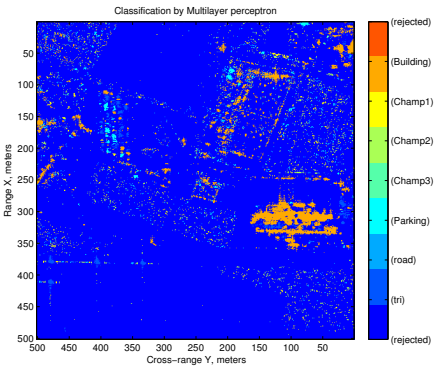

Figure 3: Classification results obtained by the Freeman Durden polarimetric time-frequency signatures

5.2 H/alpha polarimetric time-frequency signatures

The results of the H/Alpha time-frequency signatures are described on the figure 4. The trihedrals are classified as trihedral. The parking is identified by a melting pot of parking and trihedral contributions. It can be explained by their signatures. The three buildings are identified as a building. For the vegetation the results show that time-frequency analysis is not sufficient for these behaviors.

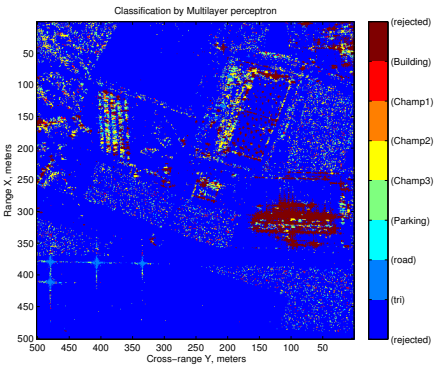

Figure 4: Classification results obtained by the H/A/Alpha polarimetric time-frequency signatures

6. CONCLUSION

A new method to classify scatterers on SAR imaging is proposed. This method is designed to work in wideband and strong angular excursion: Very High Resolution image (VHR). Indeed, it is based on stationary or non-stationary behavior of scatterers during the SAR integration. So, time-frequency analysis and polarimetric incoherent decompositions highlight this point of view. The results show that the information from polarimetric time-frequency signatures is valuable for deterministic targets (man-made targets) like trihedral, building and parking. However, this information does not allow to characterize the vegetation or the canopy.

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Annexe E

New Methods of Radar Detection Performances Analysis

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New methods of radar performances analysis

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Abstract

Original methods for radar detection performance analysis are derived for a fluctuating or non-fluctuating target embedded in additive and a priori unknown noise. This kind of noise can be, for example, the sea or ground clutter encountered in surface-based radar for the detection of low grazing angle targets and/or in high-resolution radar. In these cases, the spiky clutter tends to have a statistic which strongly differs from the Gaussian assumption. Therefore, the detection theory is no longer appropriate since the nature of statistics has to be known. The new methods proposed here are based on the parametric modelling of the moment generating function of the noise envelope by Padé approximation, and lead to a powerful estimation of its probability density function. They allow to evaluate the radar detection performances of targets embedded in arbitrary noise without knowledge of the closed form of its statistic and in the same way to take into account any possible fluctuation of the target. These methods have been tested successfully on synthetic signals and used on experimental signals such as ground clutter. © 2000 Elsevier Science B.V. All rights reserved.

Zusammenfassung

Typische Methoden zur Analyse der Eigenschaften von Radardetektoren sind für bewegte und nicht-bewegte Ziele in additivem und a priori unbekannten Rauschen hergeleitet worden. Derartiges Rauschen kann beispielsweise als See- oder Bodenclutter im oberflächen-basierten Radar für die Detektion von Zielen mit kleinen *grazing angles* und/oder beim hochauflösenden Radar angetroffen werden. In diesen Fällen tendiert das mit Spitzen versehene *clutter* zu einer Statistik, die sich von einer angenommenen Gaußverteilung nachhaltig unterscheidet. Demzufolge sind die theoretischen Betrachtungen zur Detektion aufgrund der unbekannten statistischen Eigenschaften nicht mehr geeignet. Die hier vorgestellten neuen Methoden basieren auf der parametrischen Modellierung der momentenerzeugenden Funktion mit Hilfe der Pade Approximation und führen zu einer leistungsstarken Schätzung der Wahrscheinlichkeits dichtefunktion. Diese Methoden gestatten die Überprüfung des Detektionsverhaltens bei Zielen im Umgebungsrauschen ohne Kenntnis der Statistik in geschlossener Form und unter Berücksichtigung jeglicher Quellenfluktuation. Tests mit synthetischen Signalen wurden erfolgreich mit diesen Methoden durchgeführt und auf experiementelle Signale, wie Bodenclutter angewendet. © 2000 Elsevier Science B.V. All rights reserved.

Résumé

De nouvelles méthodes d'analyse de performances de détection radar sont présentées dans le cadre d'une cible fluctuante ou non noyée dans un bruit additif *a priori* inconnu. Ce type de bruit peut être, par exemple, du fouillis de mer ou de sol rencontré dans le cadre de la détection de cible à site bas et/ou pour un radar à haute résolution distance; la nature impulsionnelle d'un tel fouillis écarte l'hypothèse gaussienne généralement retenue pour la modélisation de sa

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statistique. Les nouvelles méthodes proposées ici se basent sur la modélisation paramétrique de la fonction génératrice de moments de l'enveloppe du bruit par approximants de Padé et conduisent à une estimation de sa densité de probabilité. Elles permettent d'évaluer les performances de détection radar d'une cible noyée dans un bruit quelconque sans connaissance de sa statistique et permettent, de la même manière, de prendre en compte toute fluctuation éventuelle de la cible. Ces méthodes ont été testées avec succès sur signaux synthétiques et ont été mises en œuvre sur signaux réels, tel que du fouillis de forêt. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Radar detection; Performance analysis; Signal noise

1. Description of the problem

The radar detection of a target against a background of unwanted clutter due to echoes from sea or land is a problem of interest in the radar field. For many years, the statistics of quadrature components of the radar clutter were supposed to be jointly Gaussian because of the low radar resolution capabilities: in this case, the clutter was viewed as a sum of echoes from a very large number of elementary scatterers (Central Limit Theorem). Current systems have now improved their resolution capabilities and hence their detection performance. However, as resolution is increased, the statistics of the additive noise is no longer Gaussian. Recent experimentations conducted at ONERA and other organizations like MIT [5] indicate that large deviations from Rayleigh statistics are observed in situations such as low grazing angle illumination and/or high resolution. In such cases, due to the spiky nature of the clutter, the empirical distribution exhibits both higher tails and larger standard deviation relative to the mean than predicted by the Rayleigh distribution. Therefore, many work has been devoted to fit empirical models of distribution to experimental data. This is the case of the compound Gaussian processes [8,9], also called spherically invariant random processes (SIRP) which allow to model the multivariate probability density function (PDF) of the envelope of the clutter returns, taking into account the possible spatial or temporal correlation of the processes. The well-known log-normal, Weibull and K-distribution densities [6] belong to this class of distributions but the main problems with this kind of approach are the quality of the estimation of the SIRP parameters and the complexity of the optimal detector implementation. In this paper, we propose to analyze the performances of radar detection of a target embedded in any combination of clutter and thermal noise without knowledge of the closed form of noise densities. The estimation of the noise envelope density is only performed according to the modelisation with Padé approximation of the moment generating function (MGF) for the noise envelope. This method is based on the estimation of all n-order moments of the noise envelope, which we will suppose exactly estimated. This kind of modelisation allows to derive, for a constant false alarm rate, the simple form of the detection probability (P_d) of a target with constant or fluctuating envelope embedded in a complex noise fully characterized by the moments of its envelope.

2. General relations of the detection theory

2.1. Neymann–Pearson criterion

We consider here the basic problem of detecting the presence or absence of a complex signal $s(t)$ with envelope A in a set of measurements $y(t) = y_I(t) + iy_J(t)$ corrupted by a sum of independent additive complex noise signals $c(t)$ corresponding to the clutter echoes and white Gaussian thermal noise. This problem can be described mathematically in terms of a test between the following pair of statistical hypothesis, where $c(t)$ denotes all the unwanted noises:

$$\text{Hypothesis } H_0: \quad y(t) = c(t), \tag{1}$$

$$\text{Hypothesis } H_1: \quad y(t) = s(t) + c(t). \tag{2}$$

The Neymann–Pearson criterion fixes the probability of false alarm P_{fa} and maximizes the probability to detect signal P_d over the detection threshold θ depending on the P_{fa} value.

2.2. Statistical framework

Throughout this paper, we consider the *Envelope Detection* scheme of the complex signal $y(t)$. In a statistic framework, expressions of P_d and P_{fa} need to be derived and depend on the noise envelope statistic characterized by its probability density function (PDF). If we note $p_{H_0}(r)$ the probability density of the noise envelope $|c(t)|$ under H_0 hypothesis, the detection threshold θ is fixed by the value of the given probability of false alarm P_{fa}:

$$P_{fa} = \int_{\theta}^{+\infty} p_{H_0}(r)\,dr. \tag{3}$$

While denoting $p_{H_1}(r)$ the PDF of the envelope of the complex signal embedded in noise $|s(t) + c(t)|$ under H_1 hypothesis, the detection probability P_d is fully characterized by

$$P_d = \int_{\theta}^{+\infty} p_{H_1}(r)\,dr. \tag{4}$$

Since phases between quadrature components of the clutter, thermal noise and target are unknown, they are commonly supposed to be uniformly distributed on $[-\pi,\pi]$. This hypothesis is very important because in this case, each two-dimensional density function of the quadrature component is hence a circular symmetric distribution. So we can find [7] a very interesting relation between the PDF of the noise plus signal envelope ($p_{H_1}(r;A)$) and the PDF of the noise envelope ($p_{H_0}(r)$), using the coherent radial characteristic function of the noise process.

2.3. Basic relation

Let us consider a random complex process with real part $a(t)$ and imaginary part $b(t)$. This process is characterised by its two-dimensional density function $p(a,b)$, or dually, by a so-called *coherent* characteristic function $C(u_1,u_2)$, which is $p(a,b)$ Fourier transform:

$$C(u_1,u_2) = \iint_{-\infty}^{+\infty} p(a,b)\,e^{i(u_1 a + u_2 b)}\,da\,db. \tag{5}$$

This characteristic function can be transformed in a single radial variate ρ function (ρ represents the value of the signal envelope, $\rho = \sqrt{u_1^2 + u_2^2}$) with the assumption of a phase uniformly distributed i.e. $p_\Phi(\alpha = \arctan(b/a)) = 1/2\pi$, independent of the envelope $p_r(r = \sqrt{a^2 + b^2})$. So we have

$$p(a,b)\,da\,db = p_\Phi(\alpha)p_r(r)\,d\alpha\,dr, \tag{6}$$

and

$$C(\rho) = \int_0^{2\pi} p_\Phi(\alpha)\left[\int_0^{+\infty} p_r(r)\,e^{i\rho r\cos\alpha}\,dr\right]d\alpha$$
$$= \int_0^{+\infty} p_r(r)\left[\frac{1}{2\pi}\int_0^{2\pi} e^{i\rho r\cos\alpha}\,d\alpha\right]dr,$$

i.e.

$$C(\rho) = \int_0^{+\infty} p_r(r)J_0(\rho r)\,dr, \tag{7}$$

where $J_0(x)$ denotes the ordinary Bessel function of order 0. Inverting expression (7), we obtain $p_r(r)$ as a function of $C(\rho)$:

$$p_r(r) = \int_0^{+\infty} r\rho C(\rho)J_0(\rho r)\,d\rho. \tag{8}$$

In the following, the next two results will be used:

- For a signal with constant envelope A and an uniform phase, the PDF is characterised by

  $$p(r) = \delta(r - A),$$

  where $\delta(.)$ is the Dirac distribution, and the coherent radial characteristic function is given by

  $$C(\rho) = J_0(\rho A).$$

- The characteristic function $C_{H_1}(\rho)$ of the sum of the signal $s(t)$ and unwanted clutter $c(t)$, is equal to the product of the characteristic functions of signal $C_s(\rho)$ and of noise $C_c(\rho)$.

It is now possible to derive a relation between the density $p_{H_0}(r)$ of the envelope under hypothesis H_0 (noise only) and the density $p_{H_1}(r;A)$ of the envelope under hypothesis H_1 (noise and signal with constant envelope A), recalling that the following results are derived under the strong hypothesis of uniform phases distributions between the in-phase and quadrature components of each

process:

$$p_{H_1}(r;A) = \int_0^{+\infty} r\rho C_s(\rho)C_c(\rho)J_0(\rho r)\,d\rho$$
$$= \int_0^{+\infty} r\rho J_0(\rho A)Cc(\rho)J_0(\rho r)\,d\rho, \quad (9)$$

with

$$C_c(\rho) = \int_0^{+\infty} p_{H_0}(r)\,J_0(\rho r)\,dr. \quad (10)$$

Replacing (10) in (9) leads to the following important relation:

$$p_{H_1}(r;A) = \int_0^{+\infty}\int_0^{+\infty} r\rho J_0(\rho A)J_0(\rho r) \times J_0(\rho r')p_{H_0}(r')\,d\rho\,dr'. \quad (11)$$

In the case of fluctuating target with envelope density fluctuation law $p(A;A_0)$ (where A_0 represents the mean value of the fluctuations), relation (11) becomes more general:

$$p_{H_1}(r;A_0) = \int_0^{+\infty} p_{H_1}(r;A)p(A;A_0)\,dA$$
$$= \int_0^{+\infty}\int_0^{+\infty} r\rho J_0(\rho r)J_0(\rho r')p_{H_0}(r') \times\left[\int_0^{+\infty} J_0(\rho A)p(A;A_0)\,dA\right]d\rho\,dr'. \quad (12)$$

This relation is very important because it shows that all the expressions needed to evaluate the detection performances are related to the noise envelope statistic $p_{H_0}(r)$.

Example 1 (Gaussian noise). *Non-fluctuating target*: Relation (11) connects the PDF of the envelope of a complex Gaussian noise, i.e. a Rayleigh distribution (power $2\sigma^2$):

$$p_{H_0}(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (13)$$

to the envelope PDF of a constant signal embedded in this complex noise, which leads to the well known Rice–Nakagami distribution:

$$p_{H_1}(r;A) = \frac{r}{\sigma^2}\exp\left(-\frac{A^2+r^2}{2\sigma^2}\right)I_0\left(\frac{rA}{\sigma^2}\right), \quad (14)$$

where $I_0(x)$ is the modified Bessel function of the first kind and zero order. Replacing (14) in (4) gives a first expression of P_d for a non-fluctuating target embedded in complex Gaussian noise:

$$P_d = \int_\theta^{+\infty} p_{H_1}(r;A)\,dr = \mathcal{Q}\left(\frac{A}{\sigma},\frac{\theta}{\sigma}\right), \quad (15)$$

where $\mathcal{Q}(a,b)$ is the Marcum $\mathcal{Q}$-function defined by

$$\mathcal{Q}(a,b) = \int_b^{+\infty} x\exp\left(-\frac{x^2+a^2}{2}\right)I_0(ax)\,dx \quad (16)$$

and the detection threshold θ obtained for a given P_{fa} according to (3) and (13):

$$\theta = \sqrt{-2\sigma^2\log(P_{fa})}. \quad (17)$$

Fluctuating target: If the target is fluctuating according to a Swerling I law of power A_0^2,

$$p(A;A_0) = \frac{2A}{A_0^2}\exp\left(-\frac{A^2}{A_0^2}\right) \quad (18)$$

relation (12) becomes

$$p_{H_1}(r;A_0) = \frac{2r}{2\sigma^2+A_0^2}\exp\left(-\frac{r^2}{2\sigma^2+A_0^2}\right), \quad (19)$$

which corresponds to a Rayleigh distribution with power $2\sigma^2 + A_0^2$. Then we have (θ is derived according to (17)):

$$P_d = \int_\theta^{+\infty} p_{H_1}(r;A_0)\,dr$$
$$= \exp\left(-\frac{\theta^2}{2\sigma^2+A_0^2}\right). \quad (20)$$

Example 2 (K-distributed noise and non-fluctuating target). The K-distributed noise envelope is K-distributed with probability density function

(PDF):

$$p_{H_0}(r) = \frac{b^{v+1}}{2^{v-1}\Gamma(v)} r^v K_{v-1}(br), \tag{21}$$

cumulative density function (CDF)

$$F(r) = 1 - \frac{(br)^v}{\Gamma(v)2^{v-1}} K_v(br), \tag{22}$$

and coherent radial characteristic function

$$C(\rho) = \frac{1}{(1+\rho^2/b^2)^v}, \tag{23}$$

where $K_v(.)$ is the modified Bessel function of the third kind, v is the shape of the density and parameter b is related to the second-order moment σ^2 by:

$$b = 2\sqrt{\frac{v}{\sigma^2}}. \tag{24}$$

Smaller is the value of the shape parameter v, spikier is the K-distribution and when $v \to +\infty$, it is close to a Gaussian distribution.

Relation (11) becomes in this case

$$p_{H_1}(r;A) = \int_0^{+\infty} r\rho \frac{J_0(\rho A)J_0(\rho r)}{(1+\rho^2/b^2)^v}\,\mathrm{d}\rho.$$

In order to derive the expression of P_d we just have to integrate the above expression with respect to r. Using the tables [1] this expression becomes

$$P_d = 1 - \int_0^{+\infty} \theta \frac{J_0(\rho A)J_1(\rho\theta)}{(1+\rho^2/b^2)^v}\,\mathrm{d}\rho. \tag{25}$$

The expression of the threshold θ is detailed further in (46).

The two relations (11) and (12) are usually quite difficult to compute numerically for evaluating the detection performances for several signal-to-noise ratios.

In the next section, we will use an interesting method proposed and developed in [2–4] which allows to estimate the PDF of any noise from its n-order moments and give very useful relations to compute the pair P_d, P_{fa} for a given signal-to-noise ratio. This method is described in the following section in a general way, and then applied to the detection performance analysis.

3. Padé approximation

3.1. Description of the method

This method [2–4] is based on the parametric construction of the moment generating function (MGF) of the noise envelope by Padé approximation. The MGF $\Phi(u)$ of a random process is defined by the mono-lateral Laplace transform of its envelope PDF $p(r)$:

$$\Phi(u) = \int_0^{+\infty} p(r)\,\mathrm{e}^{-ur}\,\mathrm{d}r. \tag{26}$$

After a Taylor series expansion of e^{-ur} around $u = 0$, $\Phi(u)$ can be expressed as follows:

$$\Phi(u) = \sum_{n=0}^{\infty} \mu_n \frac{(-u)^n}{n!} = \sum_{n=0}^{\infty} c_n u^n, \quad u \to 0, \tag{27}$$

where $\mu_n = \int_0^{+\infty} r^n p(r)\,\mathrm{d}r$ denotes the nth-order moments of the process envelope.

If we assume all the moments μ_n perfectly known up to order $L + M + 1$, the main idea is to truncate the infinite series at the order $L + M + 1$ and to approximate it by a rational function $P^{[L/M]}(u)$ $(L \leqslant M)$ defined by

$$P^{[L/M]}(u) = \frac{\sum_{n=0}^{L} a_n u^n}{\sum_{n=0}^{M} b_n u^n}, \tag{28}$$

where the coefficients $\{a_n\}$ and $\{b_n\}$ are determined so that the following relation is verified:

$$\frac{\sum_{n=0}^{L} a_n u^n}{\sum_{n=0}^{M} b_n u^n} = \sum_{n=0}^{L+M} c_n u^n + \mathcal{O}(u^{L+M+1}). \tag{29}$$

The notation $\mathcal{O}(u^{L+M+1})$ simply takes into account terms of order higher than u^{L+M}. To determine the two sets of coefficients $\{a_n\}$ and $\{b_n\}$, we have to match the coefficients:

$$\sum_{n=0}^{M} b_n u^n \sum_{n=0}^{L+M} c_n u^n = \sum_{n=0}^{L} a_n u^n + \mathcal{O}(u^{L+M+1}). \tag{30}$$

The moments matching conditions fix in a first step the set of coefficients $\{b_n\}$ by solving a simple set of M linear equations for the M unknown denominator coefficients:

$$\sum_{n=0}^{M} b_n c_{L-n+j} = -c_{L+j}, \quad 1 \leqslant j \leqslant M \tag{31}$$

which can be written as matrix and linear system,

$$\begin{pmatrix} c_{L-M+1} & \cdots & c_L \\ \vdots & \vdots & \vdots \\ c_{L-M+k} & \cdots & c_{L+k-1} \\ \vdots & \vdots & \vdots \\ c_L & \cdots & c_{L+M-1} \end{pmatrix} \boldsymbol{b} = -\boldsymbol{c}, \tag{32}$$

where $\boldsymbol{b} = (b_M, \ldots, b_k, \ldots, b_1)^{\mathrm{T}}$, and $\boldsymbol{c} = (c_{L+1}, \ldots, c_{L+k+1}, \ldots, c_{L+M})^{\mathrm{T}}$.

In a second step the set $\{a_n\}$ is determined by a simple convolution of the $\{b_n\}$ and the $\{c_n\}$ coefficients:

$$a_j = c_j + \sum_{i=1}^{j} b_i c_{j-i}, \quad 0 \leqslant j \leqslant L. \tag{33}$$

The set of coefficients $\{a_n\}$ and $\{b_n\}$ determined from (32) and (33), defines, owing to the Padé Approximation, the *One Point* parametric modelling of the MGF given its power series expansion (27) at $u = 0$.

If we suppose that the rational fraction approximation has M distinct poles with negative real part to assure its convergence for $u \to \infty$, relation (28) can be rewritten as

$$P^{[L/M]}(u) = \sum_{k=1}^{M} \frac{\lambda_k}{u - \alpha_k}, \quad \mathrm{Re}(\alpha_k) < 0. \tag{34}$$

From this description, we are able to determine a random vector PDF and CDF using the inverse Laplace transform of the corresponding MGF performed by residue inversion formula and leading to a sum of weighted decaying complex exponentials.

3.2. PDF and CDF expressions

We saw (26) that MGF is the PDF Laplace transform. Using the inversion residue formula on (34) results in approximate PDF and CDF as a sum of weighted decaying complex exponentials:

$$p(r) = \sum_{k=1}^{M} \lambda_k \, \mathrm{e}^{\alpha_k r}, \quad \mathrm{Re}(\alpha_k) < 0, \tag{35}$$

$$F(r) = 1 + \sum_{k=1}^{M} \frac{\lambda_k}{\alpha_k} \mathrm{e}^{\alpha_k r}. \tag{36}$$

Given the moments of a random process, the tail of the PDF is more accurately approximated, that is, the region $u \to 0$ in $\Phi(u)$ corresponds to $r \to \infty$ in $p(r)$. This is the reason why such approximations are very interesting in radar detection studies for the P_{d} and P_{fa} computations (needing PDF tails integration).

The Padé approximation yields good results in the estimation of a PDF and an example is shown in Fig. 1 for the K-distribution PDF defined by (21) with $\nu = 0.1$, $\sigma^2 = 1$ and b related to ν and σ^2 by (24). The relative error $\varepsilon_{\mathrm{rel}}$ is defined as follows:

$$\varepsilon_{\mathrm{rel}} = \left| \frac{K_{\mathrm{approx}} - K_{\mathrm{theo}}}{K_{\mathrm{theo}}} \right|. \tag{37}$$

3.3. Extension

- *Variates generation*: As we now know the expression of a random variate CDF (36), we can generate samples X according to (35) as follows:
  1. Generate $U \sim \mathscr{U}[0,1]$.
  2. Generate $X \sim F^{-1}(U)$.

  In our case these generated samples represent variates envelope. As ones can write for a random complex process Z that $Z = |Z| \, \mathrm{e}^{\mathrm{j}\,\Phi}$ (where Φ is the assumed uniform phase between the I and Q components of the process), the complex process can be retrieved in generating Φ as an uniformely distributed variate in $[0,2\pi]$. This could be useful in the case where only data envelope are available.
- *Two-points approximation*: To obtain a more uniform approximation, we could use a two-points approximation, often about $u \to 0$ and $u \to +\infty$. This requires that the MGF be investigated in the vicinity of these two points, and not surprisingly $u \to +\infty$ in $\Phi(u)$ implies $r \to 0$ in $p(r)$. We will not use the two-points Padé approximation in this paper because our goal is to apply the method to detect problems that principally require the study of the tail of the distribution and because of the lack of knowledge about the Markov coefficients needed in the expansion at $u \to +\infty$ (see [2] for more details).
- *Stabilization*: In the case where some poles have positive real part, it is necessary to stabilize the rational fraction without changing the moments.

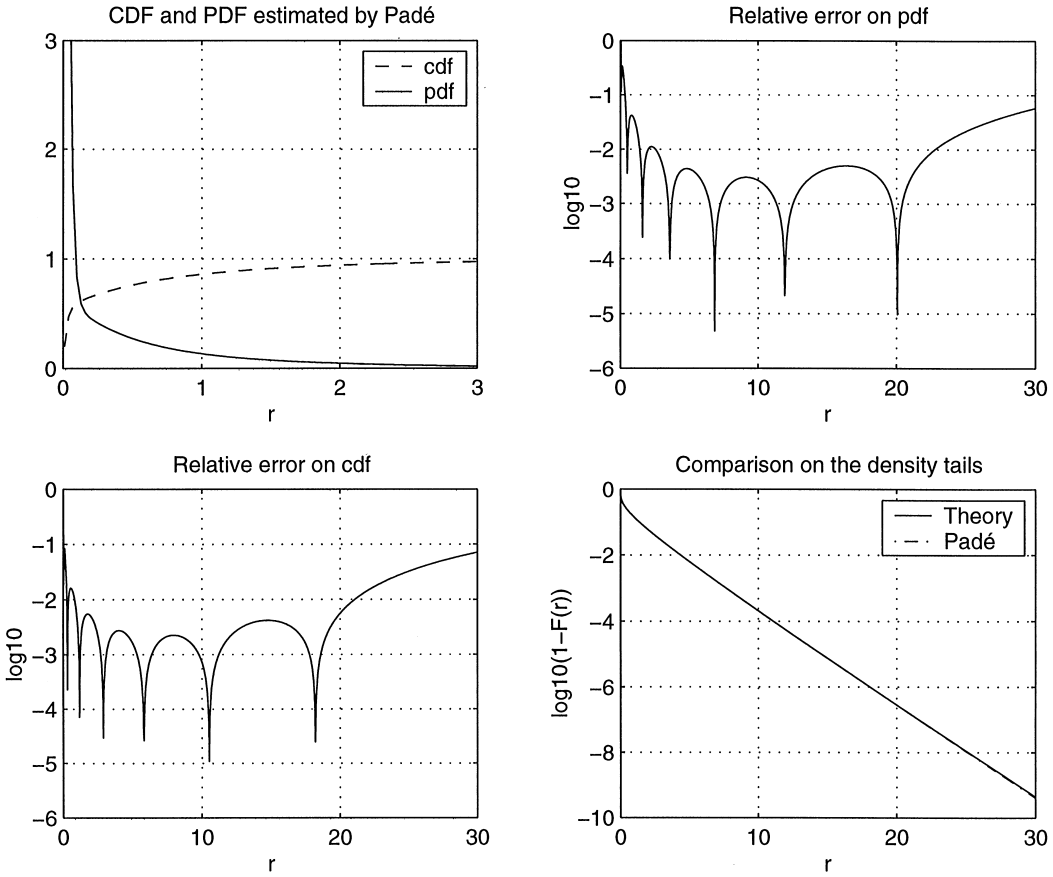

Fig. 1. Analysis of Padé K-distribution approximation ($\nu = 0.1$) and ($\sigma^2 = 1$).

Such a stabilization procedure is proposed in [2]. The task is to find a new approximation that is stable and does not sacrifice the original and extrapolated moments. This consists in rewriting the moments expression in terms of the residues and the poles and looking for the smallest change necessary to make the unstable approximation a stable one (i.e. singularities of the MGF in the left-half plane). But any change on the residues and the poles perturbes the moments and a compromise between the two effects has to be found.

In this work, we do not use this kind of procedure. A first choice for the order of the numerator L and the denominator M is $L = M - 1$. This assures the convergence of the MGF when $|u| \to +\infty$ and the resulting approximation called *sub-diagonal* is known to be one of the more accurate (with *diagonal* approximation where $L = M$). The second choice to find the value of M is to test the negativity of the poles. The initialised value of M is set to be equal to an upper value K (so $L = K - 1$) and we compute the poles and residues values. If the poles satisfy the real part negativity condition, then the approximation is found. Otherwise, we change the value of M to $M - 1$ (L to $L - 1$) and compute the new residues and poles values testing the sign of the real part of the poles. The upper value K is determined according to the rank of the Hankel matrix in (32) which is not always full (in this case the rank is full when equal to M). So we look for the highest value K which coincides with M among all the rank values coinciding with M (varying from 1 to 15). It means that for $M \geqslant K$ the rank is deficient.

3.4. Comparison with another method

This method has to be compared with a non-parametric method, based on the PDF estimation by a kernel method described in [10]. However, this one is not well adapted to our problem because of its poor quality of estimation in the tail of distribution. The mismatch of this last method can be seen in Fig. 2 where the kernel estimated PDF of the K-distribution ($\nu = 0.1$) is plotted. The relative error shows the non-efficiency of the kernel estimation if applied to our task.

In the next section, we use the method to approximate the noise PDF (that allows to simply determine the detection threshold value with wanted P_{fa}) and to take into account the target fluctuations which PDF will be also estimated by Padé approximation.

4. Evaluation of detection performances

General relations given by (11) and (12) can be simplified when using Padé approximations for the noise envelope and envelope fluctuation PDF. With the knowledge of the noise envelope of the experimental data, $p_{\mathrm{H}_0}(r)$ can be approximated using (35):

$$p_{\mathrm{H}_0}(r) = \sum_{k=1}^{M} \lambda_k \mathrm{e}^{\alpha_k r}, \tag{38}$$

where the set of coefficients $\{\lambda_k\}$ and $\{\alpha_k\}$ is determined by the Padé approximation. So, the detection threshold θ is perfectly defined by the determination (Newton find root algorithm) of the non-linear equation (see (3)):

$$P_{\mathrm{fa}} = -\sum_{k=1}^{M} \frac{\lambda_k}{\alpha_k} \mathrm{e}^{\alpha_k \theta}. \tag{39}$$

4.1. Non-fluctuating target

We always suppose that the constant envelope of the target is $|s(t)| = A$. Using (38) in (11) leads to

$$p_{\mathrm{H}_1}(r;A) = \int_0^{+\infty} \rho r J_0(\rho A) J_0(\rho r) \sum_{k=1}^{M} \frac{\lambda_k}{\sqrt{\rho^2 + \alpha_k^2}} \,\mathrm{d}\rho. \tag{40}$$

Recalling, owing to [1], that

$$\int_0^{\theta} r\, J_0(\rho r)\,\mathrm{d}r = \theta J_1(\rho\theta)/\rho,$$

$$\int_0^{+\infty} J_0(\rho y) \mathrm{e}^{\alpha_k y}\,\mathrm{d}y = \frac{1}{\sqrt{\rho^2 + \alpha_k^2}},$$

the detection probability P_{d} defined by (4) takes the simple form

$$P_{\mathrm{d}} = 1 - \int_0^{+\infty} \theta J_0(\rho A) J_1(\rho\theta) \sum_{k=1}^{M} \frac{\lambda_k}{\sqrt{\rho^2 + \alpha_k^2}} \,\mathrm{d}\rho, \tag{41}$$

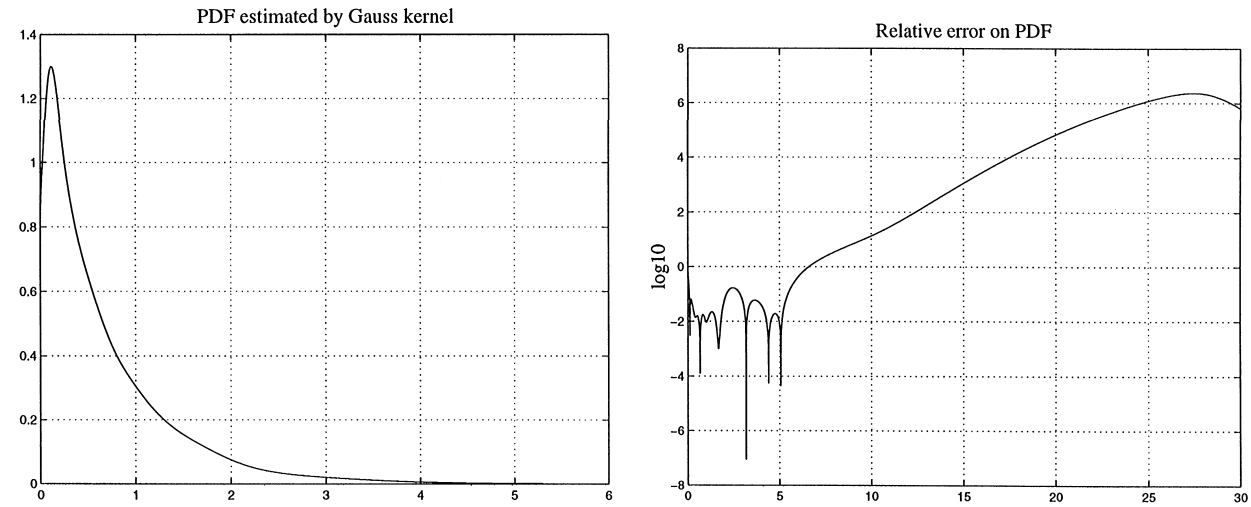

Fig. 2. Analysis of Gaussian kernel K-distribution PDF estimation ($\nu = 0.1$) and ($\sigma^2 = 1$).

where the detection threshold θ is perfectly defined by the determination of (39).

4.2. Fluctuating target

In the case of fluctuating target (Swerling fluctuations for example), it is possible to estimate the envelope fluctuation density $p(A;A_0)$ (where A_0 is mean value of fluctuation) by Padé approximation:

$$p(A;A_0) = \sum_{i=1}^{N} \frac{\gamma_i}{A_0} \mathrm{e}^{(\delta_i/A_0)A}, \tag{42}$$

where the N parameters $\{\gamma_i\}$ and $\{\delta_i\}$ are once determined when $A_0^2 = 1$. Eq. (12) can be transformed as

$$p_{\mathrm{H}_1}(r;A_0) = \int_0^{+\infty} r\rho J_0(\rho r) \sum_{k=1}^{M} \frac{\lambda_k}{\sqrt{(\rho^2 + \alpha_k^2)}} \times \sum_{i=1}^{N} \frac{\gamma_i}{\sqrt{(\rho^2 A_0^2 + \delta_i^2)}} \mathrm{d}\rho \tag{43}$$

which leads to the detection probability formula with the detection threshold θ always given by the resolution of (39):

$$P_{\mathrm{d}} = 1 - \int_0^{+\infty} \theta J_1(\rho\theta) \sum_{k=1}^{M} \frac{\lambda_k}{\sqrt{(\rho^2 + \alpha_k^2)}} \times \sum_{i=1}^{N} \frac{\gamma_i}{\sqrt{(\rho^2 A_0^2 + \delta_i^2)}} \mathrm{d}\rho. \tag{44}$$

Relations (41) and (44) are very general and can be easily computed, since we have therefore only one integration to compute instead of quadruple generalized integration without Padé approximation. The latter allows to evaluate any radar detection performance for any target embedded in any noise.

Successful tests of this method on synthetic signals allow us to be optimistic for the evaluation of PDF and CDF Padé approximation obtained on experimental forest clutter data and of their radar detection performance that we perform in the next section. We show, according to these results, the mismatch between the real hypothesis and the Rayleigh hypothesis.

5. Some results

5.1. Synthetic signals

We choose K-distributed clutter (power $\sigma^2 = 1$), as described in (21), with a small shape parameter $\nu = 0.1$ and a parameter b given by (24). Fig. 3 shows some realisations of K-distributed processes with different values of the shape parameter $\nu = 0.1, 2, 10, +\infty$ ($\nu = +\infty$ corresponds to a Rayleigh process) and power $\sigma^2 = 1$. The associated coherent radial characteristic function is

$$C(\rho) = \left(1 + \frac{\rho^2}{b^2}\right)^{-\nu}. \tag{45}$$

Detection threshold θ is defined by the determination of the non-linear equation

$$P_{\mathrm{fa}} = \frac{(b\theta)^\nu}{\Gamma(\nu)2^{\nu-1}} K_\nu(b\theta), \tag{46}$$

and so, for a non-fluctuating target we first have

$$P_{\mathrm{d}} = 1 - \int_0^{+\infty} \theta \frac{J_0(\rho A) J_1(\rho\theta)}{(1 + \rho^2/b^2)^\nu} \mathrm{d}\rho. \tag{47}$$

If we consider the target fluctuations with a Swerling I probability law (variance A_0^2),

$$p(A;A_0) = \frac{2A}{A_0^2} \exp\left(-\frac{A^2}{A_0^2}\right), \tag{48}$$

Eq. (47) becomes

$$P_{\mathrm{d}} = 1 - \int_0^{+\infty} \theta \frac{J_1(\rho\theta)\mathrm{e}^{-\rho^2 A_0^2/4}}{(1 + \rho^2/b^2)^\nu} \mathrm{d}\rho. \tag{49}$$

Fig. 4 shows theoretical curves $P_{\mathrm{d}}/P_{\mathrm{fa}}$ for different SNR (signal-to-noise ratio) varying from 10 to 50 dB, and the Fig. 5 represents the curves derived by simulation and Padé approximations. To simulate K-distributed processes we use the elegant SIRP theory (spherically invariant random process, [8,9]) which states that K-distribution ($\boldsymbol{X} = \boldsymbol{Z}S$) is derived with Gaussian vector ($\boldsymbol{Z}$) whose standard deviation is itself a gamma random variate (S), independent of $\boldsymbol{Z}$ with a non-negative PDF $f_S(S)$ so-called the characteristic PDF of the SIRV (vector). This representation is interesting to take into account the possible noise correlation since a SIRV is invariant

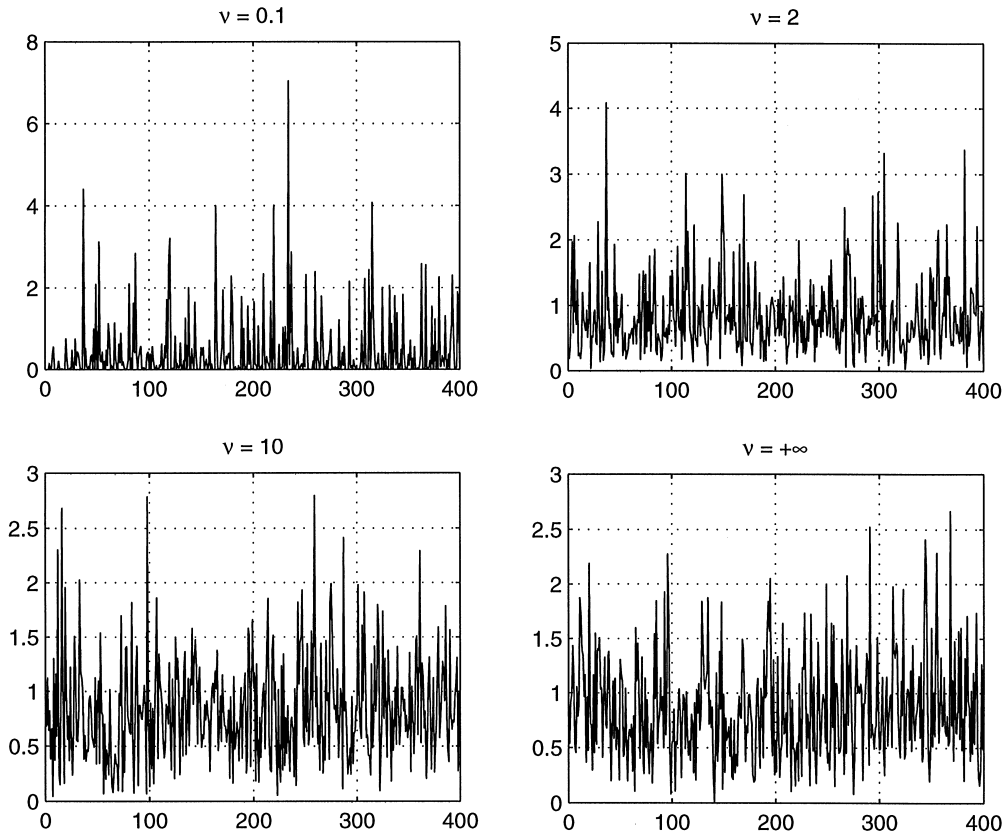

Fig. 3. Realizations of K-distributed noises for differents values of $\nu = 0.1, 2, 10, +\infty$, power $\sigma^2 = 1$.

under linear transformation, that is to say that if $\boldsymbol{X}$ is a SIRV then $\boldsymbol{Y} = \boldsymbol{AX} + \boldsymbol{B}$ is a SIRV with the same characteristic PDF ($\boldsymbol{A}$ is a non-singular matrix and $\boldsymbol{B}$ is a vector having the same dimension as $\boldsymbol{X}$).

The Rayleigh probability law is the well-known Gaussian complex vector envelope law. The fluctuations coefficients $\{\gamma_i\}$ and $\{\delta_i\}$ are derived once for $A_0^2 = 1$ and then its law is deducted for each value of the SNR from the reference ($A_0^2 = 1$). The results show the excellent quality of the approximation.

5.2. Experimental data

The signal here analysed corresponds to experimental forest clutter data envelope spatially collected at low grazing angle ($< 1°$) in 246 range bins of 0.5 m with an X-band radar. The aim of this measurements campaign was to study ground clutter scatterers and we use few of them to evaluate the radar detection performances of a virtual target embedded in such an environment with the use of a Padé approximation on the noise envelope. For convenience, the noise power has been normalized to one.

Fig. 6 shows results of PDF and CDF Padé approximation of this experimental clutter data. Fig. 7 gives detection performances of a *hypothetical* non-fluctuating target which would be embedded (in phase and amplitude) in such a noise. The curves show the mismatch between real and Rayleigh hypothesis. For the latter, we can see on the curve that the false alarm rate increases until $P_{\mathrm{fa}} = 1.34 \times 10^{-4}$ if we consider the true statistic of the analysed clutter.

The moments have been estimated from the set of complex clutter data y_i according to the

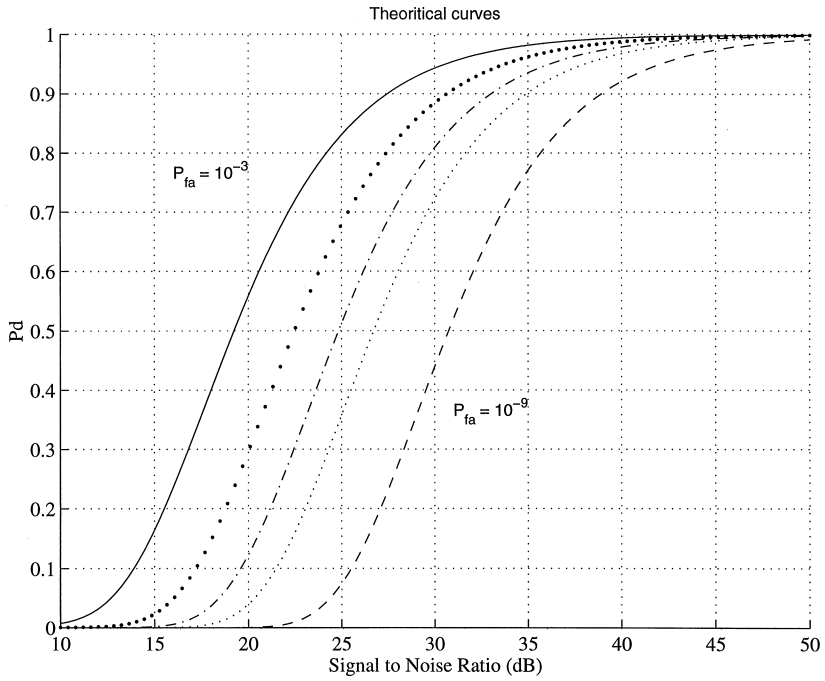

Fig. 4. Theoretical detection curves for $P_{fa} = 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-9}$: Swerling I fluctuating target embedded in K-distributed noise ($v = 0.1$ and $\sigma^2 = 1$).

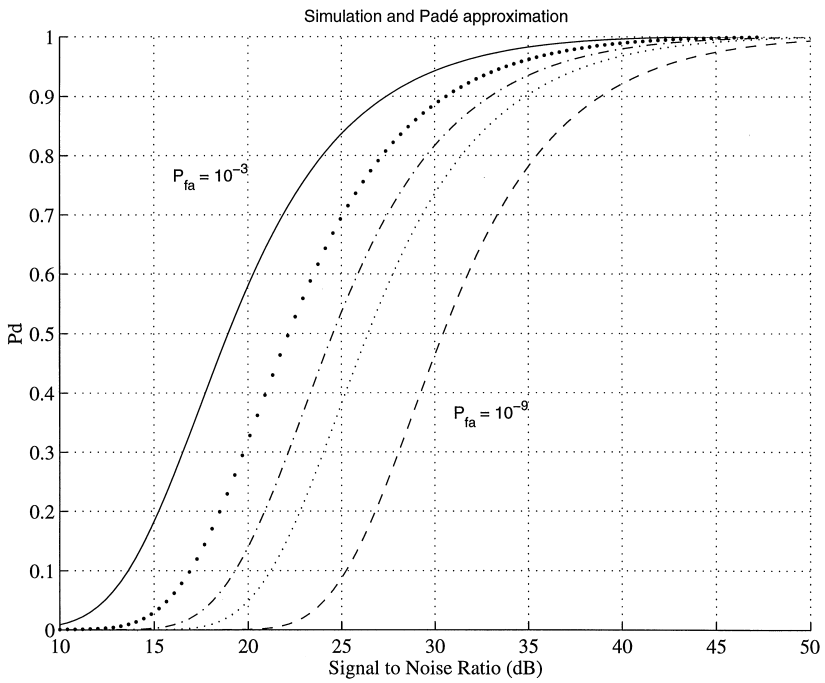

Fig. 5. Detection curves for $P_{fa} = 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-9}$: simulation and Padé approximation ($M = 4$): Swerling I fluctuating target embedded in K-distributed noise ($v = 0.1$ and $\sigma^2 = 1$).

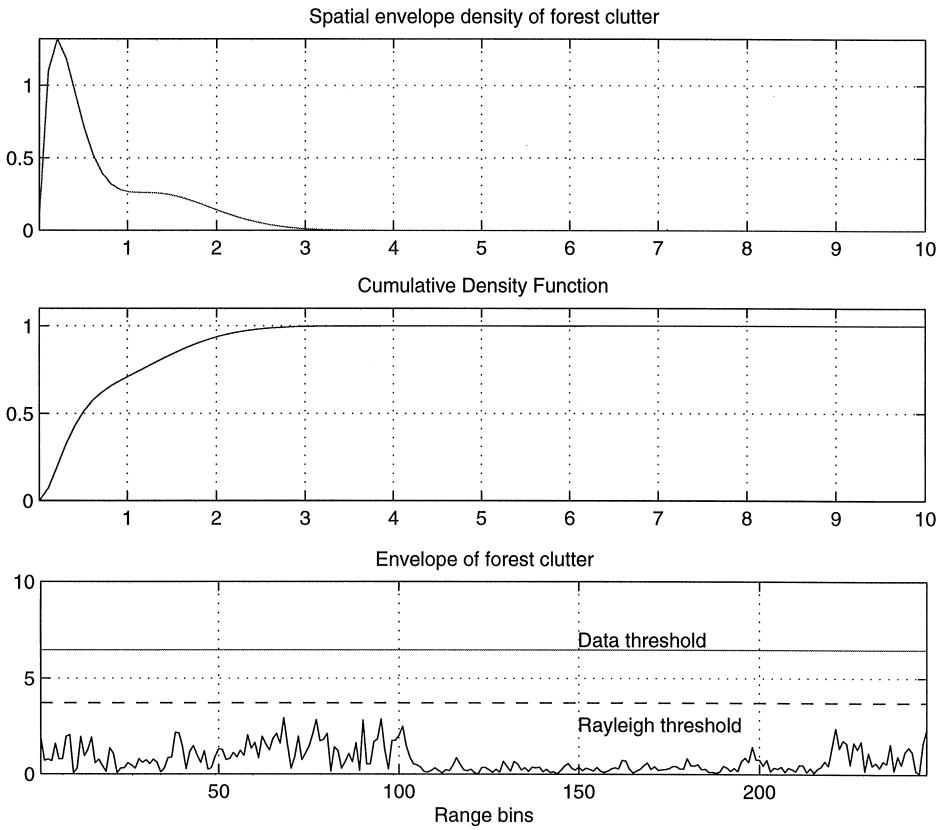

Fig. 6. Results of the approximation density obtained on the envelope of experimental data (forest clutter). The detection threshold θ for experimental data is calculated according to equation (39) with $P_{\mathrm{fa}} = 10^{-6}$ and it has to be compared with the one computed for the Rayleigh hypothesis case.

classical way:

$$\hat{\mu}_n = \frac{1}{N}\sum_{i=1}^{N} |y_i|^n. \tag{50}$$

Asymptotically, it can be shown that $\hat{\mu}_n$ tends to the real values $\mathbb{E}(|y|^n)$ of the moments as $N \to \infty$ (where $\mathbb{E}(.)$ denotes the mathematical expectation). Larger will be the number of data and better will be this estimation.

The normalised MGF of experimental data takes the form

$$\begin{aligned}\Phi(u) = {} & 1 - 0.7652u + 0.5u^2 - 0.2877u^3 \\ & + 0.1431u^4 - 0.06229u^5 + 0.02408u^6 \\ & - 0.008382u^7 + 0.00265u^8 - 0.0007672u^9 \\ & + 0.0002043u^{10} - 0.0000503u^{11}\end{aligned}$$

and the [5/6] Padé approximation becomes

$$P^{[5/6]}(u) = \sum_{k=1}^{6} \frac{\lambda_k}{u - \alpha_k} \tag{51}$$

with

$$\begin{aligned}\{\lambda_k\}_{k\in[1,6]} = {} & \{8.4237 \pm 10.244\mathrm{i}, \ldots \\ & - 0.035157 \pm 0.038976\mathrm{i}, \ldots \\ & - 8.953 \pm 48.34\mathrm{i}\}\end{aligned} \tag{52}$$

$$\begin{aligned}\{\alpha_k\}_{k\in[1,6]} = {} & \{ - 2.823 \pm 1.9382\mathrm{i}, \ldots \\ & - 1.2425 \pm 3.153\mathrm{i}, \ldots \\ & - 3.2174 \pm 0.63591\mathrm{i}\}.\end{aligned} \tag{53}$$

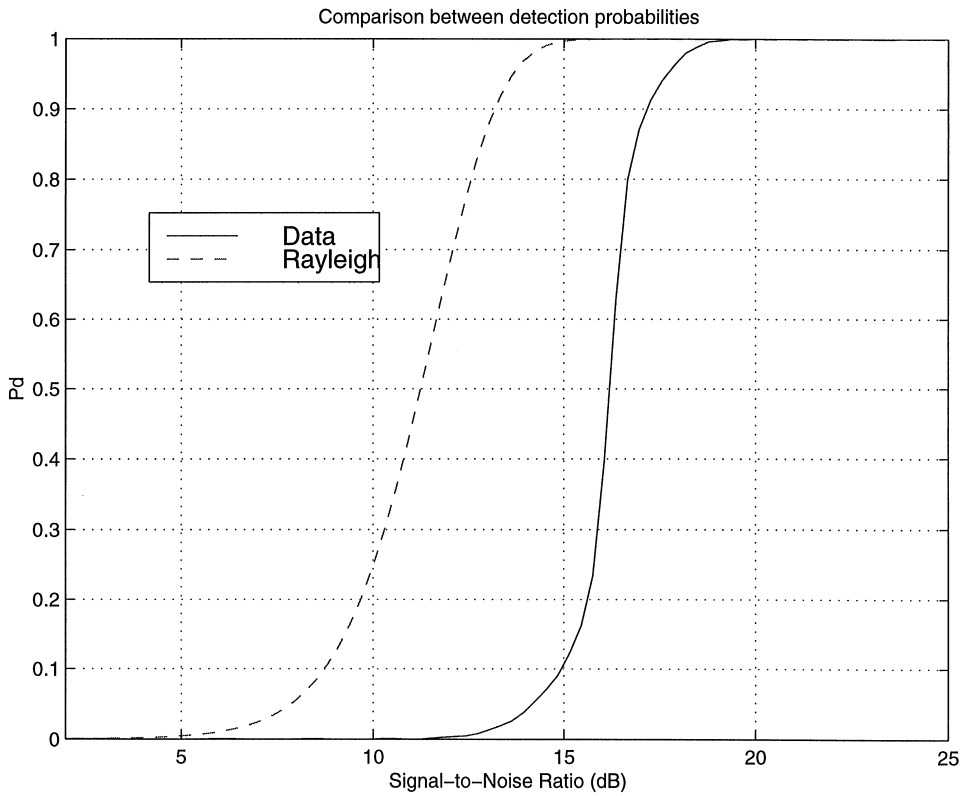

Fig. 7. Comparison between detection performances in experimental data (forest clutter) and detection performances in classical Rayleigh noise. The two noise signals have the same power and the results are shown for a probability of false alarm rate fixed to $P_{fa} = 10^{-6}$.

The last graphic on Fig. 6 shows the mismatch between the detection threshold values derived, the real one, $\theta_1 = 6.47$, by the resolution of (39), and the other, $\theta_2 = 3.71$, with the classical Gaussian hypothesis.

6. Conclusion

This paper has recalled a general modelisation method with few parameters of the true PDF of a complex process from a power series expansion of its MGF. This recalls the AR and ARMA spectral density modelisation methods and allows to derive simple and general relations of the pair (P_d, P_{fa}). This method needs the knowledge to a high degree of accuracy of the n-order moments of the envelope of the unwanted noise often unavailable. We do not solve this problem in this paper either assuming they are known or simulating a large number of variates in order to asymptotically tend to the real values. A study of the influence of an error on the moments estimation is currently investigated in further works.

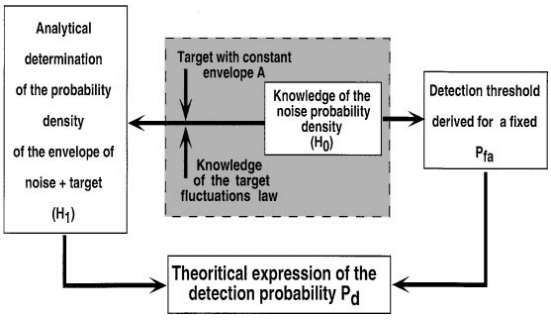

Fig. 8. Classical scheme to analyse the detection performances.

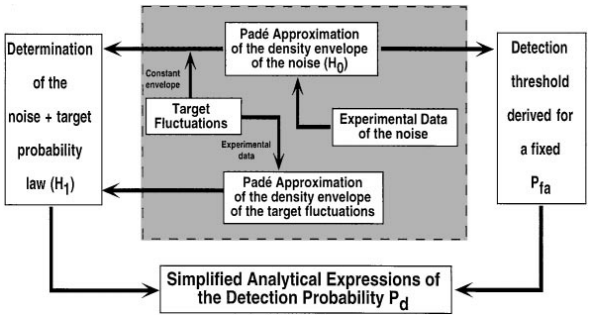

Fig. 9. Applying Padé approximation.

However, as we show in this work, Padé approximation method gives powerfull expressions for a computational aid and allows to evaluate successfully, for instance, the detection performance analysis without a priori knowledge on the noise statistic.

As a conclusion to this paper and to show the difference between the classical detection way and the use of the Padé approximation we present two schemes (Fig. 8 for the classical way; Fig. 9 for the use of Padé approximation) resuming the two different approaches.

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Annexe F

PEOD : Padé Estimated Optimum (Radar) Detector

PEOD : Padé Estimated Optimum (radar) Detector

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Abstract— **In this paper, an expression of the optimum non-Gaussian radar detector is derived from the non-Gaussian SIRP model (Spherically Invariant Random Process) clutter and a Padé approximation of the** *characteristic function* **of the SIRP. The SIRP model is used to perform coherent detection and to modelize the non-Gaussian clutter as a complex Gaussian process whose variance is itself a positive random variable (r.v.). The probability density function (PDF) of the variance characterizes the statistics of the SIRP and after performing a Padé approximation of this PDF from reference clutter cells we derive the so-called Padé Estimated Optimum (Radar) Detector (PEOD) without any knowledge about the statistics of the clutter. We evaluate PEOD performance for an unknown target signal embedded in K-distributed clutter and compare with optimum detectors performance (optimum in particular clutter statistics such as Optimum K Detector - OKD - in K-distributed clutter).**

I. Introduction

Coherent radar detection against non-Gaussian clutter has gained many interests in the radar community since experimental clutter measurements made by organizations like MIT [1] have shown to fit non-Gaussian statistical models. One of the most tractable and elegant non-Gaussian model results in the so-called *Spherically Invariant Random Process* (SIRP) theory which states that non-Gaussian random vector is the product between Gaussian random vector with a non-negative random variable (r.v.) (the variance of the Gaussian process is itself a r.v.). This model allows to derive non-Gaussian joint probability density function (PDF) and then optimum radar detector strategies. For example in [2], the optimum radar detector is derived in the presence of composite disturbance of known statistics modeled as SIRP.

The goal of this work consists in estimating the variance PDF of the noise with a Padé approximation. This allows to derive in a closed form the joint PDF of the SIRV and to perform, from the likelihood ratio test (LRT), the Padé Estimated Optimum Detector (PEOD) for non-fluctuating and unknown target signal. Padé approximation is performed after *a posteriori* variance resampling which means to deal directly with the received clutter data. It is no more necessary to have any knowledge about the clutter statistics.

II. General relations of detection theory

A. Likelihood ratio test

We consider here the basic problem of detecting the presence (H_1) or absence (H_0) of a complex signal $\mathbf{s}$ in a set of N measurements of m-complex vectors $\mathbf{y} = \mathbf{y}_{\mathrm{I}} + \mathrm{j}\,\mathbf{y}_{\mathrm{Q}}$ corrupted by a sum $\mathbf{c}$ of independent additive complex noises (noises + clutter). The problem can be described in terms of a statistical hypothesis test :

$$H_0 : \mathbf{y} = \mathbf{c} \quad (1)$$
$$H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c} \quad (2)$$

When present, the target signal $\mathbf{s}$ corresponds to a modified version of the perfectly known transmitted signal $\mathbf{t}$ and can be rewritten as $\mathbf{s} = A\,T(\underline{\theta})\,\mathbf{t}$. A is the target amplitude and we suppose determined all the other parameters $\underline{\theta}$ which characterize the target after the transformation T (Doppler frequency, time delay, ...). In the following, $\mathbf{p} = T(\underline{\theta})\,\mathbf{t}$. The observed vector $\mathbf{y}$ is used to form the LRT $\Lambda(\mathbf{y})$ which is compared to a threshold η in order to reach a desired false alarm probability (P_{fa}) value :

$$\Lambda(\mathbf{y}) = \frac{p_{\mathbf{y}}(\mathbf{y}/H_1)}{p_{\mathbf{y}}(\mathbf{y}/H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (3)$$

LRT performance follow from the statistics of the data. P_{fa} is the probability of choosing H_1 when the target is absent, and the detection probability (P_d) is the probability of choosing H_1 when the target is present, that is :

$$P_{fa} = \mathbb{P}(\Lambda(\mathbf{y}) \underset{H_0}{>} \eta) \quad \text{and} \quad P_d = \mathbb{P}(\Lambda(\mathbf{y}) \overset{H_1}{>} \eta), \quad (4)$$

where $\mathbb{P}(X > a)$ is the probability of X being greater than a under the PDF of the r.v. X.

B. Gaussian clutter case

When the clutter $\mathbf{c}$ is supposed to be complex Gaussian-distributed ($\mathcal{CN}(\mathbf{0}, 2\sigma^2\mathbf{M})$), the so-called Optimum Gaussian Detector (OGD) gives from (3) and for a known signal $\mathbf{s}$:

$$\Re(\mathbf{y}^\dagger\,\mathbf{M}^{-1}\mathbf{s}) \underset{H_0}{\overset{H_1}{\gtrless}} \sigma^2\,\lambda + \frac{\mathbf{s}^\dagger\,\mathbf{M}^{-1}\mathbf{s}}{2}. \quad (5)$$

$\lambda = \log(\eta)$, $\Re(z)$ denotes the real part of z and † is the transpose conjugate operator.

When the target signal $\mathbf{s}$ is unknown and non-fluctuating, the generalized likelihood ratio test (GLRT) yields to :

$$|\mathbf{p}^\dagger\,\mathbf{M}^{-1}\,\mathbf{y}|^2 \underset{H_0}{\overset{H_1}{\gtrless}} 2\sigma^2\lambda\,\mathbf{p}^\dagger\mathbf{M}^{-1}\mathbf{p}. \tag{6}$$

Statistic tests under H_0 and H_1 are respectively exponential and Rice-Nagakami-distributed and corresponding P_{fa} and P_d expressions are given in [3]. For fluctuating target, P_d expression has to be integrated over the fluctuation PDF, generally Swerling-K fluctuations (K is an integer), and expressions are given in [3].

C. Non-Gaussian clutter case - SIRV model

In the case of non-Gaussian clutter, detection strategies can be derived if we consider a particular clutter nature, i.e. if an *a priori* hypothesis is made on the clutter statistic.
Non-Gaussian clutter and general radar detector expressions come from the SIRP representation ([4], [5], [6]).
SIRV (Vector) model interpretes each element of the clutter vector $\mathbf{c}$ as the product of a m-complex Gaussian vector $\mathbf{x}$ ($\mathcal{CN}(\mathbf{0}, \in \mathbf{M})$) with a positive r.v. τ, that is $\mathbf{c} = \mathbf{x}\sqrt{\tau}$.

The PDF of the variable τ is called the *characteristic function* of the SIRV and the so formed vector $\mathbf{c}$ is, conditionnally to τ, a complex Gaussian random process ($\mathcal{CN}(\mathbf{0}, \in \tau\,\mathbf{M})$) with joint PDF $p(\mathbf{c}/\tau)$. The joint PDF of the SIRV gives :

$$p(\mathbf{c}) = \int_0^{+\infty} \frac{\tau^{-m}}{(2\pi)^m|\mathbf{M}|}\exp\left(-\frac{\mathbf{c}^\dagger\mathbf{M}^{-1}\mathbf{c}}{2\tau}\right)p(\tau)d\tau. \tag{7}$$

From (7), $p_{\mathbf{y}}(\mathbf{y}/H_0) = p_{\mathbf{c}}(\mathbf{y})$ and $p_{\mathbf{y}}(\mathbf{y}/H_1) = p_{\mathbf{y}}(\mathbf{y} - \mathbf{s}/H_0)$ for a known target signal $\mathbf{s}$. The LRT becomes in this case :

$$\int_0^{+\infty}\left[\exp\left(-\frac{q_1(\mathbf{y})}{2\tau}\right) - \exp\left(\lambda - \frac{q_0(\mathbf{y})}{2\tau}\right)\right]\frac{p(\tau)}{\tau^m}\,d\tau \underset{H_0}{\overset{H_1}{\gtrless}} 0, \tag{8}$$

where $q_0(\mathbf{y}) = \mathbf{y}^\dagger\mathbf{M}^{-1}\mathbf{y}$, $q_1(\mathbf{y}) = q_0(\mathbf{y}-\mathbf{s})$ for a known signal $\mathbf{s}$ and $\lambda = \ln(\eta)$.

When the target signal $\mathbf{s}$ is unknown, the detection strategy is given by (8) where now :

$$q_1(\mathbf{y}) = \mathbf{y}^\dagger\mathbf{M}^{-1}\mathbf{y} - \frac{|\mathbf{p}^\dagger\mathbf{M}^{-1}\mathbf{y}|^2}{\mathbf{p}^\dagger\mathbf{M}^{-1}\mathbf{p}}. \tag{9}$$

D. Optimum K Detector (OKD)

In the case of K-distributed clutter (size m) with parameters ν and b, the r.v. τ is Gamma-distributed with parameters ν and $\beta = 2/b^2$. Following the same processes with (8), the expression of the so-called Optimum K-distributed Detector (OKD) becomes $\forall m \geq 2$:

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{\frac{\nu-m}{2}} \cdot \frac{K_{\nu-m}\left(b\sqrt{q_1(\mathbf{y})}\right)}{K_{\nu-m}\left(b\sqrt{q_0(\mathbf{y})}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{10}$$

where $q_0(\mathbf{y}) = \mathbf{y}^\dagger\,\mathbf{M}^{-1}\mathbf{y}$ and $q_1(\mathbf{y})$ is given by (9) for unknown signal $\mathbf{s}$.
For $m = 1$ the expression is given in [3].

Padé PDF approximation of $p(\tau)$ provides an expression in terms of a sum of weighted complex decaying exponential which allows to perform integration over $p(\tau)$ in (8) to give the so-called Padé Estimated Optimum (radar) Detector (PEOD). Padé approximation method is presented in details in [7], [8]. In the next section we just recall the resulting expressions of PDF and CDF (cumulative density function) approximation.

III. Padé Estimated Optimum (radar) Detector (PEOD)

Padé approximation method provides the following PDF and cumulative density function (CDF) expressions :

$$\tilde{p}(z) = \sum_{k=1}^{M}\lambda_k\,e^{-\alpha_k\,z} \quad \text{and} \quad \tilde{F}(z) = 1 - \sum_{k=1}^{M}\frac{\lambda_k}{\alpha_k}\,e^{-\alpha_k\,z}. \tag{11}$$

Padé coefficients $\{\alpha_k\}_{k=1}^M$ and $\{\lambda_k\}_{k=1}^M$ are in pairs conjugate when complex or real and all $\{\alpha_k\}$ real parts are positive (to assure the PDF convergence towards zero when z tends to $+\infty$).

When $p(\tau)$ is replaced by $\tilde{p}(\tau)$ in (8), integration becomes henceforth tractable [9] and yields to the PEOD expression :

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{\frac{1-m}{2}} \cdot \frac{\sum_{k=1}^{M}\lambda_k(\alpha_k)^{\frac{m-1}{2}}K_{1-m}\left(\sqrt{B_k^1(\mathbf{y})}\right)}{\sum_{k=1}^{M}\lambda_k(\alpha_k)^{\frac{m-1}{2}}K_{1-m}\left(\sqrt{B_k^0(\mathbf{y})}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{12}$$

where $q_0(\mathbf{y}) = \mathbf{y}^\dagger\,\mathbf{M}^{-1}\,\mathbf{y}$, $q_1(\mathbf{y}) = q_0(\mathbf{y}-\mathbf{s})$ for $\mathbf{s}$ known, $q_1(\mathbf{y})$ is given by (9) for $\mathbf{s}$ unknown, $B_k^j(\mathbf{y}) = 2\,\alpha_k q_j(\mathbf{y})$, $j = 0, 1$.

Based on moment generating function (MGF) approximation, Padé approximation method requires knowledge of the moments of the r.v. τ up to the order $L + M + 1$ or estimation of the moments from samples of $p(\tau)$ (the integer L is less than M (generally $L = M - 1$) and needed to perform Padé approximation. See [7], [8] for more details). But neither true moments nor samples from $p(\tau)$ are available.

In the next section we propose to regenerate variance samples according to the *a posteriori* PDF (APDF) of the variance

(from reference clutter cells and with a non-informative variance prior) and to perform Padé approximation with the estimated moments of the resamples.

We have made comparisons between the two approaches via simulations and it can be shown that errors due to the moments estimation does not involve significant loss of performance in PEOD.

A. Variance resampling

From N_r reference clutter cells of size m, $\mathbf{R} = [\mathbf{r}_1, \cdots, \mathbf{r}_{N_r}]^T$ where $\mathbf{r}_i = [r_i(1), \cdots, r_i(m)]^T$ and with the Bayes'rule, we regenerate N_r variance samples according to the variance APDF.
The Bayes'rule provides us directly the APDF :

$$p(\tau/\mathbf{r}_i) = \frac{p(\mathbf{r}_i/\tau)g(\tau)}{p(\mathbf{r}_i)}. \tag{13}$$

$p(\mathbf{r}_i)$ is the normalization constant calculated in integrating the numerator of (13) over τ :

$$p(\mathbf{r}_i) = \int_0^{+\infty} p(\mathbf{r}/\tau_i)g(\tau)d\tau. \tag{14}$$

$g(\tau)$ is the prior density of the variance for the reference clutter cells. As we do not have any knowledge (except the variance positivity) about the variance density, we choose a non-informative prior, called Jeffreys' prior, which is proportional to the square root of Fisher's information measure.
So, we have :

$$g(\tau) = \frac{1}{\tau}. \tag{15}$$

After calculation, the expression of the variance APDF becomes an inverse gamma PDF ($\mathcal{IG}$ PDF) and the N_r variance are resampled according to :

$$\tau_{i=1}^{N_r} \sim \mathcal{IG}\left(m, \frac{2}{\mathbf{r}_i^\dagger \mathbf{M}^{-1} \mathbf{r}_i}\right) \tag{16}$$

where $\mathcal{IG}(.)$ is the inverse Gamma distribution (i.e. the PDF of the inverse of a gamma r.v.).

We can also choose a *conjugate* prior density in our case because of the form of the likelihood of the data.
A prior density is called *conjugate* if the resulting APDF own to the same PDF family than the prior density. So, we could choose an inverse gamma PDF for the prior density with parameters a_p and b_p (the subscript p is for prior) :

$$g(\tau) = \frac{1}{b_p^{a_p}\Gamma(a_p)}\tau^{-a_p-1}e^{-\frac{1}{\tau b_p}}, \tag{17}$$

and the N_r variance are so resampled according to :

$$\tau_{i=1}^{N_r} \sim \mathcal{IG}\left(m + a_p, \frac{2b_p}{2 + b_p\mathbf{r}_i^\dagger \mathbf{M}^{-1} \mathbf{r}_i}\right). \tag{18}$$

We have to choose a_p and b_p values to keep the prior density as non-restrictive as possible.
On figures (1) and (2) different $\mathcal{IG}$ PDF are plotted for different values of the parameters a_p and b_p.
If $a_p \longrightarrow 0$ and $b_p \longrightarrow +\infty$ then $\mathcal{IG}(\tau; a_p, b_p) \propto a_p/\tau$. That is, the conjugate prior density tends to a non-informative density.
This is shown on figure (3) and we choose $a_p = 10^{-3}$ and $b_p = 3.65$.

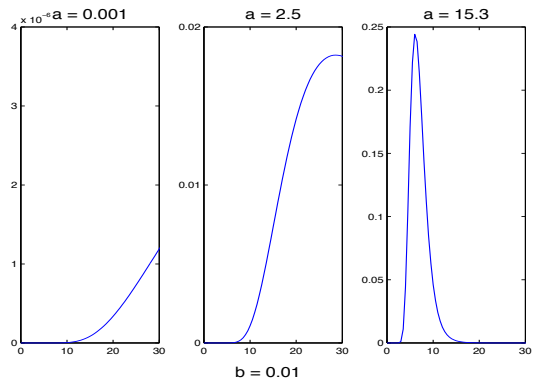

Fig. 1. Inverse gamma(x;a_p,b_p) PDF for $b_p = 0.01$ and $a_p = 0.001, 2.5, 15.3$.

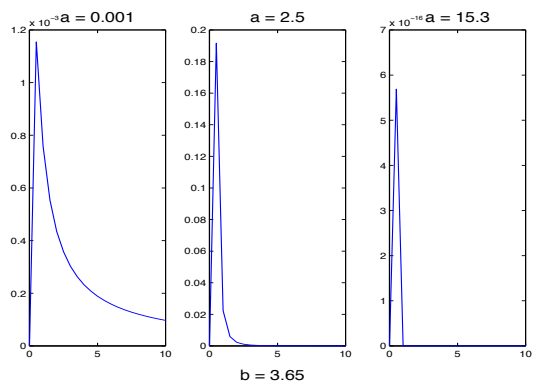

Fig. 2. Inverse gamma(x;a_p,b_p) PDF for $b_p = 3.65$ and $a_p = 0.001, 2.5, 15.3$.

All the $L + M + 1$ moments are then computed empirically from which M Padé coefficients $\{\alpha\}_k$ and $\{\lambda\}_k$ are derived in order to perform PEOD.
Variance resampling allows to deal directly with the data without statistical assumption on the clutter and PEOD performances are shown below. PEOD expression stands even if the

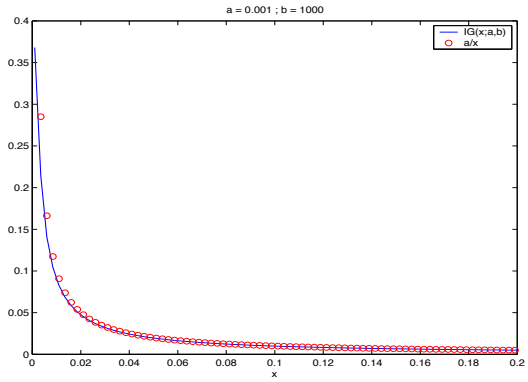

Fig. 3. Comparison between IG(x;a_p,b_p) and a/x for small a_p value ($a_p = 10^{-3}$) and high b_p value ($b_p = 10^3$).

characteristic function $p(\tau)$ of the SIRV is theoritically unknown like for Weibull clutter.
This is shown on **figure (6)**.

IV. Simulations

We compare the performances of the PEOD with those of OGD and OKD.
K-distributed clutter is generated with different values of the form parameter $\nu = 0.5,\ 20$. Smaller is the value of ν and spikier is the clutter. Inversely, when ν is high, K-PDF tends to a Gaussian distribution and this fact is confirmed through OGD and OKD performances on **figure (5)**.
Moreover, from PEOD expression, we can deduce OGD and OKD expression for particular variance PDF.
On **figure (6)**), we compare PEOD, OGD and OKD performances (OKD with different values of ν) for an unknown target signal embedded in Weibull clutter.

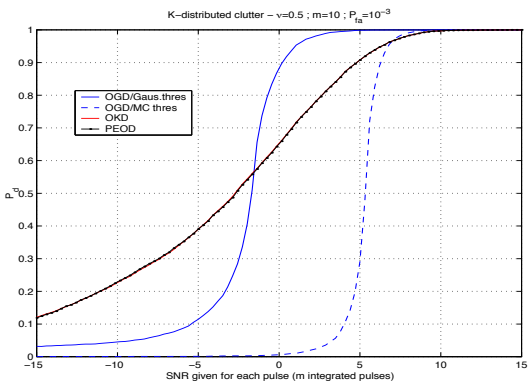

Fig. 4. Performances comparison between the OGD, OKD and PEOD for K-distributed clutter ($\nu = 0.5, P_{fa} = 10^{-3}, m = 10$ integrated pulses)

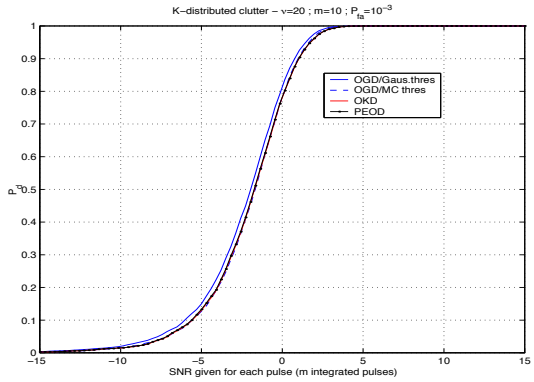

Fig. 5. Performances comparison between the OGD, OKD and PEOD for K-distributed clutter ($\nu = 20, P_{fa} = 10^{-3}, m = 10$ integrated pulses)

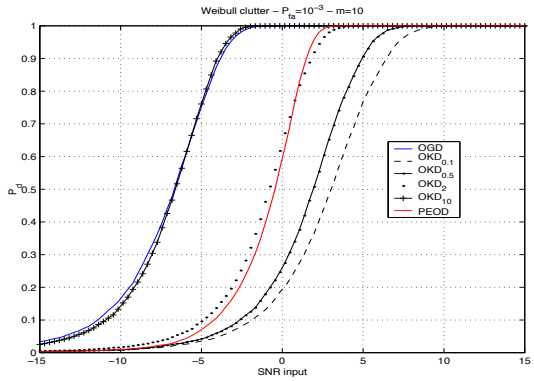

Fig. 6. Performances comparison between the OGD, OKD and PEOD for an unknown target signal in Weibull-distributed clutter ($a = 0.2$, $b = 2$; $P_{fa} = 10^{-3}$, $m = 10$ integrated pulses and $\nu = 0.1, 0.5, 2, 10$ for OKD). SNR given on x-axis is for one pulse : 0 dB before coherent post-integration corresponds to $10 \log 10(m)$ dB after the coherent pulse integration.

V. Conclusions

The present paper has addressed the contribution of Padé approximation method to the problem of coherent radar detection of a target embedded in a clutter with unknown statistics. The simple expression of PEOD (Padé Estimated Optimum Detector) allows to build the optimum radar detector and to evaluate its detection performances without having the knowledge of the clutter statistic.

VI. Outlooks : improving PEOD

At that time we are now able to give an improvement to PEOD, that is to say, to avoid the Padé approximation step of the variance PDF in estimating this PDF thanks to a bayesian estimator from reference clutter cells and the same variance prior density.

The resulting expression is called BORD (for Bayesian Optimum Radar Detector) and will be found in further papers.

BORD is "self-adaptative" to the clutter statistics because of its only dependence of the data (references and observed data). BORD performances reach OKD performances for K-distributed clutter and can apply for any clutter statistics (see on figures (7) and (8)). The only hypothesis made at the beginning of the study is the clutter SIRP modelization.

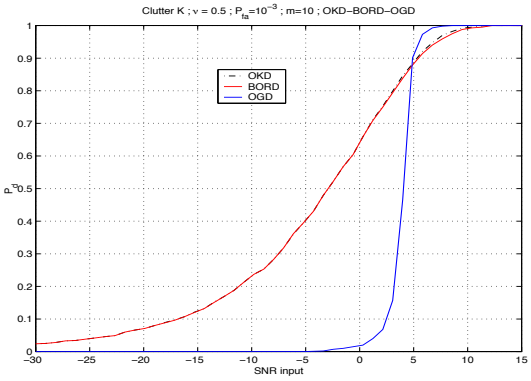

Fig. 7. Performances comparison between the OGD, OKD and BORD for K-distributed clutter ($\nu = 0.5$, $P_{fa} = 10^{-3}$, $m = 10$ integrated pulses).

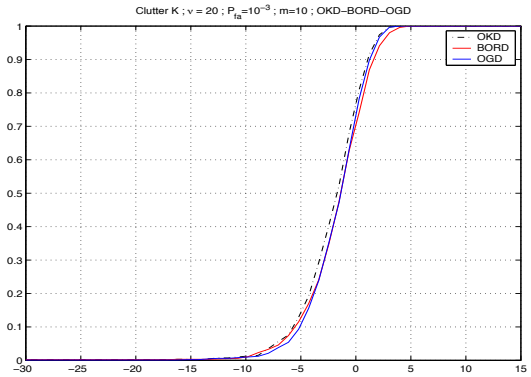

Fig. 8. Performances comparison between the OGD, OKD and BORD for K-distributed clutter ($\nu = 20$, $P_{fa} = 10^{-3}$, $m = 10$ integrated pulses).

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Annexe G

BORD : Bayesian Optimum Radar Detector

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BORD: bayesian optimum radar detector

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Abstract

We derive the expression of an optimum non-Gaussian radar detector from the non-Gaussian spherically invariant random process (SIRP) clutter model and a bayesian estimator of the SIRP *characteristic density*. SIRP modelizes non-Gaussian process as a complex Gaussian process whose variance, the so-called *texture*, is itself a positive random variable (r.v.). After performing a bayesian estimation of the *texture* probability density function (PDF) from reference clutter cells we derive the so-called bayesian optimum radar detector (BORD) without any knowledge about the clutter statistics. We also derive the asymptotic expression of BORD (in law convergence), the so-called asymptotic BORD, as well as its theoretical performance (analytical threshold expression). BORD performance curves are shown for an unknown target signal embedded in correlated K-distributed and are compared with those of the optimum K-distributed detector. These results show that BORD reach optimal detector performances.
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Keywords: SIRP Model; Radar detection; Bayesian estimation

1. Introduction

Coherent radar detection against non-Gaussian clutter has gained many interests in the radar community since experimental clutter measurements made by organizations like MIT [17,18,4] have shown to fit non-Gaussian statistical models. One of the most tractable and elegant non-Gaussian models results in the so-called *spherically invariant random process* (SIRP) which states that many non-Gaussian random processes are the product of a Gaussian random process (called the *speckle*) with a non-negative random variable (r.v.) (the so-called *texture*), so that an SIRP is a compound Gaussian process. This model is the base of many results like Gini et al.'s works [8] in which the optimum detector in the presence of composite disturbance of known statistics modeled as SIRP is derived.

In this paper, a bayesian approach is proposed to determine the probability density function (PDF) of the *texture* from N_{ref} reference clutter cells. We use Bayes' rule and a Monte Carlo estimation given a *non-informative* prior on the *texture*. This approach exploits the SIRP model particularity to describe non-Gaussian processes as compound processes and allows one to derive the expression of the optimum detector called bayesian optimum radar detector

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(BORD). Henceforth, it is no more necessary to have any knowledge about the clutter statistics and BORD deals directly with the received data. In Sections 2 and 3 we briefly recall the formulation of a detection problem and describe how the SIRP model clutter yields to general and particular optimum detector. In Section 4, we explain the bayesian approach used to determine a bayesian estimator to the *texture* PDF and give the expression of the resulting BORD. In this section the asymptotic expression of BORD (the so-called asymptotic BORD) and the analytical performance of the Asymptotic BORD are also derived. Section 5 is devoted to the simulations description to evaluate BORD performance (compared with optimum detectors performance). Conclusions and outlooks are given in Section 6.

2. General relations of detection theory

We consider here the basic problem of detecting the presence (H_1) or absence (H_0) of a complex signal $\mathbf{s}$ in a set of N_{ref} measurements of m-complex vectors $\mathbf{y} = \mathbf{y}_{\mathrm{I}} + \mathrm{j}\mathbf{y}_{\mathrm{Q}}$ corrupted by a sum $\mathbf{c}$ of independent additive complex noises (noises+clutter). The problem can be described in terms of a statistical hypothesis test:

$$\mathrm{H}_0 : \mathbf{y} = \mathbf{c}, \tag{1}$$

$$\mathrm{H}_1 : \mathbf{y} = \mathbf{s} + \mathbf{c}. \tag{2}$$

When present, the target signal $\mathbf{s}$ corresponds to a modified version of the perfectly known transmitted signal $\mathbf{t}$ and can be rewritten as $\mathbf{s} = AT(\underline{\theta})\mathbf{t}$. A is the target complex amplitude and we suppose to have determined all the others parameters ($\underline{\theta}$) which characterize the target (Doppler frequency, time delay, etc). In the following, $\mathbf{p} = T(\underline{\theta})\mathbf{t}$.

The observed vector $\mathbf{y}$ is used to form the likelihood ratio test (LRT) $\Lambda(\mathbf{y})$ which is compared with a threshold η set to a desired false alarm probability (P_{fa}) value:

$$\Lambda(\mathbf{y}) = \frac{p_{\mathbf{y}}(\mathbf{y}/\mathrm{H}_1)}{p_{\mathbf{y}}(\mathbf{y}/\mathrm{H}_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta. \tag{3}$$

LRT performances follow from the statistics of the data: P_{fa} is the probability of choosing H_1 when the target is absent, and the detection probability (P_{d}) is the probability of choosing H_1 when the target is present, that is,

$$P_{\mathrm{fa}} = \mathbb{P}(\Lambda(\mathbf{y}) > \eta | \mathrm{H}_0)$$

and

$$P_{\mathrm{d}} = \mathbb{P}(\Lambda(\mathbf{y}) > \eta | \mathrm{H}_1).$$

3. Non-Gaussian clutter case: SIRV and optimum radar detector

In the case of non-Gaussian clutter, detection strategies can be derived if we consider a particular clutter nature, i.e. if an a priori hypothesis is made on the clutter statistics. To model non-Gaussian clutter and derive general detector expressions, we use the SIRP representation [1,8,13,19].

3.1. Description and general expressions

A spherically invariant random vector (SIRV) is a vector issued from a SIRP which modelizes each element of the clutter vector $\mathbf{c}$ as the product of a m-complex Gaussian vector $\mathbf{x}$ (the *speckle*) ($\mathscr{CN}(\mathbf{0}, 2\mathbf{M})$) with a positive r.v. τ (the *texture*), that is $\mathbf{c} = \mathbf{x}\sqrt{\tau}$. The so-formed vector $\mathbf{c}$ is, conditionally to τ, a complex Gaussian random vector ($\mathscr{CN}(\mathbf{0}, 2\tau\mathbf{M})$) with multivariate PDF $p(\mathbf{c}/\tau)$. The PDF of the clutter is then

$$p(\mathbf{c}) = \int_0^{+\infty} \frac{\tau^{-m}}{(2\pi)^m |\mathbf{M}|} \exp\left(-\frac{\mathbf{c}^\dagger \mathbf{M}^{-1}\mathbf{c}}{2\tau}\right) p(\tau)\,\mathrm{d}\tau, \tag{4}$$

where $\dagger$ is the transpose conjugate operator, and $|\mathbf{M}|$ is the determinant of the matrix $\mathbf{M}$. This general expression leads, for a known $p(\tau)$, to multivariate PDFs of non-Gaussian random vectors. For example, joint K-distributed PDF is obtained if $p(\tau)$ is a Gamma PDF (see further).

3.2. Optimum SIRV detector

Applied to the detection problem, expression (4) gives $p_{\mathbf{C}}(\mathbf{y}/\mathrm{H}_0)$ and $p_{\mathbf{C}}(\mathbf{y}/\mathrm{H}_1) = p_{\mathbf{C}}(\mathbf{y} - \mathbf{s}/\mathrm{H}_0)$ when

the target signal $\mathbf{s}$ is known. The LRT becomes

$$\frac{\int_0^{+\infty} \tau^{-m} \exp(-q_1(\mathbf{y})/2\tau)\,\mathrm{d}\tau}{\int_0^{+\infty} \tau^{-m} \exp(-q_0(\mathbf{y})/2\tau)\,\mathrm{d}\tau} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \lambda, \tag{5}$$

where

$$q_0(\mathbf{y}) = \mathbf{y}^\dagger \mathbf{M}^{-1}\mathbf{y}, \tag{6}$$

$q_1(\mathbf{y}) = q_0(\mathbf{y}-\mathbf{s})$ for a known signal $\mathbf{s}$ and $\lambda = \ln(\eta)$. When the target complex amplitude A is unknown, ML estimation of A is performed [8]:

$$\hat{A}_{\mathrm{ML}} = \frac{\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{y}}{\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{p}}, \tag{7}$$

and the detection strategy is given by (5) where now [8]

$$q_1(\mathbf{y}) = \mathbf{y}^\dagger \mathbf{M}^{-1}\mathbf{y} - \frac{|\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{y}|^2}{\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{p}}. \tag{8}$$

With (8), expression (5) is called generalized LRT (GLRT).

3.3. Examples: various optimum SIRV detectors

All the following optimum SIRV detectors are given for an unknown amplitude target whose value is estimated is the ML sense. So, the quadratic forms $q_0(\mathbf{y})$ and $q_1(\mathbf{y})$ are, respectively, given by (6) and (8).

3.3.1. OKD: optimum K detector

In the case of K-distributed clutter (size m) with parameters v and b, *texture* PDF is a Gamma$(v, 2/b^2)$ PDF with expression

$$p(\tau) = \frac{\tau^{v-1} b^{2v}}{\Gamma(v) 2^v} \exp\left(-\frac{\tau b^2}{2}\right). \tag{9}$$

The PDF of $\mathbf{y}$ under H_0 hypothesis is then given by

$$p_{\mathbf{y}}(\mathbf{y}/\mathrm{H}_0) = \frac{2b^{v+m} q_0(\mathbf{y})^{(v-m)/2}}{\pi^m |\mathbf{M}| \Gamma(v) 2^{v+m}} K_{v-m}\left(b\sqrt{q_0(\mathbf{y})}\right), \tag{10}$$

where $K_v(.)$ is the modified Bessel function of order v and $\Gamma(.)$ is the Gamma function. The value of v determines the spikiness of the distribution. Following the same processes with (5) expression of the so-called optimum K-distributed detector (OKD) becomes $\forall m \geqslant 2$

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{(v-m)/2} \frac{K_{v-m}(b\sqrt{q_1(\mathbf{y})})}{K_{v-m}(b\sqrt{q_0(\mathbf{y})})} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta. \tag{11}$$

For $m = 1$, $q_0(\mathbf{y}) = |y|^2$ (where y is the scalar vector we observe) and expression (11) becomes

$$|y|^{v-1} K_{v-1}(b|y|) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \frac{2^{v-1}\Gamma(v)}{2b^{v-1} v\eta}. \tag{12}$$

3.3.2. OLD: optimum Laplace detector

This detector can be considered as a particular case of OKD since the K-distribution becomes a Laplace PDF when $v = 1$. The *texture* PDF is then an exponential PDF with parameter $b^2/2$, with expression

$$p(\tau) = \frac{b^2}{2} \exp\left(-\frac{b^2\tau}{2}\right), \tag{13}$$

and the resulting expression of the OLD becomes $\forall m \geqslant 2$

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{(1-m)/2} \frac{K_{1-m}(b\sqrt{q_1(\mathbf{y})})}{K_{1-m}(b\sqrt{q_0(\mathbf{y})})} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta. \tag{14}$$

For $m = 1$, $q_0(\mathbf{y}) = |y|^2$ and we just have

$$K_0(b|y|) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \frac{1}{2\eta}. \tag{15}$$

3.3.3. OStD: optimum Student-t detector

Student-$t(v, b)$ marginal PDF is obtained when the *texture* PDF is an inverse gamma PDF $\mathcal{IG}(v, 2/b^2)$ whose expression is

$$p(\tau) = \frac{b^{2v}\tau^{-v-1}}{2^v \Gamma(v)} \exp\left(-\frac{b^2}{2\tau}\right). \tag{16}$$

The resulting expression of the OStD is then

$$\left(\frac{b^2 + q_0(\mathbf{y})}{b^2 + q_1(\mathbf{y})}\right)^{m+v} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta. \tag{17}$$

3.3.4. OχD: optimum χ detector

As for the other examples given from now, the *texture* PDF of a χ SIRV (the marginal PDF is a χ PDF with v degrees of freedom and parameter b, σ^2 is

the variance) is analytically unknown. It is, however, possible [13] to derive the optimum χ detector for $v \leqslant 1$, which is

$$\frac{\sum_{k=1}^{m}(-1)^{k-1}C_{m-1}^{k-1}(b^2\sigma^2)^{k-1}D_k(q_1(\mathbf{y}),v,b\sigma)}{\sum_{k=1}^{m}(-1)^{k-1}C_{m-1}^{k-1}(b^2\sigma^2)^{k-1}D_k(q_0(\mathbf{y}),v,b,\sigma)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta, \tag{18}$$

where for $j = 0, 1$,

$$D_k(q_j(\mathbf{y}),v,b,\sigma) = \prod_{i=1}^{m-1}(v-i)(q_j(\mathbf{y}))^{v-k}\mathrm{e}^{-b^2\sigma^2 q_j(\mathbf{y})}$$

and

$$C_{m-1}^{k-1} = \frac{(m-1)!}{(k-1)!(m-k)!}.$$

3.3.5. OgRD: optimum generalized Rayleigh detector

As said previously, the *texture* PDF of a generalized Rayleigh SIRV (the marginal PDF is a generalized Rayleigh PDF with parameters η and β, σ^2 is the variance) is analytically unknown but the optimum generalized Rayleigh detector is given for $\eta \leqslant 2$ by

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{1-m} \exp\left(\frac{\sigma^\eta}{\beta^\eta}[q_0(\mathbf{y})^{\eta/2} - q_1(\mathbf{y})^{\eta/2}]\right) \times \frac{\sum_{k=1}^{m-1}(D_k(\eta)/k!)(\sigma\sqrt{q_1(\mathbf{y})}/\beta)^{\eta k}}{\sum_{k=1}^{m-1}(D_k(\eta)/k!)(\sigma\sqrt{q_0(\mathbf{y})}/\beta)^{\eta k}} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \xi, \tag{19}$$

where ξ is an threshold notation other than η,

$$D_k(\eta) = \sum_{j=1}^{k}(-1)^j C_k^j \prod_{i=0}^{j-1}\left(\frac{j\eta}{2} - i\right),$$

and $C_k^j = (k)!/(j)!(k-j)!$.

3.3.6. OWD: optimum Weibull detector

For a Weibull SIRV, the *texture* PDF can be written as an integral form depending on the G-Meijer's functions. Its expression can be found in [3,9] and the optimum Weibull detector is given for $0 \leqslant b \leqslant 2$ by

$$\left(\frac{q_0(\mathbf{y})}{q_1(\mathbf{y})}\right)^m \times \frac{\sum_{k=1}^{m-1}(D_k(b)/k!)(a\sigma^b)^k F_1^{kb}(\mathbf{y})\exp(-a\sigma^b F_1^{kb}(\mathbf{y}))}{\sum_{k=1}^{m-1}(D_k(b)/k!)(a\sigma^b)^k F_0^{kb}(\mathbf{y})\exp(-a\sigma^b F_0^{kb}(\mathbf{y}))} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta, \tag{20}$$

where a, b and σ^2 are, respectively, the two parameters of the marginal SIRV Weibull and its variance. In the OWD expression, we also have

$$F_p^{kb} = \sqrt{q_p(\mathbf{y})}^{kb} \quad \text{for } p = 0, 1,$$

$$D_k(b) = \sum_{j=1}^{k}(-1)^j C_k^j \prod_{i=0}^{j-1}\left(\frac{jb}{2} - i\right),$$

and

$$C_k^j = \frac{(k)!}{(j)!(k-j)!}.$$

3.3.7. ORD: optimum rice detector

The Rice marginal PDF of an SIRV, with parameter ρ and variance σ^2, gives rise to the optimum rice detector whose expression is given for $0 < \rho \leqslant 1$ by

$$\exp[K(q_0(\mathbf{y}) - q_1(\mathbf{y}))] \times \frac{\sum_{k=1}^{m}(-1)^k C_{m-1}^k(\rho/2)^k D_k(q_1(\mathbf{y}),\sigma,\rho)}{\sum_{k=1}^{m}(-1)^k C_{m-1}^k(\rho/2)^k D_k(q_0(\mathbf{y}),\sigma,\rho)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta, \tag{21}$$

where $K = \sigma^2/2(1-\rho^2)$, and for $j = 0, 1$

$$D_k(q_j(\mathbf{y})) = \sum_{l=0}^{k} C_k^l I_{k-2l}(K\rho q_j(\mathbf{y})).$$

$I_v(.)$ is the first kind modified Bessel function of order v, and $C_k^l = (k)!/(l)!(k-l)!$.

3.3.8. OCpGD: optimum clutter plus Gaussian detector

When thermal noise is separately accounted in the received data, the general expression (5) has to be changed because the total correlation matrix of the noise plus clutter depends on the *texture* since we have

$$\mathbf{M}_{y/\tau} = \tfrac{1}{2}\mathbb{E}(\mathbf{y}\mathbf{y}^{\dagger}/\tau) = \tau\mathbf{M}_x + \sigma_b^2\mathbf{I}, \tag{22}$$

where $\mathbf{I}$ is the identity matrix. GLRT expression (5) becomes

$$\frac{\int_0^{+\infty}(1/|\tau\mathbf{M}_x + \sigma_b^2\mathbf{I}|)\exp(-\frac{q_1(\mathbf{y},\tau)}{2})p(\tau)\,\mathrm{d}\tau}{\int_0^{+\infty}(1/|\tau\mathbf{M}_x + \sigma_b^2\mathbf{I}|)\exp(-\frac{q_0(\mathbf{y},\tau)}{2})p(\tau)\,\mathrm{d}\tau} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta, \tag{23}$$

where now

$$q_0(\mathbf{y},\tau) = \mathbf{y}^{\dagger}(\tau\mathbf{M}_x + \sigma_b^2\mathbf{I})^{-1}\mathbf{y}, \tag{24}$$

and $q_1(\mathbf{y},\tau)$ is deduced from $q_0(\mathbf{y},\tau)$ as in the previous case (known or unknown target amplitude).

This case was introduced by Gini et al. in [8] where the authors consider K-distributed clutter plus thermal noise to derive OKGD (optimum K plus Gaussian detector). This detector cannot be derived in a simple analytical form but theoretical performances are given in an integral form.

From expressions (5) and (23) we can see that SIRV detectors depends on the *texture* PDF. In [9,10] the authors used a Padé approximation to estimate the *texture* PDF and then to derive an adaptive detector. This method depends on the quality of the moment estimation and requires reference clutter cells to estimate the Padé coefficients.

In the next section we propose to use a bayesian estimator of the *texture* PDF which comes from the Bayes'rule and Monte Carlo integration. Then, BORD expression is derived.

4. Bayesian optimum radar detector

4.1. Bayesian study of the problem

As we have said in the previous section, for a known *texture* PDF $p(\tau)$, it is possible to derive the associated detector expression. The idea of a bayesian approach is to determine, from N_{ref} reference clutter cells of size m, $\mathbf{R} = [\mathbf{r}_1,\ldots,\mathbf{r}_{N_{\mathrm{ref}}}]^{\mathrm{T}}$ where $\mathbf{r}_i = [r_i(1),\ldots,r_i(m)]^{\mathrm{T}}$, a bayesian estimator for $p(\tau)$. We write $p(\tau)$ as follows:

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r})p(\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{25}$$

Given $\mathbf{r}_{i=1}^{N_{\mathrm{ref}}}$ a Monte Carlo estimation of (25) is

$$\hat{p}_{N_{\mathrm{ref}}}(\tau) = \frac{1}{N_{\mathrm{ref}}}\sum_{i=1}^{N_{\mathrm{ref}}} p(\tau/\mathbf{r}_i). \tag{26}$$

This estimate is unbiased and as the samples $\{\mathbf{r}_i,\ i = 1,\ldots,N_{\mathrm{ref}}\}$ are statistically independent, the strong law of large numbers applies and gives

$$\lim_{N_{\mathrm{ref}}\to+\infty} \hat{p}_{N_{\mathrm{ref}}}(\tau) \overset{\mathrm{a.s.}}{\to} p(\tau). \tag{27}$$

To evaluate (26) we have to know the expression of the a posteriori PDF of τ given the N_{ref} reference cells, that is the expression of $p(\tau/\mathbf{r}_i)$. Using the Bayes'rule, we have

$$p(\tau/\mathbf{r}_i) = \frac{p(\mathbf{r}_i/\tau)p(\tau)}{p(\mathbf{r}_i)}, \tag{28}$$

and as $p(\tau)$ is unknown (as well as $p(\mathbf{r}_i)$) we replace it by a prior distribution, called $g(\tau)$. This processing step works as an information processor that updates the prior density function $g(\tau)$ into the posterior PDF $p(\tau/\mathbf{r}_i)$. The Bayes'rule can be interpreted as a relevant mechanism to provide a rational solution of how to learn from the reference cells about the quantity of interest τ. By this way, Eq. (26) becomes

$$\hat{p}_{N_{\mathrm{ref}}}(\tau) = \frac{1}{N_{\mathrm{ref}}}\sum_{i=1}^{N_{\mathrm{ref}}} \frac{p(\mathbf{r}_i/\tau)g(\tau)}{p(\mathbf{r}_i)}, \tag{29}$$

where the normalization constant $p(\mathbf{r}_i)$ is obtained by integrating the numerator in (26) over $g(\tau)$ and is given by

$$p(\mathbf{r}_i) = \int_0^{+\infty} p(\mathbf{r}_i/\tau)g(\tau)\,\mathrm{d}\tau. \tag{30}$$

Replacing $\hat{p}_{N_{\mathrm{ref}}}(\tau)$ in (5), the expression of the so-called Bayesian Optimum Radar Detector can be derived.

Other methods are investigated in [7,15] to attain detector adaptivity but only with respect to parameters of $p(\tau)$. The proposed method makes the detector adaptive with respect to the unknown $p(\tau)$ and it is no

more necessary to have knowledge about the clutter statistics.

4.2. BORD expression

The N_{ref} reference clutter cells $[\mathbf{r}_1,\ldots,\mathbf{r}_{N_{\text{ref}}}]^{\text{T}}$ are supposed to be modeled as SIRV and so we have

$$p(\mathbf{r}_i/\tau)=\frac{\tau^{-m}}{(2\pi)^m|\mathbf{M}|}\exp\left(-\frac{Z_i}{2\tau}\right), \tag{31}$$

where $Z_i=\mathbf{r}_i^{\dagger}\mathbf{M}^{-1}\mathbf{r}_i$. If we consider this likelihood function as a function of τ it is proportional to an inverse gamma PDF, $\mathcal{IG}(m+1,2/Z_i)$ (an inverse gamma PDF is the PDF of the inverse of a Gamma-distributed variable). In (28) a prior distribution is required for the reference clutter *texture* and in this case we could choose an inverse gamma prior, $\mathcal{IG}(a_p,b_p)$ (parameters a_p and b_p would have to be chosen in order to make the prior as non-restrictive as possible). This prior would be the so-called *conjugate prior* because the resulting PDF would belong to the same PDF family as the likelihood function (see [14,16] for more details).

As the clutter statistics is unknown we choose a non-informative prior distribution $g(\tau)=1/\tau$, called Jeffrey's prior which is proportional to the square root of the Fischer's information measure and which is also an asymptotical case of the inverse gamma PDF when parameters $a_p\to 0$ and $b_p\to+\infty$.

With the non-informative prior, the a posteriori PDF of τ given the N_{ref} reference cells can be derived and (28) becomes

$$p(\tau/\mathbf{r}_i)=\frac{\tau^{-m-1}}{(2\pi)^m|\mathbf{M}|\,p(\mathbf{r}_i)}\exp\left(-\frac{Z_i}{2\tau}\right). \tag{32}$$

The normalization constant $p(\mathbf{r}_i)$ is computed as follows:

$$\begin{aligned}p(\mathbf{r}_i)&=\int_0^{+\infty}p(\mathbf{r}_i/\tau)g(\tau)\,\mathrm{d}\tau\\&=\int_0^{+\infty}\frac{\tau^{-m-1}}{(2\pi)^m|\mathbf{M}|}\exp\left(-\frac{Z_i}{2\tau}\right)\mathrm{d}\tau\\&=\frac{\Gamma(m)}{\pi^m|\mathbf{M}|Z_i^m},\end{aligned} \tag{33}$$

and (32) becomes

$$p(\tau/\mathbf{r}_i)=\frac{Z_i^m}{2^m\Gamma(m)}\tau^{-m-1}\exp\left(-\frac{Z_i}{2\tau}\right). \tag{34}$$

This expression is exactly an inverse gamma PDF with parameters m and $2/Z_i$. So, we have

$$\hat{p}_{N_{\text{ref}}}(\tau)=\frac{\tau^{-m-1}}{2^m\Gamma(m)N_{\text{ref}}}\sum_{i=1}^{N_{\text{ref}}}Z_i^m\exp\left(-\frac{Z_i}{2\tau}\right), \tag{35}$$

where we recall that $Z_i=\mathbf{r}_i^{\dagger}\mathbf{M}^{-1}\mathbf{r}_i$. Replacing (35) in (4) for each observed vector $\mathbf{y}_{\text{obs}}$ (size m) and given the N_{ref} reference clutter vectors $\mathbf{r}_{i=1}^{N_{\text{ref}}}$ we have to compute the following expression under H_j $(j=0,1)$ to form the GLRT:

$$\begin{aligned}&A_m^{N_{\text{ref}}}\sum_{i=1}^{N_{\text{ref}}}Z_i^m\int_0^{+\infty}\tau^{-2m-1}\exp\left(-\frac{W_{i,j}(\mathbf{y}_{\text{obs}})}{2\tau}\right)\mathrm{d}\tau\\&\quad=2^{2m}A_m^{N_{\text{ref}}}\Gamma(2m)\sum_{i=1}^{N_{\text{ref}}}\frac{Z_i^m}{(W_{i,j}(\mathbf{y}_{\text{obs}}))^{2m}},\end{aligned} \tag{36}$$

where Z_i is given previously and

$$(A_m^{N_{\text{ref}}})^{-1}=(2\pi)^m|\mathbf{M}|N_{\text{ref}}2^m\Gamma(m),$$

$$W_{i,j}(\mathbf{y}_{\text{obs}})=q_j(\mathbf{y}_{\text{obs}})+Z_i.$$

The so-called BORD expression becomes

$$\Lambda_{N_{\text{ref}}}(\mathbf{y}_{\text{obs}})=\frac{\sum_{i=1}^{N_{\text{ref}}}\left[\frac{Z_i}{(q_1(\mathbf{y}_{\text{obs}})+Z_i)^2}\right]^m}{\sum_{i=1}^{N_{\text{ref}}}\left[\frac{Z_i}{(q_0(\mathbf{y}_{\text{obs}})+Z_i)^2}\right]^m}\underset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}}\lambda. \tag{37}$$

Both of the quadratic forms q_0 and q_1 are, respectively, given by (6) and (8).

BORD expression depends only on the reference clutter cells which provide all the necessary information about the clutter statistics. That makes itself "self-adaptive" if the correlation matrix is determined from the reference cells of the clutter. This problem was investigated in [5,7] where the authors use the normalized sample covariance matrix estimator (NSCM). Given that $\mathbf{M}=\mathbf{M}_r/\mathbb{E}(\tau)=\mathbb{E}(\mathbf{r}\mathbf{r}^{\dagger})/\mathbb{E}(\mathbf{r}^{\dagger}\mathbf{r})$, the NSCM estimate of $\mathbf{M}$ is given by

$$\hat{\mathbf{M}}=\frac{m}{N_{\text{ref}}}\sum_{k=1}^{N_{\text{ref}}}\frac{\mathbf{r}_k\mathbf{r}_k^{\dagger}}{\mathbf{r}_k^{\dagger}\mathbf{r}_k}, \tag{38}$$

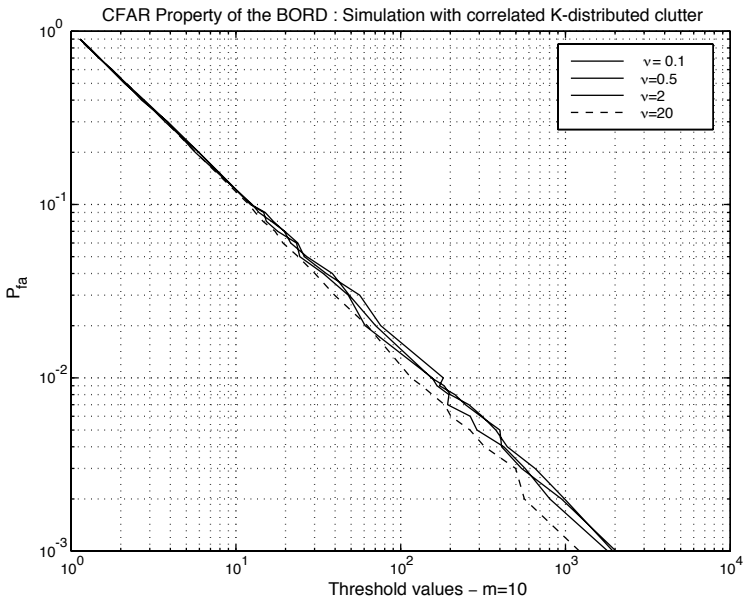

Fig. 1. CFAR property of the BORD with respect to the *texture* PDF shown for correlated K-distributed environments with parameters $v = 0.1$, 0.5, 2, 20 for $m = 10$.

where $\mathbf{r}_k$ $(k = 1, \ldots, N_{\text{ref}})$ are the N_{ref} reference clutter cells of size m.

4.3. CFAR property of the BORD

We can show by simulations that the BORD is CFAR (Constant False Alarm Rate) with respect to the *texture* PDF. It means that whatever the clutter statistics is (dually whatever the *texture* PDF is), BORD probability of false alarm is the same. In other words the BORD law is independent of the clutter statistics. This property is shown in Figs. 1 and 2 where the detection threshold η is plotted against different P_{fa} values. What ever the clutter statistics is (K-distributed with various parameters, Gaussian, Student-t or whose *texture* PDF is a Weibull PDF), the calculation of the detection threshold for a fixed P_{fa} is the same for all.

4.4. Asymptotical result of the BORD: AsBORD

BORD expression comes after a Monte Carlo estimation of (25) given N_{ref} reference clutter vector $\mathbf{r}_{i=1}^{N_{\text{ref}}}$. Given $Z_i = \mathbf{r}_i^{\dagger}\mathbf{M}^{-1}\mathbf{r}_i$, which is a positive r.v. with PDF $p(Z)$, BORD expression can be considered as the Monte Carlo estimation of

$$\frac{\int_0^{+\infty}(Z^m/(q_1(\mathbf{y}_{\text{obs}}) + Z)^{2m})p(Z)\,\mathrm{d}Z}{\int_0^{+\infty}(Z^m/(q_0(\mathbf{y}_{\text{obs}}) + Z)^{2m})p(Z)\,\mathrm{d}Z}. \tag{39}$$

Given that $\mathbf{r} = \sqrt{\tau}\mathbf{x}$ where $\mathbf{x}$ is a complex Gaussian vector of size m with covariance matrix $2\mathbf{M}$, we have $Z = \mathbf{r}^{\dagger}\mathbf{M}^{-1}\mathbf{r} = \tau\mathbf{x}^{\dagger}\mathbf{M}^{-1}\mathbf{x}$.

The quadratic form $Q = \mathbf{x}^{\dagger}\mathbf{M}^{-1}\mathbf{x}$ is χ^2_{2m} distributed $(\chi^2_{2m} = \mathcal{G}(m, 2))$.

So, Z/τ is $\mathcal{G}(m, 2\tau)$ and the PDF of Z is derived by integrating $p(Z/\tau)$ over the prior $g(\tau)$. Finally, we can show that

$$\lim_{N_{\text{ref}} \to +\infty} \Lambda_{N_{\text{ref}}}(\mathbf{y}_{\text{obs}}) = \left(\frac{q_0(\mathbf{y}_{\text{obs}})}{q_1\mathbf{y}_{obs})}\right)^m. \tag{40}$$

This asymptotical result, called asymptotic BORD (AsBORD), coincides with the GLRT given, for example, in [15]. It was obtained after replacing the *texture* τ by its two maximum likelihood estimates (the one under H_0 and the other under H_1) in the optimum detection structure (5), where τ was considered

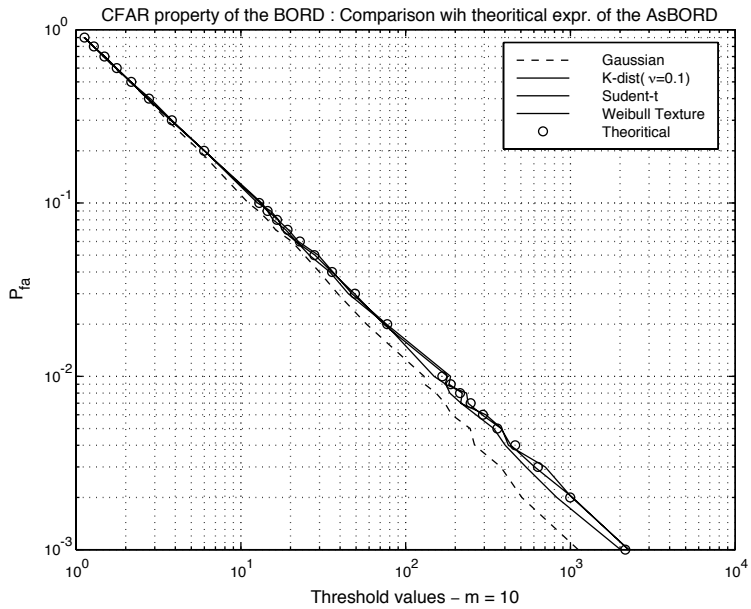

Fig. 2. CFAR property of the BORD with respect to the *texture* PDF shown for different correlated environments: K-distributed clutter with parameter $\nu = 0.1$, Gaussian clutter, Student-t clutter and unknown clutter whose *texture* PDF is a Weibull PDF. $m = 10$. Comparison with the theoretical expression of the AsBORD threshold.

as an unknown deterministic parameter. This was also obtained after different calculations in [2] and for Gaussian clutter in [11,12]. Moreover, with the NSCM estimate of the correlation matrix this expression is also called adaptive linear quadratic (ALQ) in [5,6].

4.5. CFAR property of the AsBORD

Replacing both of the quadratic forms q_0 and q_1 in (40) and after simple modification, the AsBORD expression can be written as follows:

$$\frac{|\mathbf{p}^{\dagger}\mathbf{M}^{-1}\mathbf{x}_{\text{obs}}|^2}{(\mathbf{p}^{\dagger}\mathbf{M}^{-1}\mathbf{p})(\mathbf{x}_{\text{obs}}^{\dagger}\mathbf{M}^{-1}\mathbf{x}_{\text{obs}})} \underset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}} \frac{\sqrt[m]{\eta} - 1}{\sqrt[m]{\eta}}, \tag{41}$$

where $\mathbf{x}_{\text{obs}}$ is the (Gaussian) *speckle* of the observed SIRV.

The left term of (41) is independent of the *texture*. Then AsBORD PDF is statistically independent of the *texture* PDF that makes the AsBORD to be CFAR with respect of the *texture* PDF. In this case it is possible to derive the AsBORD PDF.

4.6. AsBORD PDF

According to (41) the AsBORD expression depends on Gaussian vectors and on a related quadratic form.

Extending the Cochran's theorem to complex Gaussian vectors and after simple computation, the AsBORD PDF is a Beta PDF with parameters 1 and $m - 1$ whose expression is [9]

$$p(u) = (m-1)(1-u)^{m-2}. \tag{42}$$

Then the expression of the threshold value depends only on P_{fa} and m, the size of the observed vector [9]:

$$\eta = P_{\text{fa}}^{(m/1-m)}. \tag{43}$$

Applying Cochran's theorem implies that the covariance matrix of the observed vector is non-singular. Under this condition AsBORD PDF applies also to the BORD as shown in Fig. 2.

In [9] the AsBORD PDF in the case of real vectors is also derived.

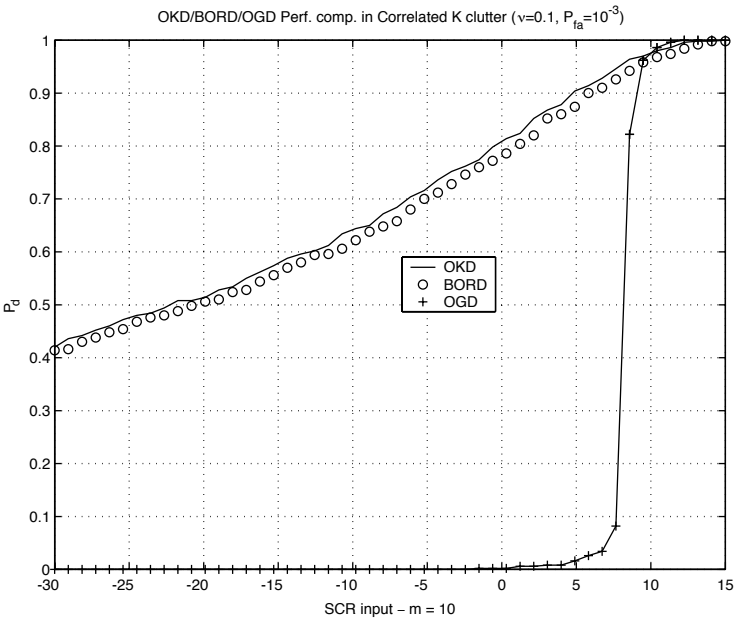

Fig. 3. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($v = 0.1, P_{\text{fa}} = 10^{-3},\ m = 10$).

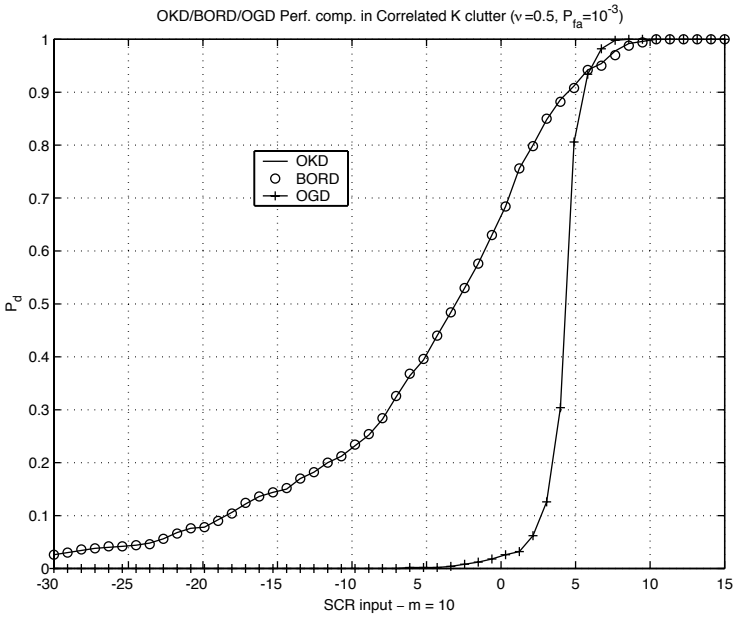

Fig. 4. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($v = 0.5$, $P_{\text{fa}} = 10^{-3}$, $m = 10$).

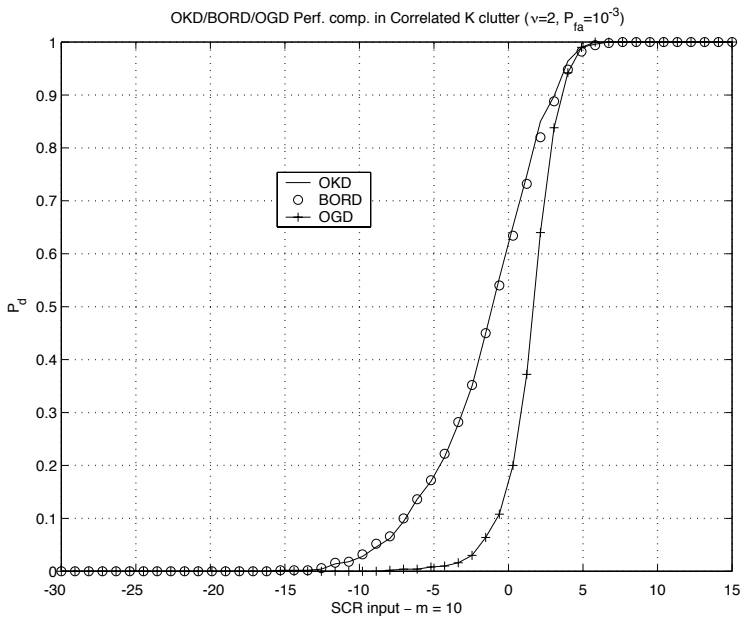

Fig. 5. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($v = 2$, $P_{\mathrm{fa}} = 10^{-3}$, $m = 10$).

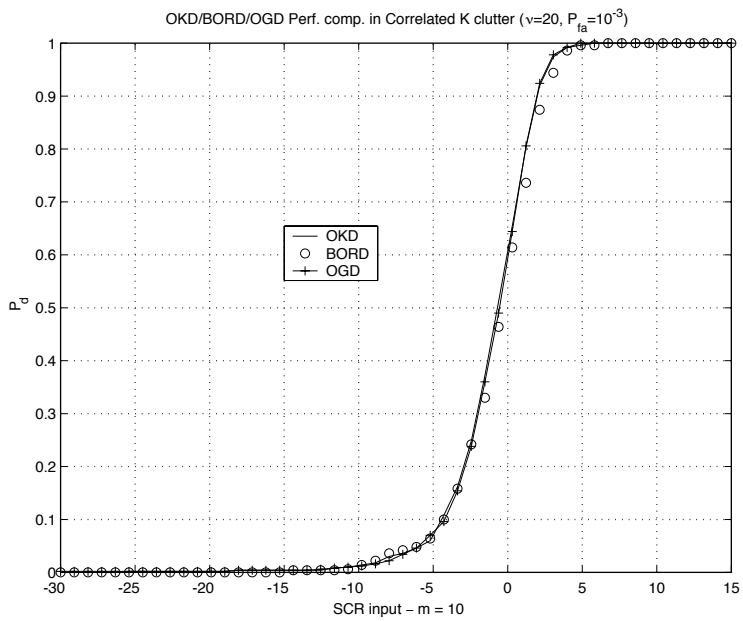

Fig. 6. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($v = 20$, $P_{\mathrm{fa}} = 10^{-3}$, $m = 10$).

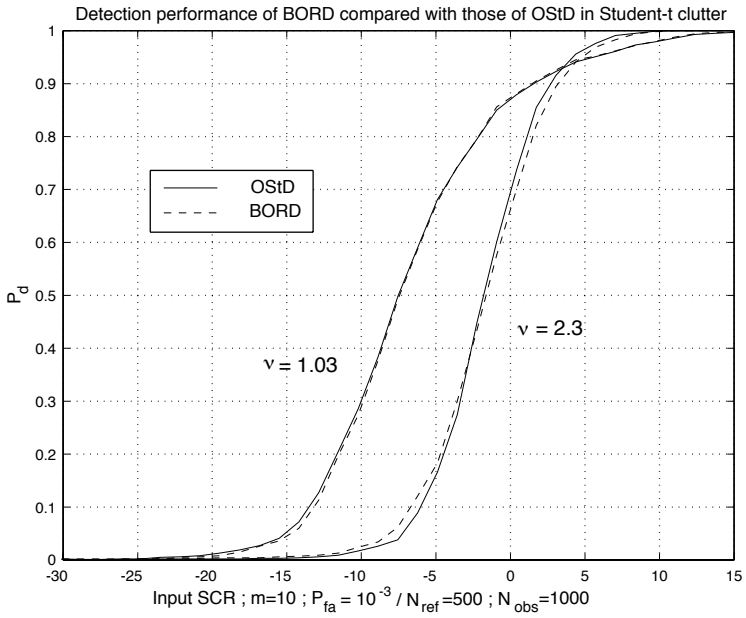

Fig. 7. Performance comparison between OStD and BORD for an unknown target complex amplitude in correlated Student-t clutter ($v = 1.03, 2.3$, $P_{\text{fa}} = 10^{-3}$, $m = 10$).

5. Detection performances of BORD

The correlation matrix $\mathbf{M}$ is considered to be known. $\mathbf{M}$ values come from a given Gaussian power spectral density and is an $m \times m$ Toeplitz matrix whose first auto-correlation coefficient is $\rho_1 = 0.0098$.

We compare BORD performance with those of optimum detectors such OKD, OGD (optimum Gaussian detector, optimum for Gaussian clutter) or OStD for an unknown target signal embedded in K-distributed clutter or Student-t clutter.

In the case of K-distributed clutter, OKD is optimum and we see that BORD performance reach OKD performance whatever the value of v is. Different values of the shape parameter are tested, $v = 0.1, 0.5, 2, 20$. When $v \to +\infty$ K-PDF tends to a Gaussian PDF which is confirmed in the series of Figs. 3–6.

In the case of Student-t clutter (shown in Fig. 7) the conclusions are the same: BORD reach optimum performances given by those of the optimum detector OStD.

All the curves represent the detection probability P_{d} versus the signal-to-clutter ratio (SCR) given for one pulse. As $m = 10$ pulses are considered, the total SCR is $10 \log_{10}(m) = 10$ dB more than for one pulse. The detection threshold is previously computed via Monte Carlo simulation for each of the detectors to keep a false alarm rate equal to $P_{\text{fa}} = 10^{-3}$.

Once the detection threshold is obtained we evaluate performance with $N_{\text{ref}} = 1000$ samples of reference clutter and for $N_{\text{obs}} = 500$ samples of observed clutter data (which corresponds in fact to the number of Monte Carlo trials used for the evaluation of the probability of detection) in which an unknown complex target signal for a different SCR is embedded.

6. Conclusions and outlooks

The present paper has addressed a bayesian approach to the determination of the clutter statistics if we consider the clutter vector modeled as a SIRV. By this way a bayesian estimator of the *texture* PDF of the SIRV has been derived from reference clutter cells and the resulting BORD expression depends only on these reference cells. For example, in the case of

CFAR (Constant False Alarm Rate) detector th reference clutter cells are the cells adjacent to the cell under test.

We derive also the asymptotic expression of BORD, called AsBORD, whose PDF is derived in a closed form. With this latter expression we are able to compute the threshold value to set to verify a desired P_{fa} in the case where the correlation matrix is non-singular. Under this assumption, the theoretical threshold expression applies also to BORD. In this paper, we use Monte Carlo simulation to set the threshold value of BORD. In further work we will compare the performances of BORD and AsBORD for experimental ground clutter data and study the applicability of the theoretical threshold expression with respect to the correlation of the data.

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Annexe H

Covariance Structure Maximum-Likelihood Estimates in Compound Gaussian Noise : Existence and Algorithm Analysis

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Covariance Structure Maximum-Likelihood Estimates in Compound Gaussian Noise: Existence and Algorithm Analysis

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***Abstract*—Recently, a new adaptive scheme [Conte *et al.* (1995), Gini (1997)] has been introduced for covariance structure matrix estimation in the context of adaptive radar detection under non-Gaussian noise. This latter has been modeled by compound-Gaussian noise, which is the product c of the square root of a positive unknown variable τ (deterministic or random) and an independent Gaussian vector x, $\mathrm{c} = \sqrt{\tau}\mathrm{x}$. Because of the implicit algebraic structure of the equation to solve, we called the corresponding solution, the fixed point (FP) estimate. When τ is assumed deterministic and unknown, the FP is the exact maximum-likelihood (ML) estimate of the noise covariance structure, while when τ is a positive random variable, the FP is an approximate maximum likelihood (AML). This estimate has been already used for its excellent statistical properties without proofs of its existence and uniqueness. The major contribution of this paper is to fill these gaps. Our derivation is based on some likelihood functions general properties like homogeneity and can be easily adapted to other recursive contexts. Moreover, the corresponding iterative algorithm used for the FP estimate practical determination is also analyzed and we show the convergence of this recursive scheme, ensured whatever the initialization.**

***Index Terms*—Adaptive detection, compound Gaussian, constant false alarm rate (CFAR) detector, maximum-likelihood (ML) estimate, spherically invariant random vectors (SIRV).**

I. Introduction

THE basic problem of detecting a complex signal embedded in an additive Gaussian noise has been extensively studied during last decades. In these contexts, adaptive detection schemes required an estimate of the noise covariance matrix generally obtained from signal-free data traditionally called secondary or reference data. The resulting adaptive detectors, as those proposed by [7] and [8], are all based on the Gaussian assumption for which the maximum-likelihood (ML) estimate of the covariance matrix is given by the sample covariance matrix. However, these detectors may exhibit poor performance when the additive noise is no more Gaussian [6].

This is the case in radar detection problems where the additive noise is due to the superposition of unwanted echoes reflected by the environment and traditionally called the clutter. Indeed, experimental radar clutter measurements showed that these data are non-Gaussian. This fact arises for example when the illuminated area is nonhomogeneous or when the number of scatterers is small. This kind of non-Gaussian noises is usually described by distributions such as K-distribution, Weibull, etc. Therefore, this non-Gaussian noise characterization has gained a lot of interest in the radar detection community.

One of the most general and elegant non-Gaussian noise model is provided by the compound-Gaussian process which includes the so-called *spherically invariant random vectors* (SIRVs). These processes encompass a large number of non-Gaussian distributions mentioned previously and include, of course, Gaussian processes. They have been recently introduced, in radar detection, to model clutter for solving the basic problem of detecting a known signal. This approach resulted in the adaptive detectors development such as the generalized likelihood tatio test–linear quadratic (GLRT-LQ) in [1] and [2] or the Bayesian optimum Radar dtector (BORD) in [3] and [4]. These detectors require an estimate of the covariance matrix of the noise Gaussian component. In this context, ML estimates based on secondary data have been introduced in [11] and [12], together with a numerical procedure supposed to obtain them. However, as noticed in [12, p. 1852], "*existence of the ML estimate and convergence of iteration* [...] *is still an open problem*."

To the best of our knowledge, the proofs of existence, uniqueness of the ML estimate,and convergence of the algorithm proposed in [1] have never been established. The main purpose of this paper is to fill these gaps.

This paper is organized as follows. In Section II, we present the two main models of interest in our ML estimation framework. Both models lead to ML estimates which are solution of a transcendental equation. Section IV presents the main results of this paper while a proofs outline is given in Section V: for presentation clarity, full demonstrations are provided in Appendices I–VIII. Finally, Section VI gives some simulations results which confirm the theoretical analysis.

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II. State of the Art and Problem Formulation

A compound-Gaussian process $\mathbf{c}$ is the product of the square root of a positive scalar quantity τ called the texture and a m-dimensional zero mean complex Gaussian vector $\mathbf{x}$ with covariance matrix $\mathbf{M} = \mathbb{E}(\mathbf{x}\mathbf{x}^H)$ usually normalized according to $\mathrm{Tr}(\mathbf{M}) = m$, where H denotes the conjugate transpose operator and $\mathrm{Tr}(.)$ stands for the trace operator

$$\mathbf{c} = \sqrt{\tau}\mathbf{x}. \tag{1}$$

This general model leads to two distinct approaches: the well-known SIRV modeling where the texture is considered random and the case where the texture is treated as an unknown nuisance parameter.

Generally, the covariance matrix $\mathbf{M}$ is not known and an estimate $\widehat{\mathbf{M}}$ is required for the likelihood-ratio (LR) computation. Classically, such an estimate $\widehat{\mathbf{M}}$ is obtained from ML theory, well known for its good statistical properties. In this problem, estimation of $\mathbf{M}$ must respect the previous $\mathbf{M}$-normalization $\mathrm{Tr}(\widehat{\mathbf{M}}) = m$. This estimate $\widehat{\mathbf{M}}$ will be built using N independent realizations of $\mathbf{c}$ denoted $\mathbf{c}_i = \sqrt{\tau_i}\mathbf{x}_i$ for $i = 1, \ldots, N$.

It straightforwardly appears that the likelihood will depend on the assumption relative to texture. The two most often met cases are presented in Sections II-A and II-B.

A. SIRV Case

Let us recap that an SIRV [5] is the product of the square root of a positive random variable τ (*texture*) and a m-dimensional independent complex Gaussian vector $\mathbf{x}$ (*speckle*) with zero mean normalized covariance matrix $\mathbf{M}$. This model led to many investigations [1]–[4].

To obtain the ML estimate of $\mathbf{M}$, with no proofs of existence and uniqueness, Gini *et al.* derived in [12] an AML estimate $\widehat{\mathbf{M}}$ as the solution of the following:

$$\widehat{\mathbf{M}} = f(\widehat{\mathbf{M}}) \tag{2}$$

where f is given by

$$f(\widehat{\mathbf{M}}) = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{c}_i\mathbf{c}_i^H}{\mathbf{c}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{c}_i}. \tag{3}$$

B. Unknown Deterministic τ Case

This approach has been developed in [13], where the τ_i's are assumed to be unknown deterministic quantities. The corresponding likelihood function to maximize with respect to $\mathbf{M}$ and τ_i's, is given by

$$p_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M}, \tau_1, \ldots, \tau_N) = \frac{1}{(\pi)^{mN}|\mathbf{M}|^N} \times \prod_{i=1}^{N}\frac{1}{\tau_i^m}\exp\left(-\frac{\mathbf{c}_i^H\mathbf{M}^{-1}\mathbf{c}_i}{\tau_i}\right) \tag{4}$$

where $|\mathbf{M}|$ denotes the determinant of matrix $\mathbf{M}$.

Maximization with respect to τ_i's, for a given $\mathbf{M}$, leads to $\hat{\tau}_i = (\mathbf{c}_i^H\mathbf{M}^{-1}\mathbf{c}_i)/m$, and then by replacing the τ_i's in (4) by their ML estimates $\hat{\tau}_i$'s, we obtain the reduced likelihood function

$$\hat{p}_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M}) = \frac{1}{(\pi)^{mN}|\mathbf{M}|^N}\prod_{i=1}^{N}\frac{m^m\exp(-m)}{\left(\mathbf{c}_i^H\mathbf{M}^{-1}\mathbf{c}_i\right)^m}.$$

Finally, maximizing $\hat{p}_C(\mathbf{c}_1, \ldots, \mathbf{c}_N; \mathbf{M})$ with respect to $\mathbf{M}$ is equivalent to maximize the following function F, written in terms of $\mathbf{x}_i$'s and τ_i's thanks to (1):

$$F(\mathbf{M}) = \frac{1}{|\mathbf{M}|^N}\prod_{i=1}^{N}\frac{1}{\tau_i^m\left(\mathbf{x}_i^H\mathbf{M}^{-1}\mathbf{x}_i\right)^m}. \tag{5}$$

By cancelling the gradient of F with respect to $\mathbf{M}$, we obtain the following:

$$\widehat{\mathbf{M}} = f(\widehat{\mathbf{M}}) \tag{6}$$

where f is given again by (3) and whose solution is the ML estimator in the deterministic texture framework.

Note that f can be rewritten from (1) as

$$f(\widehat{\mathbf{M}}) = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{x}_i\mathbf{x}_i^H}{\mathbf{x}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{x}_i}. \tag{7}$$

Equation (7) shows that $f(\widehat{\mathbf{M}})$ does not depend on the texture τ but only on the Gaussian vectors $\mathbf{x}_i$'s.

C. Problem Formulation

It has been shown in [12] and [13] that estimation schemes developed under both the stochastic case (Section II-A) and the deterministic case (Section II-B) lead to the analysis of the same equation [(2) and (6)], whose solution is a fixed point (FP) of f (7). A first contribution of this paper is to establish the existence and the uniqueness, up to a scalar factor, of this FP $\widehat{\mathbf{M}}_{\mathrm{FP}}$ which is the AML estimate under the stochastic assumption and the exact ML under the deterministic assumption.

Moreover, a second contribution is to analyze an algorithm based on the key (6), which defines $\widehat{\mathbf{M}}_{\mathrm{FP}}$. The convergence of this algorithm will be established. Then, numerical results of Section VI will illustrate the computational efficiency of the algorithm for obtaining the FP estimate.

Finally, the complete statistical properties investigation of the corresponding ML estimate will be addressed in a forthcoming paper.

III. Statement of the Main Result

We first provide some notations. Let m and N be positive integers such that $m < N$. We use $\mathbb{R}^{+*}$ to denote the set of strictly positive real scalars, $M_m(\mathbb{C})$ to denote the set of $m \times m$ complex matrices, and $\mathcal{G}$ the subset of $M_m(\mathbb{C})$ defined by the positive–definite Hermitian matrices. For $\mathbf{M} \in M_m(\mathbb{C})$, $\|\mathbf{M}\| := \mathrm{Tr}(\mathbf{M}^H\mathbf{M})^{1/2}$ the Frobenius norm of $\mathbf{M}$ which is the norm associated to an inner product on $M_m(\mathbb{C})$. Moreover, from the statistical independence hypothesis of the N complex m-vectors $\mathbf{x}_i$, it is natural to assume the following.

H) Let us set $\mathbf{x}_i = \mathbf{x}_i^{(1)} + j\mathbf{x}_i^{(2)}$. Any $2m$ distinct vectors taken in

$$\left\{ \begin{pmatrix} \mathbf{x}_1^{(1)} \\ \mathbf{x}_1^{(2)} \end{pmatrix}, \ldots, \begin{pmatrix} \mathbf{x}_N^{(1)} \\ \mathbf{x}_N^{(2)} \end{pmatrix}, \begin{pmatrix} -\mathbf{x}_1^{(2)} \\ \mathbf{x}_1^{(1)} \end{pmatrix}, \ldots, \begin{pmatrix} -\mathbf{x}_N^{(2)} \\ \mathbf{x}_N^{(1)} \end{pmatrix} \right\}$$

are linearly independent.

From (5) and (7), one has

$$\begin{aligned} F : \mathcal{G} &\longrightarrow \mathbb{R}^{+*} \\ \mathbf{M} &\longrightarrow F(M) = \frac{1}{|\mathbf{M}|^N} \prod_{i=1}^{N} \frac{1}{\tau_i^m \left(\mathbf{x}_i^H \mathbf{M}^{-1} \mathbf{x}_i\right)^m} \end{aligned}$$

and

$$\begin{aligned} f : \mathcal{G} &\longrightarrow \mathcal{G} \\ \mathbf{M} &\longrightarrow f(M) = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^H}{\mathbf{x}_i^H \mathbf{M}^{-1} \mathbf{x}_i}. \end{aligned}$$

Theorem III.1:

1) There exists $\widehat{\mathbf{M}}_{\mathrm{FP}} \in \mathcal{G}$ with unit norm such that, for every $\alpha > 0$, f admits a unique FP of norm $\alpha > 0$ equal to $\alpha \widehat{\mathbf{M}}_{\mathrm{FP}}$. Moreover, F reaches its maximum over $\mathcal{G}$ only on $\mathcal{L}_{\widehat{\mathbf{M}}_{\mathrm{FP}}}$, the open half-line spanned by $\widehat{\mathbf{M}}_{\mathrm{FP}}$.
2) Let $(S)_{\mathrm{dis}}$ be the discrete dynamical system defined on $\mathcal{D}$ by

$$(S)_{\mathrm{dis}} : \mathbf{M}_{k+1} = f(\mathbf{M}_k). \tag{8}$$

Then, for every initial condition $\mathbf{M}_0 \in \mathcal{G}$, the resulting sequence $(\mathbf{M}_k)_{k \geq 0}$ converges to an FP of f, i.e., to a point where F reaches its maximum.
3) Let $(S)_{\mathrm{cont}}$ be the continuous dynamical system defined on $\mathcal{G}$ by

$$(S)_{\mathrm{cont}} : \dot{\mathbf{M}} = \nabla F(\mathbf{M}). \tag{9}$$

Then, for every initial condition $\mathbf{M}(0) = \mathbf{M}_0 \in \mathcal{G}$, the resulting trajectory $\mathbf{M}(t), t \geq 0$, converges when t tends to $+\infty$, to the point $||\mathbf{M}_0|| \widehat{\mathbf{M}}_{\mathrm{FP}}$, i.e., to a point where F reaches its maximum.

Consequently to 1), $\widehat{\mathbf{M}}_{\mathrm{FP}}$ is the unique positive–definite $m \times m$ matrix of norm one satisfying

$$\widehat{\mathbf{M}}_{\mathrm{FP}} = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^H}{\mathbf{x}_i^H \widehat{\mathbf{M}}_{\mathrm{FP}}^{-1} \mathbf{x}_i}. \tag{10}$$

Proof: The same problem and the same result can be formulated with real numbers instead of complex numbers and symmetric matrices instead of Hermitian matrices, while hypothesis H) becomes hypothesis H2). The proof of Theorem III.1 breaks up into two stages. We first show in Appendix I how to derive Theorem III.1 from the corresponding real results. Then, the rest of this paper is devoted to the study of the real case. ■

IV. Notations and Statements of the Results in the Real Case

A. Notations

In this paragraph, we introduce the main notations of this paper for the real case. Notations already defined in the complex case are translated in the real one. Moreover, real results will be valid for every integer m. For every positive integer n, $[1, n]$ denotes the set of integers $\{1, \ldots, n\}$. For vectors of $\mathbb{R}^m$, the norm used is the Euclidean one. Throughout this paper, we will use several basic results on square matrices, especially regarding diagonalization of real symmetric and orthogonal matrices. We refer to [14] for such standard results.

We use $M_m(\mathbb{R})$ to denote the set of $m \times m$ real matrices, $SO(m)$ to denote the set of $m \times m$ orthogonal matrices, and $\mathbf{M}^\top$, the transpose of $\mathbf{M}$. We denote the identity matrix of $M_m(\mathbb{R})$ by $\mathbf{I}_m$.

In the following, we define and list the several sets of matrices used in the sequel:

- $\mathcal{D}$, the subset of $M_m(\mathbb{R})$ defined by the symmetric positive–definite matrices;
- $\overline{\mathcal{D}}$, the closure of $\mathcal{D}$ in $M_m(\mathbb{R})$, i.e., the subset of $M_m(\mathbb{R})$ defined by the symmetric nonnegative matrices;
- for every $\alpha > 0$

$$\begin{cases} \mathcal{D}(\alpha) = \{\mathbf{M} \in \mathcal{D} | \, ||\mathbf{M}|| = \alpha\} \\ \overline{\mathcal{D}}(\alpha) = \{\mathbf{M} \in \overline{\mathcal{D}} | \, ||\mathbf{M}|| = \alpha\} \end{cases}.$$

It is obvious that $\overline{\mathcal{D}}(\alpha)$ is compact in $M_m(\mathbb{R})$.

For $\mathbf{M} \in \mathcal{D}$, we use $\mathcal{L}_{\mathbf{M}}$ to denote the open half-line spanned by $\mathbf{M}$ in the cone $\mathcal{D}$, i.e., the set of points $\lambda \mathbf{M}$, with $\lambda > 0$. Recall that the order associated with the cone structure of $\mathcal{D}$ is called the Loewner order for symmetric matrices of $M_m(\mathbb{R})$ and is defined as follows. Let $\mathbf{A}$ and $\mathbf{B}$ be two symmetric $m \times m$ real matrices. Then, $\mathbf{A} \leq \mathbf{B}$ ($\mathbf{A} < \mathbf{B}$, respectively) means that the quadratic form defined by $\mathbf{B} - \mathbf{A}$ is nonnegative (positive definite, respectively), i.e., for every nonzero $\mathbf{x} \in \mathbb{R}^m$ and $\mathbf{x}^\top(\mathbf{A} - \mathbf{B})\mathbf{x} \leq 0$, ($>0$, respectively). Using that order, one has $\mathbf{M} \in \mathcal{D}$ ($\in \overline{\mathcal{D}}$, respectively) if and only if $\mathbf{M} > \mathbf{0}$ ($\mathbf{M} \geq \mathbf{0}$, respectively).

As explained in Appendix I, we will study in this section the applications F and f (same notations as in the complex case) defined as follows:

$$\begin{aligned} F : \mathcal{D} &\longrightarrow \mathbb{R}^{+*} \\ \mathbf{M} &\longrightarrow \frac{1}{|\mathbf{M}|^N} \prod_{i=1}^{N} \frac{1}{\tau_i^m \left(\mathbf{x}_i^\top \mathbf{M}^{-1} \mathbf{x}_i\right)^m} \end{aligned}$$

and

$$\begin{aligned} f : \mathcal{D} &\longrightarrow \mathcal{D} \\ \mathbf{M} &\longrightarrow \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^\top}{\mathbf{x}_i^\top \mathbf{M}^{-1} \mathbf{x}_i}. \end{aligned}$$

Henceforth, F and f stay for the real formulation. In the previous, the vectors $(\mathbf{x}_i)$, $1 \leq i \leq N$, belong to $\mathbb{R}^m$ and verify the following two hypotheses:

H1) $||\mathbf{x}_i|| = 1, 1 \leq i \leq N$;

H2) for any m 2×distinct indices $i(1) < \ldots < i(m)$ chosen in $[1, N]$, the vectors $\mathbf{x}_{i(1)}, \ldots, \mathbf{x}_{i(m)}$ are linearly independent.

Consequently, the vectors $\mathbf{c}_1, \ldots, \mathbf{c}_m$ verify H2).

Hypothesis H1) stems from the fact that function f does not depend on $\mathbf{x}_i$'s norm.

Let us already emphasize that hypothesis H2) is the key assumption for getting all our subsequent results. Hypothesis H2) has the following trivial but fundamental consequence that we state as a remark.

Remark IV.1: For every n vectors $\mathbf{x}_{i(1)}, \ldots, \mathbf{x}_{i(n)}$ (respectively, $\mathbf{c}_{i(1)}, \ldots, \mathbf{c}_{i(n)}$) with $1 \leq n \leq m$, $1 \leq i \leq N$, the vector space generated by $\mathbf{x}_{i(1)}, \ldots, \mathbf{x}_{i(n)}$ (respectively, $\mathbf{c}_{i(1)}, \ldots, \mathbf{c}_{i(n)}$) has dimension n.

In the sequel, we use f^n, $n \geq 1$, to denote the nth iterate of f, i.e., $f^n := f \circ \ldots \circ f$, where f is repeated n times. We also adopt the following standard convention $f^0 := Id_{\mathcal{D}}$.

The two functions F and f are related by the following relation, which is obtained after an easy computation. For every $\mathbf{M} \in \mathcal{D}$, let $\nabla F(\mathbf{M})$ be the gradient of F at $\mathbf{M} \in \mathcal{D}$, i.e., the unique symmetric matrix verifying, for every matrix $M \in \mathcal{S}$

$$\nabla F(\mathbf{M}) = NF(\mathbf{M})\mathbf{M}^{-1}\left(f(\mathbf{M}) - \mathbf{M}\right)\mathbf{M}^{-1}.$$

Clearly, $\mathbf{M}$ is an FP of f if and only if M is a critical point of the vector field defined by ∇F on $\mathcal{D}$.

B. Statements of the Results

The goal of this paper is to establish the following theorems whose proofs are outlined in Section V.

Theorem IV.1: There exists $\widehat{\mathbf{M}}_{\mathrm{FP}} \in \mathcal{D}$ with unit norm such that, for every $\alpha > 0$, f admits a unique FP of norm $\alpha > 0$ equal to $\alpha\widehat{\mathbf{M}}_{\mathrm{FP}}$. Moreover, F reaches its maximum over $\mathcal{D}$ only on $\mathcal{L}_{\widehat{\mathbf{M}}_{\mathrm{FP}}}$, the open half-line spanned by $\widehat{\mathbf{M}}_{\mathrm{FP}}$.

Consequently, $\widehat{\mathbf{M}}_{\mathrm{FP}}$ is the unique positive–definite $m \times m$ matrix of norm one satisfying

$$\widehat{\mathbf{M}}_{\mathrm{FP}} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{x}_i\mathbf{x}_i^\top}{\mathbf{x}_i^\top\widehat{\mathbf{M}}_{\mathrm{FP}}^{-1}\mathbf{x}_i}. \tag{11}$$

Remark IV.2: Theorem IV.1 relies on the fact that F reaches its maximum on $\mathcal{D}$. Roughly speaking, that issue is proved as follows. The function F is continuously extended by the zero function on the boundary of D, excepted on the zero matrix. Since F is positive and bounded on $\mathcal{D}$, we conclude. Complete argument is provided in Appendix II.

As a consequence of Theorem IV.1, one obtains the next result.

Theorem IV.2:

- Let $(S)_{\mathrm{dis}}$ be the discrete dynamical system defined on $\mathcal{D}$ by

$$(S)_{\mathrm{dis}} : \mathbf{M}_{k+1} = f(\mathbf{M}_k). \tag{12}$$

  Then, for every initial condition $\mathbf{M}_0 \in \mathcal{D}$, the resulting sequence $(\mathbf{M}_k)_{k\geq 0}$ converges to an FP of f, i.e., to a point where F reaches its maximum.

- Let $(S)_{\mathrm{cont}}$ be the continuous dynamical system defined on $\mathcal{D}$ by

$$(S)_{\mathrm{cont}} : \dot{\mathbf{M}} = \nabla F(\mathbf{M}). \tag{13}$$

  Then, for every initial condition $\mathbf{M}(0) = \mathbf{M}_0 \in \mathcal{D}$, the resulting trajectory $\mathbf{M}(t)$, $t \geq 0$, converges, when t tends to $+\infty$, to the point $\|\mathbf{M}_0\|\widehat{\mathbf{M}}_{\mathrm{FP}}$, i.e., to a point where F reaches its maximum.

The last theorem can be used to characterize numerically the points where F reaches its maximum and the value of that maximum.

Notice that algorithm defined by (12) does not allow the control of the FP norm. Therefore, for practical convenience, we propose a slightly modified algorithm in which the $\mathbf{M}$-normalization is applied at each iteration. This is summarized in Corollary IV.1.

Corollary IV.1: The following scheme:

$$\mathbf{M}'_{k+1} = \frac{f\left(\mathbf{M}'_k\right)}{\mathrm{Tr}\left(f\left(\mathbf{M}'_k\right)\right)} \tag{14}$$

yields the matrices sequence $\{\mathbf{M}'_0, \ldots, \mathbf{M}'_k\}$, which is related to the matrices sequence $\{\mathbf{M}_0, \ldots, \mathbf{M}_k\}$, provided by (12), for $1 \leq i \leq k$, by

$$\mathbf{M}'_i = \frac{\mathbf{M}_i}{\mathrm{Tr}(\mathbf{M}_i)}.$$

This algorithm converges to $\widehat{\mathbf{M}}_{\mathrm{FP}}$ up to a scaling factor which is $1/\mathrm{Tr}(\widehat{\mathbf{M}}_{\mathrm{FP}})$.

As a consequence of Theorem IV.1, we can prove a matrix inequality which is interesting on its own. It simply expresses that the Hessian computed at a critical point of F is nonpositive. We also provide an example showing that, in general, the Hessian is not definite negative. Therefore, in general, the convergence rate to the critical points of F for the dynamical systems $(S)_{\mathrm{dis}}$ and $(S)_{\mathrm{cont}}$ is not exponential.

Proposition IV.1: Let m, N be two positive integers with $m < N$ and $\mathbf{x}_1, \ldots, \mathbf{x}_N$ be unit vectors of $\mathbb{R}^m$ subject to H2) and such that

$$\frac{m}{N}\sum_{i=1}^{N}\mathbf{x}_i\mathbf{x}_i^\top = \mathbf{I}_m. \tag{15}$$

Then, for every matrix $\mathbf{M}$ of $M_m(\mathbb{R})$, we have

$$\frac{m}{N}\sum_{i=1}^{N}\left(\mathbf{x}_i^\top\mathbf{M}\mathbf{x}_i\right)^2 \leq \|\mathbf{M}\|^2. \tag{16}$$

Assuming Theorem IV.1, the proof of the proposition is short enough to be provided next.

We may assume $\mathbf{M}$ to be symmetric since it is enough to prove the result for $(\mathbf{M} + \mathbf{M}^T)/2$, the symmetric part of $\mathbf{M}$.

Applying Theorem IV.1, it is clear that the function F associated to the $\mathbf{x}_i$'s reaches its maximum over $\mathcal{D}$ at $\mathbf{I}_m$. The expression of $H_{\mathbf{I}_m}$, the Hessian of F at $\mathbf{I}_m$ is the following. For every symmetric matrix $\mathbf{M}$, we have

$$H_{\mathbf{I}_m}(\mathbf{M},\mathbf{M}) = NF(\mathbf{I}_m)\left(\frac{m}{N}\sum_{i=1}^{N}\left(\mathbf{x}_i^\top \mathbf{M}\mathbf{x}_i\right)^2 - \|\mathbf{M}\|^2\right).$$

Since $H_{\mathbf{I}_m}$ is nonpositive, (16) follows. Note that a similar formula can be given if, instead of (15), the $\mathbf{x}_i$'s verify the more general (11).

Because of the homogeneity properties of F and f and in order to prove that the rates of convergence of both $(S)_{\text{dis}}$ and $(S)_{\text{cont}}$ are not exponential, one must prove that the Hessian $H_{\mathbf{I}_m}$ is not negative definite on the orthogonal to $\mathbf{I}_m$ in the set of all symmetric matrices. The latter is simply the set of symmetric matrices with null trace. We next provide a numerical example describing that situation. Here, $m = 3$, $N = 4$, and

$$\mathbf{x}_1 = \begin{pmatrix} \frac{2\sqrt{2}}{3} \\ 0 \\ \frac{1}{3} \end{pmatrix} \quad \mathbf{x}_2 = \begin{pmatrix} -\frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{3} \end{pmatrix}$$

$$\mathbf{x}_3 = \begin{pmatrix} -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{3} \end{pmatrix} \quad \mathbf{x}_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Then, H1), H2), and (15) are satisfied. Moreover, it is easy to see that, for every diagonal matrix $\mathbf{D}$, we have equality in (16).

V. Proofs Outline

In this section, we give Theorems IV.1 and IV.2 proofs. Each proof is decomposed in a sequence of lemmas and propositions whose arguments are postponed in the Appendices I–VIII.

A. Proof of Theorem IV.1

Theorem conclusions are the consequences of several propositions whose statements are listed in the following.

First, it is clear that F is homogeneous of degree zero and f is homogeneous of degree one, i.e., for every $\lambda > 0$ and $\mathbf{M} \in \mathcal{D}$, one has

$$F(\lambda\mathbf{M}) = F(\mathbf{M}) \quad f(\lambda\mathbf{M}) = \lambda f(\mathbf{M}).$$

The first proposition is the following.

Proposition V.1: The supremum of F over $\mathcal{D}$ is finite and is reached at a point $\widehat{\mathbf{M}}_{\text{FP}} \in \mathcal{D}$ with $\|\widehat{\mathbf{M}}_{\text{FP}}\| = 1$. Therefore, f admits the open half-line $\mathcal{L}_{\widehat{\mathbf{M}}_{\text{FP}}}$ as FPs.

Proof: See Appendix II. ■

It remains to show that there are no other FPs of f except $\mathcal{L}_{\widehat{\mathbf{M}}_{\text{FP}}}$. For that purpose, one must study the function f. We first establish the following result.

Proposition V.2: The function f verifies the following properties.

P1) For every $\mathbf{M}, \mathbf{Q} \in \mathcal{D}$, if $\mathbf{M} \leq \mathbf{Q}$, then $f(\mathbf{M}) \leq f(\mathbf{Q})$ (also true with strict inequalities).

P2) For every $\mathbf{M}, \mathbf{Q} \in \mathcal{D}$, then

$$f(\mathbf{M}+\mathbf{Q}) \geq f(\mathbf{M}) + f(\mathbf{Q}) \tag{17}$$

and equality occurs if and only if $\mathbf{M}$ and $\mathbf{Q}$ are colinear.

Proof: See Appendix III. ■

The property of f described in Proposition V.3 turns out to be basic for the proofs of both theorems.

Proposition V.3: The function f is eventually strictly increasing, i.e., for every $\mathbf{Q}, \mathbf{P} \in \mathcal{D}$ such that $\mathbf{Q} \geq \mathbf{P}$ and $\mathbf{Q} \neq \mathbf{P}$, then $f^m(\mathbf{Q}) > f^m(\mathbf{P})$.

Proof: See Appendix IV. ■

We next proceed by establishing another property of f, which can be seen as an intermediary step towards the conclusion.

Recall that the *orbit* of f associated to $\mathbf{M} \in \mathcal{D}$ is the trajectory of $(S)_{\text{dis}}$ (12) starting at $\mathbf{M}$.

Proposition V.4: The following statements are equivalent:

1) f admits an FP;
2) f has one bounded orbit in $\mathcal{D}$;
3) every orbit of f is bounded in $\mathcal{D}$.

Proof: See Appendix V. ■

From Proposition V.1, f admits an FP. Thus, Proposition V.4 ensures that every orbit of f is bounded in $\mathcal{D}$.

Finally, using Proposition V.3, we get Corollary V.1, which concludes the proof of Theorem IV.1.

Corollary V.1: Assume that every orbit of f is bounded in $\mathcal{D}$. The following holds true.

C1) Let $\mathbf{P} \in \mathcal{D}$ and $n \geq 1$ such that $\mathbf{P}$ can be compared with $f^n(\mathbf{P})$, i.e., $\mathbf{P} \geq f^n(\mathbf{P})$ or $\mathbf{P} \leq f^n(\mathbf{P})$. Then, $\mathbf{P} = f^n(\mathbf{P})$. In particular, if $\mathbf{P} \geq f(\mathbf{P})$ or $\mathbf{P} \leq f(\mathbf{P})$, then $\mathbf{P}$ is an FP of f.

C2) All the FPs of f are colinear.

Proof: See Appendix VI. ■

To summarize, Proposition V.1 establishes the existence of an FP while Corollary V.1 ensures the uniqueness of the unit norm FP.

B. Proof of Theorem IV.2

1) Convergence Results for $(S)_{\text{dis}}$: In Section V-A, we already proved several important facts relative to the trajectories of $(S)_{\text{dis}}$ defined by (12), i.e., the orbits of f. Indeed, since f has FPs, then all the orbits of f are bounded in $\mathcal{D}$. It remains to show now that each of them is convergent to an FP of f.

For this purpose, we consider, for every $\mathbf{M} \in \mathcal{D}$, the positive limit set $\omega(\mathbf{M})$ associated to $\mathbf{M}$, i.e., the set made of the cluster points of the sequence $(\mathbf{M}_k)_{k\geq 0}$, where $\mathbf{M}_{k+1} = f(\mathbf{M}_k)$ with $\mathbf{M}_0 = \mathbf{M}$. Since the orbit of f associated to $\mathbf{M}$ is bounded in $\mathcal{D}$, the set $\omega(\mathbf{M})$ is a compact of $\mathcal{D}$ and is invariant by f, for every $\mathbf{P} \in \omega(\mathbf{M})$, $f(\mathbf{P}) \in \omega(\mathbf{M})$. It is clear that the sequence $(\mathbf{M}_k)_{k\geq 0}$ converges if and only if $\omega(\mathbf{M})$ reduces to a single point.

The last part of the proof is divided into Lemmas V.1 and V.2.

Lemma V.1: For every $\mathbf{M} \in \mathcal{D}$, $\omega(\mathbf{M})$ contains a periodic orbit of f (i.e., contain a finite number of points).

Proof: See Appendix VII. ■

Lemma V.2: Let $\mathbf{M}_1$ and $\mathbf{M}_2 \in \mathcal{D}$ be such that their respective orbits are periodic. Then, $\mathbf{M}_1$ and $\mathbf{M}_2$ are colinear and are both FPs of f.

Proof: See Appendix VIII. ■

We now complete the proof of Theorem IV.2 in the discrete case.

Let $\mathbf{M} \in \mathcal{D}$. Using both lemmas, it is easy to deduce that $\omega(\mathbf{M})$ contains an FP of f, which will be denoted by $\mathbf{Q}$. Notice that there exists a compact $\mathcal{K}$ containing both the orbit of f associated to $\mathbf{M}$ and $\omega(\mathbf{M})$. We next prove that, for every $\epsilon > 0$, there exists a positive integer $n_\epsilon > 0$ such that

$$(1-\varepsilon)\mathbf{Q} \leq f^{n_\varepsilon}(\mathbf{M}) \leq (1+\varepsilon)\mathbf{Q}. \tag{18}$$

Indeed, since $\mathbf{Q} \in \omega(\mathbf{M})$, for every $\epsilon > 0$, there exists a positive integer $n_\epsilon > 0$ such that

$$||f^{n_\varepsilon}(\mathbf{M}) - \mathbf{Q}|| \leq \varepsilon.$$

After standard computations, one can see that there exists a constant $K > 0$, only depending on the compact $\mathcal{K}$, such that, for $\epsilon > 0$ small enough

$$(1-K\varepsilon)\mathbf{Q} \leq f^{n_\varepsilon}(\mathbf{M}) \leq (1+K\varepsilon)\mathbf{Q}.$$

The previous inequality implies at once (18).

Applying f^l, $l \geq 0$, to (18), and taking into account that $\mathbf{Q}$ is an FP of f, one deduces that

$$(1-\varepsilon)\mathbf{Q} \leq f^{l+n_\varepsilon}(\mathbf{M}) \leq (1+\varepsilon)\mathbf{Q}.$$

This is nothing else but the definition of the convergence of the sequence $(f^l(\mathbf{M}))_{l\geq 0}$ to $\mathbf{Q}$. ■

2) Convergence Results for $(S)_{\text{cont}}$*:* Let $t \to \mathbf{M}(t)$, $t \geq 0$, be a trajectory of $(S)_{\text{cont}}$ with initial condition $\mathbf{M}_0 \in \mathcal{D}$.

Thanks to (B.27) which appears in the proof of Proposition V.1 in Appendix II, we have for every trajectory $\mathbf{M}(t)$ of $(S)_{\text{cont}}$

$$\frac{\mathrm{d}}{\mathrm{d}t}||\mathbf{M}||^2 = 2\mathrm{Tr}(\mathbf{M}\dot{\mathbf{M}}) = 2\mathrm{Tr}\,(\nabla F(\mathbf{M}).\mathbf{M}) = 0.$$

Then, for every $t \geq 0$, $\mathbf{M}(t)$ keeps a constant norm equal to $||\mathbf{M}_0||$. Moreover, one has for every $t \geq 0$

$$F(\mathbf{M}(t)) - F(\mathbf{M}(0)) = \int_0^t \frac{\mathrm{d}}{\mathrm{d}t}F(\mathbf{M}) = \int_0^t ||\nabla F(\mathbf{M})||^2 > 0.$$

Since F is bounded over $\mathcal{D}(||\mathbf{M}_0||)$, we deduce that

$$\int_0^{+\infty} ||\nabla F(\mathbf{M})||^2 < +\infty. \tag{19}$$

In addition, since $t \to F(\mathbf{M}(t))$ is an increasing function, then $\mathbf{M}(t)$ remains in a compact subset $\mathcal{K}$ of $\mathcal{D}(||\mathbf{M}_0||)$, which is independent of the time t. As $\mathcal{D}(||\mathbf{M}_0||)$ contains a unique equilibrium point of $(S)_{\text{cont}}$, we proceed by proving Theorem IV.2 in the continuous case

$$\forall \mathbf{M}_0 \in \mathcal{D}, \mathbf{M}(t) \xrightarrow[t\to+\infty]{} ||\mathbf{M}_0||\widehat{\mathbf{M}}_{\text{FP}}. \tag{20}$$

Without loss of generality, we assume that $||\mathbf{M}_0|| = 1$. Let F_0 be the limit of $F(\mathbf{M}(t))$ as t tends to $+\infty$. Thanks to Theorem IV.1 and the fact that $||\mathbf{M}(t)||$ is constant, it is easy to see that (20) follows if one can show that $F_0 = F(\widehat{\mathbf{M}}_{\text{FP}})$. We assume the contrary and will reach a contradiction.

Indeed, if we assume that $F_0 < F(\widehat{\mathbf{M}}_{\text{FP}})$, then there exists ϵ_0 such that $||\mathbf{M}(t) - \widehat{\mathbf{M}}_{\text{FP}}|| \geq \epsilon_0$, for every $t \geq 0$. This implies together with the fact that $\widehat{\mathbf{M}}_{\text{FP}}$ is the unique FP of f in $\mathcal{D}(1)$ and $||\nabla F(\mathbf{M})||$ is continuous and that there exists C_0 such that $||\nabla F(\mathbf{M})|| \geq C_0$, for every $t \geq 0$. Then, $\int_{t_0}^{+\infty} ||\nabla F(\mathbf{M})||^2 = +\infty$, which contradicts (19). Therefore, (20) holds true. ■

VI. Simulations

The main purpose of this section is to give some tools for computing the FP estimate regardless of its statistical properties; in particular, we investigate the numerical accuracy and the algorithm convergence in different contexts for the complex case.

The following two algorithms presented in Section IV will be compared:

- the discrete case algorithm of Theorem IV.2, called Algorithm 1 in the sequel, defined by (12) and whose convergence to the FP estimate has been proved in Section V;
- the normalized algorithm, called Algorithm 2 in the sequel, defined by (14).

The first purpose of simulations is to compare the two algorithms in order to choose the best one in terms of convergence speed.

Second, we study the parameters influence in the retained algorithm: the order m of matrix $\mathbf{M}$, the number N of reference data $(\mathbf{c}_1, \ldots, \mathbf{c}_N)$, and the algorithm starting point. Note that the distribution of the $\mathbf{c}_i$'s has no influence on the simulations because of the independence of (3) (which completely defines the FP estimate) with respect to the distribution of the τ_i's. Thus, without loss of generality, the Gaussian distribution will be used in the sequel.

Convergence will be analyzed by evaluating the widely used criterion C

$$C(k) = \frac{||\widehat{\mathbf{M}}_{k+1} - \widehat{\mathbf{M}}_k||}{||\widehat{\mathbf{M}}_k||} \tag{21}$$

as a function of algorithm iteration k. The numerical limit of C (when algorithm has converged) is called the floor level.

Section VI-A compares Algorithms 1 and 2 while Section VI-B studies parameters influence.

A. Comparison of the Two Algorithms

This section is devoted to the comparison of Algorithm 1 and 2 for Toeplitz matrices which are met when the processes are stationary. We will use the set of Toeplitz matrices $\mathbf{M}$ defined by the following widely used structure:

$$M_{ij} = \rho^{|i-j|} \tag{22}$$

for $1 \leq i, j \leq m$, and for $0 < \rho < 1$. Notice that the covariance matrix $\mathbf{M}$ is fully defined by the parameter ρ, which characterizes the correlation of the data.

1) Convergence Behavior for Different Values of ρ*:* Fig. 1 displays the criterion $C(k)$ versus the iterations number k for the following set of parameters: $m = 10$, $N = 20$, and the starting point $\mathbf{M}_0 = \mathbf{I}_m$. Three typical cases are investigated: weak correlation [$\rho = 10^{-5}$, Fig. 1(a)], medium correlation

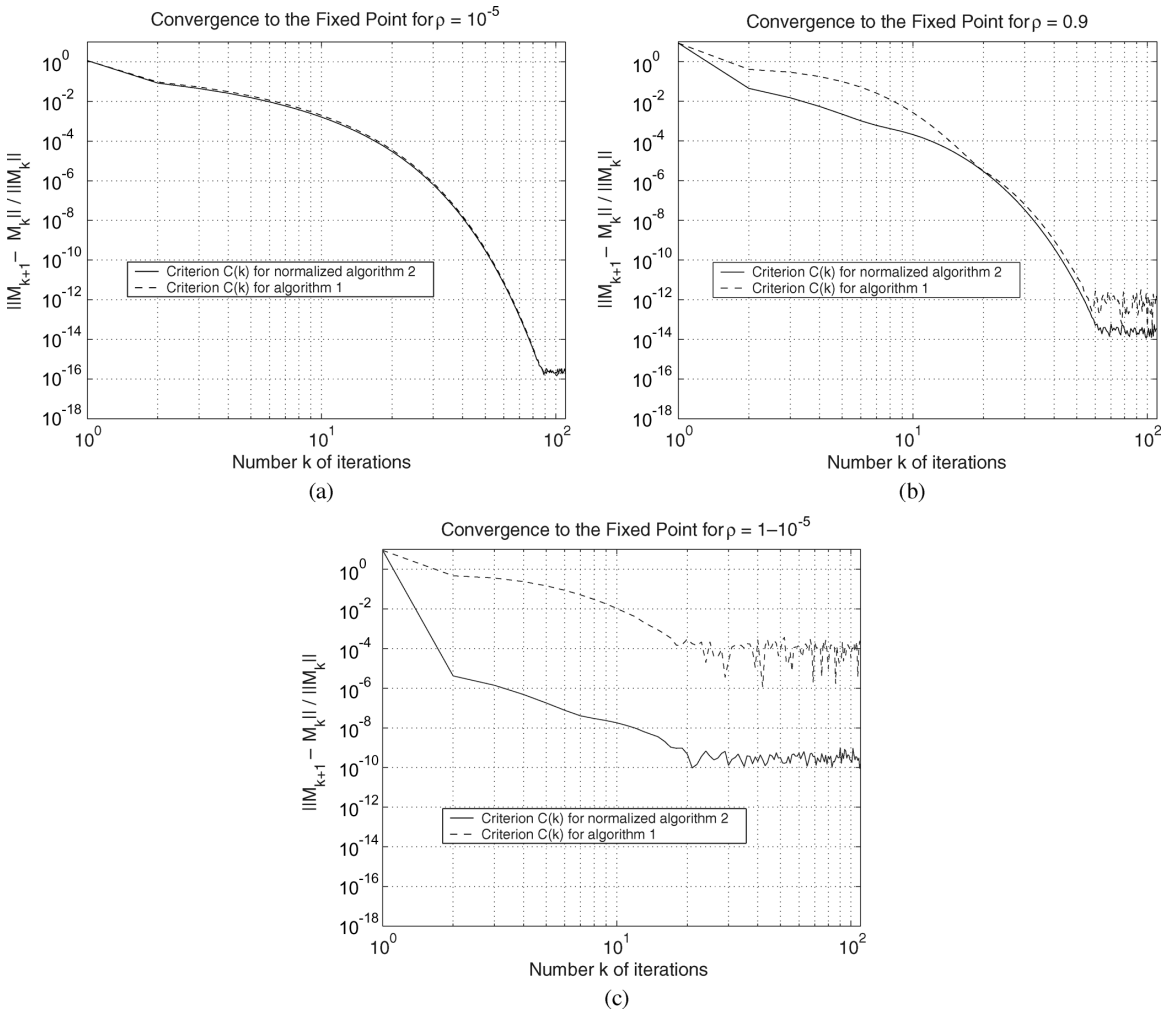

Fig. 1. Convergence to the FP for three different ρ: (a) $\rho = 10^{-5}$, (b) $\rho = 0.9$, and (c) $\rho = 1 - 10^{-5}$.

[$\rho = 0.9$, Fig. 1(b)] and strong correlation [$\rho = 1 - 10^{-5}$, Fig. 1(c)].

Fig. 1 leads to four main comments.

- For a given ρ, both algorithms' numerical convergence occurs for the same iteration number. Moreover, Algorithm 2 always presents a better accuracy (in terms of floor level).
- Higher the ρ is, faster the convergence is; for $\rho = 10^{-5}$, convergence is reached around 90 iterations, for $\rho = 0.9$, 60 iterations are enough, and for $\rho = 1 - 10^{-5}$, only 20 iterations are required.
- Stronger the correlation is, lower the limit accuracy becomes.
- The improvement of Algorithm 2 in term of accuracy increases with ρ.

With this first analysis, we infer that Algorithm 2 is better than Algorithm 1.

In Fig. 2, we have plotted the criterion C versus ρ when the convergence has occurred. Floor level is evaluated at the 150th iteration. Both algorithms exhibit the same behavior: the floor level gets worth when correlation parameter ρ increases. Floor level is always better for the normalized algorithm than for the Algorithm 1. Moreover, the distance between the two curves increases with ρ.

Fig. 3 shows the required iteration number k to achieve a relative error C equal to 10^{-5}. Plots are given as a function of correlation parameter ρ. Algorithm 1 is quite insensitive to the correlation parameter influence. The number of iteration k is always close to 21. Conversely, for Algorithm 2, the iteration number k decreases with ρ, starting at $k = 20$ for small ρ and ending at $k = 8$ for ρ close to 1. Surprisingly, the more data are correlated, faster the convergence is [but according to Fig. 1(c), the floor level gets worse].

These results allow to conclude that Algorithm 2 (normalized algorithm) is the best in all situations. That is why, in the sequel, we will study parameters influence on the normalized algorithm.

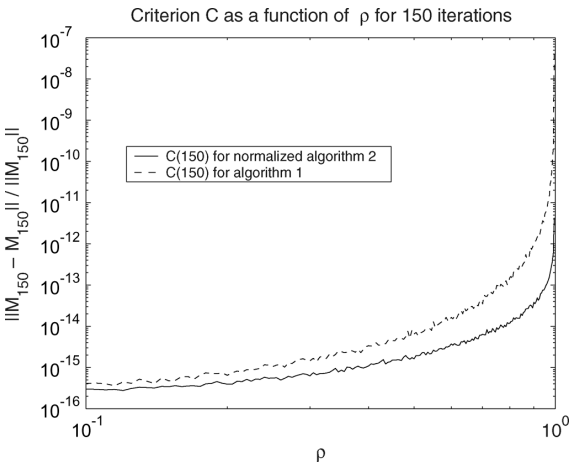

Fig. 2. Floor level $C(150)$ against ρ.

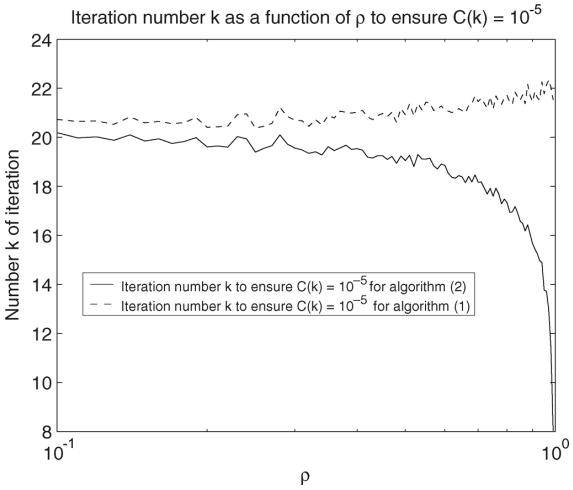

Fig. 3. Required iteration number k to achieve the relative error $C = 10^{-5}$.

B. Parameters Influence

This section studies the influence on the normalized algorithm of the starting point $\mathbf{M}_0$ and the number N of reference data.

Fig. 4(a) shows the criterion $C(k)$ for four different initial conditions $\mathbf{M}_0$ and a medium correlation parameter $\rho = 0.9$: the well-known sample covariance matrix estimate (SCME), the true covariance matrix $\mathbf{M}$, a random matrix whose elements are uniformly distributed, and the identity matrix $\mathbf{I}_m$. Floor level and convergence speed are independent of the algorithm initialization; after ten iterations, all the curves merge. Fig. 4(b) represents $C(k)$ for various values of N: 20, 200, 2000, and 4000. Notice that convergence speed increases with N, while the floor level is almost independent of N.

VII. Conclusion

In this paper, we have considered the problem of covariance matrix estimation for adaptive radar detection in compound-Gaussian clutter. The corresponding ML estimate of the covariance matrix built with secondary data is known to be the solution (if such a solution exists and is unique) of an equation for which no closed-form solution is available. We have established in this paper a sound demonstration of the existence and uniqueness of this ML estimate, called fixed point estimator (FPE). We have also derived two algorithms for obtaining the FPE. The convergence of each algorithm has been theoretically proved and emphasized by extensive simulations which have shown the superiority of one of them, the so-called normalized algorithm. The numerical behavior of the two algorithms in realistic scenario has been also investigated as a function of main parameters, correlation and number of reference data, highlighting their fast convergence and, therefore, their great practical interests. These important results will allow the use of the FPE in real radar detection scheme [15]. It remains now to analyze the statistical behavior of the FPE; the preliminary results in that direction have been already obtained in [16].

Appendix I
Reduction of the Complex Case to the Real Case

Let $\mathcal{G}$ be the set of $m \times m$ definite–positive Hermitian matrices and $\mathcal{S}$ the set of 2 m $\times$ 2 m symmetric matrices. Let us define the function g by

$$g : \mathcal{G} \longrightarrow \widetilde{\mathcal{D}} = g(\mathcal{G}) \subset \mathcal{S}$$
$$\mathbf{M} \longrightarrow g(\mathbf{M}) = \begin{pmatrix} \mathbf{M}^{(1)} & -\mathbf{M}^{(2)} \\ \mathbf{M}^{(2)} & \mathbf{M}^{(1)} \end{pmatrix}$$

where $\mathbf{M} = \mathbf{M}^{(1)} + j\mathbf{M}^{(2)}$ with $\mathbf{M}^{(1)}$, symmetric matrix, the real part of $\mathbf{M}$ and $\mathbf{M}^{(2)}$, antisymmetric matrix, the imaginary part. It is obvious that g is a bijection between $\mathcal{G}$ and $\widetilde{\mathcal{D}}$. Moreover, we have Proposition A.1.

Proposition A.1:

$$\forall \mathbf{M} \in \mathcal{G}, g\left(f(\mathbf{M})\right) = f_R\left(g(\mathbf{M})\right)$$

where f is given by (7) and f_R by

$$f_R(\mathbf{M}_r) = \frac{m}{N} \sum_{i=1}^{2N} \frac{\mathbf{w}_i \mathbf{w}_i^\top}{\mathbf{w}_i^\top \mathbf{M}_r^{-1} \mathbf{w}_i}$$

with $\mathbf{M}_r \in \widetilde{\mathcal{D}}$ and the $2m$-vectors $\mathbf{w}_1, \ldots, \mathbf{w}_{2N}$ are defined as follows:

- for the N first vectors $\mathbf{w}_1, \ldots, \mathbf{w}_N$ (called $\mathbf{u}_i$ for clarity), $\mathbf{w}_i = \mathbf{u}_i = \binom{\mathbf{x}_i^{(1)}}{\mathbf{x}_i^{(2)}}$;
- for the N last vectors $\mathbf{w}_{N+1}, \ldots, \mathbf{w}_{2N}$ (called $\mathbf{v}_i$), $\mathbf{w}_{N+i} = \mathbf{v}_i = \binom{-\mathbf{x}_i^{(2)}}{\mathbf{x}_i^{(1)}}$.

Proof: We have

$$g\left(f(\mathbf{M})\right) = \frac{m}{N} \sum_{i=1}^{N} \frac{g\left(\mathbf{x}_i \mathbf{x}_i^H\right)}{\mathbf{x}_i^H \mathbf{M}^{-1} \mathbf{x}_i}.$$

Thanks to the following results: $g(\mathbf{M}^{-1}) = g(\mathbf{M})^{-1}$, $g(\mathbf{x}_i \mathbf{x}_i^H) = \mathbf{u}_i \mathbf{u}_i^\top + \mathbf{v}_i \mathbf{v}_i^\top$, and $\mathbf{x}_i^H \mathbf{M}^{-1} \mathbf{x}_i = \mathbf{u}_i^\top g(\mathbf{M}^{-1}) \mathbf{u}_i = \mathbf{v}_i^\top g(\mathbf{M}^{-1}) \mathbf{v}_i$, Proposition A.1 follows straightforwardly. ■

Hypothesis H) of Section III implies hypothesis H2) of linear independence for the real problem just defined in $\mathbb{R}^{2m}$. Thanks

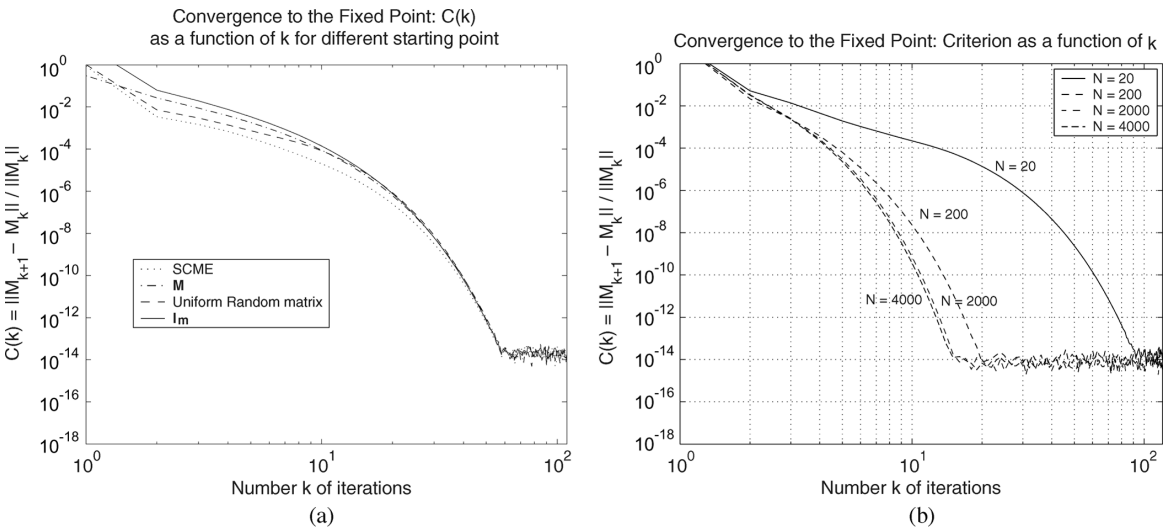

Fig. 4. Convergence to the FP. (a) $C(k)$ as a function of k for different starting points $\mathbf{M}_0$. (b) $C(k)$ as a function of k for various values of N: 20, 200, 2000, and 4000. (a) Error influence with points. (b) Error influence with the number N of secondary data.

to Theorem IV.1, there exists a unique FP $\mathbf{M}_R^{\mathrm{FP}}$ (up to scalar factor) in $\mathcal{S}$. Thus, it remains to show that $\mathbf{M}_R^{\mathrm{FP}}$ belongs to $\widetilde{D}$. Thanks to Proposition A.1, if initialization of algorithm defined in Theorem IV.2, (12) belongs to $\widetilde{D}$, the resulting sequence $\{\mathbf{M}_0, \ldots, \mathbf{M}_k\}$ obviously belongs to $\widetilde{D}$. Since this sequence converges in $\mathcal{S}$, by elementary topological considerations, the limit belongs to $\widetilde{D}$.

Now, since f_R admits a unique FP $\mathbf{M}_R^{\mathrm{FP}}$ (up to a scalar factor) in $\widetilde{D}$, the proof of Theorem III.1 is completed. Indeed, there exists a unique matrix $\mathbf{M}^{\mathrm{FP}}$ (up to a scalar factor) which verifies

$$\begin{aligned}\mathbf{M}^{\mathrm{FP}} &= g^{-1}\left(\mathbf{M}_R^{\mathrm{FP}}\right) = g^{-1}\left(f_R\left(\mathbf{M}_R^{\mathrm{FP}}\right)\right)\\ &= g^{-1}\left(f_R\left(g(\mathbf{M}^{\mathrm{FP}})\right)\right) = g^{-1}\left(g\left(f(\mathbf{M}^{\mathrm{FP}})\right)\right)\\ &= f(\mathbf{M}^{\mathrm{FP}}).\end{aligned}$$

Appendix II
Proof of Proposition V.1

If such a $\widehat{\mathbf{M}}_{\mathrm{FP}}$ exists, then for every $\lambda > 0$, $\lambda\widehat{\mathbf{M}}_{\mathrm{FP}}$ is also an FP of f, since f is homogeneous of degree one. We start by demonstrating Lemma B.1.

Lemma B.1: The function F can be extended as a continuous function of $\overline{\mathcal{D}}\backslash\{\mathbf{0}\}$ so that, for every noninvertible $\mathbf{M} \in \overline{\mathcal{D}}\backslash\{\mathbf{0}\}$, $F(\mathbf{M}) = \mathbf{0}$.

Proof: It is enough to show that, for every noninvertible $\mathbf{M} \in \overline{\mathcal{D}} \setminus \{\mathbf{0}\}$ and every sequence $(\mathbf{Q}^{(k)})_{k\geq 0}$ in $\overline{\mathcal{D}}$ converging to zero and so that $\mathbf{M} + \mathbf{Q}^{(k)}$ is invertible, we have

$$\forall \mathbf{Q} \in \mathcal{D}, \lim_{k\to\infty} F\left(\mathbf{M} + \mathbf{Q}^{(k)}\right) = 0.$$

Since F is smooth, we may assume that $\mathbf{Q}^{(k)} \in \mathcal{D}$ for every $k \geq 0$. We introduce the notation $F^{\mathbf{c}}$ for the function F in order to emphasize the dependence of F with respect to the N-tuple $\mathbf{c} = (\mathbf{c}_1, \ldots, \mathbf{c}_N)$. If $\mathbf{R}$ is an invertible matrix, let $\mathbf{R.c}$ be the N-tuple $\mathbf{R.c} := (\mathbf{R}\mathbf{c}_1, \ldots, \mathbf{R}\mathbf{c}_N)$. Clearly, one has for every $\mathbf{M} \in \mathcal{D}$

$$F^{\mathbf{c}}(\mathbf{M}) = |\mathbf{R}|^{2N} F^{\mathbf{R.c}}(\mathbf{R}\mathbf{M}\mathbf{R}^T).$$

Fix now a symmetric matrix $\mathbf{M}$ such that $\mathbf{M} \geq 0$ and the rank of $\mathbf{M}$, $\mathrm{rk}(\mathbf{M})$, is equal to l with $0 < l < m$. Thanks to the previous equation, we may assume that $\mathbf{M} = \mathbf{J}_l$, with $\mathbf{J}_l := \mathrm{diag}(\mathbf{I}_l\mathbf{0}\cdots\mathbf{0})$, where $\mathbf{0}$ is repeated $m-l$ times. For $i \in [1, N]$, we write $\mathbf{c}_i$ as

$$\mathbf{c}_i = \begin{pmatrix}\mathbf{c}_i^1\\ \mathbf{c}_i^2\end{pmatrix}, \quad \text{with} \quad \mathbf{c}_i^1 \in \mathrm{Im}(\mathbf{J}_l) \text{ and } \mathbf{c}_i^2 \in \mathrm{Ker}(\mathbf{J}_l).$$

According to that orthogonal decomposition, we write $\mathbf{Q}^{(k)}$ by blocks

$$\mathbf{Q}^{(k)} = \begin{pmatrix}\mathbf{Q}_1^{(k)} & \left(\mathbf{Q}_2^{(k)}\right)^T\\ \mathbf{Q}_2^{(k)} & \mathbf{Q}_3^{(k)}\end{pmatrix}.$$

Then

$$\mathbf{M} + \mathbf{Q}^{(k)} = \begin{pmatrix}\mathbf{I}_l + \mathbf{Q}_1^{(k)} & \left(\mathbf{Q}_2^{(k)}\right)^T\\ \mathbf{Q}_2^{(k)} & \mathbf{Q}_3^{(k)}\end{pmatrix}.$$

For every $k \geq 0$, set $\mathbf{P}_k := (\mathbf{I}_l + \mathbf{Q}_1^{(k)})^{-1}$ and $\mathbf{R}_k := \mathbf{Q}_3^{(k)} - \mathbf{Q}_2^{(k)}\mathbf{P}_k(\mathbf{Q}_2^{(k)})^T$. Then, for every $k \geq 0$, one has, after standard computations using the Schur complement formula (cf. [14], for instance), that

$$\begin{aligned}&\left(\mathbf{M} + \mathbf{Q}^{(k)}\right)^{-1}\\ &= \begin{pmatrix}\mathbf{P}_k + \mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\mathbf{Q}_2^{(k)}\mathbf{P}_k & -\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\\ -\mathbf{R}_k^{-1}\mathbf{Q}_2^{(k)}\mathbf{P}_k & \mathbf{R}_k^{-1}\end{pmatrix}\end{aligned}$$

and $|\mathbf{M}+\mathbf{Q}^{(k)}| = |\mathbf{I}_l+\mathbf{Q}_1^{(k)}||\mathbf{R}_k|$. We next compute $\mathbf{c}_i^T(\mathbf{M}+\mathbf{Q}^{(k)})^{-1}\mathbf{c}_i$ for $i \in [1,N]$ and $k \geq 0$. We get

$$\mathbf{c}_i^T\left(\mathbf{M}+\mathbf{Q}^{(k)}\right)^{-1}\mathbf{c}_i = \left(\mathbf{c}_i^1\right)^T\left(\mathbf{P}_k+\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\mathbf{Q}_2^{(k)}\mathbf{P}_k\right)$$
$$\mathbf{c}_i^1 - 2\left(\mathbf{c}_i^1\right)^T\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\mathbf{c}_i^2 + \left(\mathbf{c}_i^2\right)^T\mathbf{R}_k^{-1}\mathbf{c}_i^2. \quad \text{(B.23)}$$

Lemma B.2: With the previous notations, we have

$$\lim_{k\to\infty}\mathbf{P}_k + \mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\mathbf{Q}_2^{(k)}\mathbf{P}_k = \mathbf{I}_l$$

and, if $\mathbf{c}_i^2 \neq \mathbf{0}$, then

$$\lim_{k\to\infty}\frac{\mathbf{c}_i^T\left(\mathbf{M}+\mathbf{Q}^{(k)}\right)^{-1}\mathbf{c}_i}{\mathbf{c}_i^2\mathbf{R}_k^{-1}\mathbf{c}_i^2} = 1. \quad \text{(B.24)}$$

Proof: Both results are a consequence of the following fact:

$$\lim_{k\to\infty}\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\mathbf{Q}_2^{(k)}\mathbf{P}_k = 0. \quad \text{(B.25)}$$

To see that, first recall that $\mathbf{S}_k := \mathbf{Q}_3^{(k)} - \mathbf{Q}_2^{(k)}(\mathbf{Q}_1^{(k)})^{-1}(\mathbf{Q}_2^{(k)})^T$ is definite positive, since $\mathbf{Q}^{(k)}$ is positive definite. Next, we write

$$\mathbf{R}_k = \mathbf{S}_k + \mathbf{Q}_2^{(k)}\left(\mathbf{Q}_1^{(k)}\right)^{-1}\left(\mathbf{Q}_2^{(k)}\right)^T - \mathbf{Q}_2^{(k)}\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T$$
$$= \mathbf{Q}_2^{(k)}\left(\mathbf{Q}_1^{(k)}\right)^{-1}\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T$$

and we then have

$$\mathbf{P}_k\left(\mathbf{Q}_2^{(k)}\right)^T\mathbf{R}_k^{-1}\mathbf{Q}_2^{(k)}\mathbf{P}_k = \mathbf{P}_k^{1/2}\left(\mathbf{Q}_1^{(k)}\right)^{1/2}$$
$$\times\mathbf{B}_k^T\left(\mathbf{I}_l+\mathbf{B}_k\mathbf{B}_k^T\right)^{-1}\mathbf{B}_k\left(\mathbf{Q}_1^{(k)}\right)^{1/2}\mathbf{P}_k^{1/2}$$

where $\mathbf{B}_k := \mathbf{S}_k^{-1/2}\mathbf{Q}_2^{(k)}(\mathbf{Q}_1^{(k)})^{-1/2}\mathbf{P}_k^{1/2}$.

It is now clear that (B.25) holds true if the $l \times l$ symmetric nonnegative matrix $\mathbf{B}_k^T(\mathbf{I}_l+\mathbf{B}_k\mathbf{B}_k^T)^{-1}\mathbf{B}_k$ is bounded. Computing the norm, we end up with

$$\left\|\mathbf{B}_k^T\left(\mathbf{I}_l+\mathbf{B}_k\mathbf{B}_k^T\right)^{-1}\mathbf{B}_k\right\|^2 = \left\|(\mathbf{I}_l+\mathbf{T}_k)^{-1}\mathbf{T}_k\right\|^2$$

where $\mathbf{T}_k := \mathbf{B}_k\mathbf{B}_k^T \in \overline{\mathcal{D}}$. Since $(\mathbf{I}_l+\mathbf{T}_k)^{-1}\mathbf{T}_k \leq \mathbf{I}_l$, we conclude the proof of Lemma B.2. ■

We next consider the diagonalization of $\mathbf{R}_k$ in an orthonormal basis, given by

$$\mathbf{R}_k = \mathbf{U}_k^T\mathbf{D}_k\mathbf{U}_k, \qquad \text{for } k \geq 0$$

with $\mathbf{U}_k \in SO(m-l)$ and $\mathbf{D}_k = \mathrm{diag}(\epsilon_k^{(l+1)}, \cdots, \epsilon_k^{(m)})$. By definition, $\lim_{k\to\infty}\epsilon_k^{(j)} = 0^+$, for every $j \in [l+1, m]$, and, with no loss of generality, we will assume that $\epsilon_k^{(m)} = \min_{l+1\leq j\leq m}\epsilon_k^{(j)}$ and $\lim_{k\to\infty}\mathbf{U}_k = \mathbf{U} \in SO(m-l)$.

We next establish Lemma B.3.

Lemma B.3: Let $\mathbf{E}_m = (0\cdots01)^T$ with 0 repeated $m-l-1$ times. With the previous notations, there exist $C > 0$ and $i^* \in [1,N]$ such that, for $k \geq 0$ large enough, we have

$$\left|\mathbf{E}_m^T\mathbf{U}_k\mathbf{c}_{i^*}^2\right| \geq C. \quad \text{(B.26)}$$

Proof: By a continuity argument, it is enough to show the existence of an index i^* so that $\mathbf{E}_m^T\mathbf{U}\mathbf{c}_{i^*}^2 \neq 0$. Moreover, according to hypothesis H2), it is not possible to find m vectors $\mathbf{c}_{i(1)}, \cdots, \mathbf{c}_{i(m)}$ linearly independent such that

$$\mathbf{e}_m^T\overline{\mathbf{U}}\mathbf{c}_i = \mathbf{E}_m^T\mathbf{U}\mathbf{c}_i^2 = 0$$

where $\mathbf{e}_m = (0\cdots01)^T \in \mathbb{R}^m$ and $\overline{\mathbf{U}} = \mathrm{diag}(\mathbf{I}_l, \mathbf{U})$. (Otherwise, there exist m vectors $\mathbf{c}_{i(1)}, \cdots, \mathbf{c}_{i(m)}$ linearly independent belonging to the orthogonal of $\overline{\mathbf{U}}\mathbf{e}_m$, which has dimension $m-1$.)

By a simple counting argument, the index i^*, therefore, exists. Indeed, otherwise, the N vectors $\mathbf{c}_i$'s, with $i \notin S$ verify $\mathbf{e}_m^T\overline{\mathbf{U}}\mathbf{c}_i = 0$, meaning that all the vectors $\mathbf{c}_i$, $1 \leq i \leq N$, are orthogonal to $\overline{\mathbf{U}}^T\mathbf{e}_m$, which is impossible. The proof of Lemma B.3 is complete. ■

We can now finish the proof of Lemma B.1. Let $\mathbf{c}^*$ be the $(N-1)$-tuple made of the $\mathbf{c}_i$'s for $i \in [1,N]/\{i^*\}$. For every $k \geq 0$, we have

$$F^{\mathbf{c}}\left(\mathbf{M}+\mathbf{Q}^{(k)}\right) = \frac{1}{\left|\mathbf{M}+\mathbf{Q}^{(k)}\right|}\left(\frac{1}{\mathbf{c}_{i^*}^T\left(\mathbf{M}+\mathbf{Q}^{(k)}\right)^{-1}\mathbf{c}_{i^*}}\right)^m$$
$$\times F^{\mathbf{c}^*}\left(\mathbf{M}+\mathbf{Q}^{(k)}\right).$$

Since $N-1 \geq m$, we apply the result of [13] which states that the supremum of $F^{\mathbf{c}^*}$ over $\mathcal{D}$ is finite, i.e., there exists a positive constant C^* such that, for every $\mathbf{R} \in \mathcal{D}$, $F^{\mathbf{c}^*}(\mathbf{R}) \leq C^*$. Therefore, the conclusion holds true if

$$\lim_{k\to\infty}\frac{1}{\left|\mathbf{M}+\mathbf{Q}^{(k)}\right|}\left(\frac{1}{\mathbf{c}_{i^*}^T\left(\mathbf{M}+\mathbf{Q}^{(k)}\right)^{-1}\mathbf{c}_{i^*}}\right)^m = 0.$$

Thanks to (B.24), this amounts to show that

$$\lim_{k\to\infty}\frac{1}{|\mathbf{D}_k|}\left(\frac{1}{(\mathbf{c}_{i^*}^2)^T(\mathbf{R}_k)^{-1}\mathbf{c}_{i^*}^2}\right)^m = 0.$$

It is clear that $|\mathbf{D}_k| \geq (\epsilon_k^{(m)})^{m-l}$. In addition, by using Lemma B.3, we can write

$$\left(\mathbf{c}_{i^*}^2\right)^T(\mathbf{R}_k)^{-1}\mathbf{c}_{i^*}^2 = \left(\mathbf{U}_k\mathbf{c}_{i^*}^2\right)^T(\mathbf{D}_k)^{-1}\mathbf{U}_k\mathbf{c}_{i^*}^2$$
$$= \xi_k\frac{\left(\mathbf{E}_m^T\mathbf{U}_k\mathbf{c}_{i^*}^2\right)^2}{\varepsilon_k^{(m)}}$$

where ξ_k is bounded below and above by positive constants independent on k. We finally get that

$$\frac{1}{|\mathbf{D}_k|}\left(\frac{1}{(\mathbf{c}_{i^*}^2)^T(\mathbf{R}_k)^{-1}\mathbf{c}_{i^*}^2}\right)^m \leq C\left(\varepsilon_k^{(m)}\right)^l$$

with a positive constant C independent of k. By letting k go to infinity, we conclude the proof of Lemma B.1. ■

End of the Proof of Proposition V.1: Recall that $\overline{\mathcal{D}}(1)$ is a compact subset of $\overline{\mathcal{D}} \setminus \{\mathbf{0}\}$. Then, F is well defined on $\mathcal{D}(1)$ and is continuous. The application F reaches its maximum over $\mathcal{D}(1)$ at a point $\widehat{\mathbf{M}}_{\mathrm{FP}}$. Since F is strictly positive on $\mathcal{D}(1)$ and equal to zero on $\mathcal{D}(1) \setminus \mathcal{D}(1)$, then $F(\widehat{\mathbf{M}}_{\mathrm{FP}}) > 0$, implying that $\widehat{\mathbf{M}}_{\mathrm{FP}} \in \mathcal{D}(1)$. We complete the proof of Proposition V.1 by establishing Lemma B.4.

Lemma B.4: Let $\widehat{\mathbf{M}}_{\mathrm{FP}} \in \mathcal{D}(1)$ be defined as previously. Then, $\nabla F(\widehat{\mathbf{M}}_{\mathrm{FP}}) = \mathbf{0}$, which implies that $\widehat{\mathbf{M}}_{\mathrm{FP}}$ is an FP of f.

Proof: By definition of $\widehat{\mathbf{M}}_{\mathrm{FP}}$, one has $F(\widehat{\mathbf{M}}_{\mathrm{FP}}) = \max_{\mathbf{M}\in\mathcal{D}(1)} F(\mathbf{M})$. By standard calculus, it results in that $\nabla F(\widehat{\mathbf{M}}_{\mathrm{FP}})$ and $\nabla \mathcal{N}(\widehat{\mathbf{M}}_{\mathrm{FP}})$ are colinear, where $\mathcal{N}(\mathbf{M}) = \|\mathbf{M}\|^2$ for every $\mathbf{M} \in \mathcal{D}$. Since $\widehat{\mathbf{M}}_{\mathrm{FP}} \in \mathcal{D}$, there exists a real number μ such that $\nabla F(\widehat{\mathbf{M}}_{\mathrm{FP}}) = \mu \widehat{\mathbf{M}}_{\mathrm{FP}}$. Recall that, since F is homogeneous of degree zero, then

$$\forall \mathbf{M} \in \mathcal{D}, \qquad \nabla(\mathbf{M}).\mathbf{M} = 0. \tag{B.27}$$

One deduces that $\mu = \mu \|\widehat{\mathbf{M}}_{\mathrm{FP}}\|^2 = \nabla F(\widehat{\mathbf{M}}_{\mathrm{FP}}).\widehat{\mathbf{M}}_{\mathrm{FP}} = 0$. The proof of Lemma B.4 is complete. ■

Appendix III
Proof of Proposition V.2

We start by establishing P1). Let $\mathbf{M}$ and $\mathbf{Q} \in \mathcal{D}$ with $\mathbf{M} \leq \mathbf{Q}$. Then, $\mathbf{M}^{-1} \geq \mathbf{Q}^{-1}$ and, for every $1 \leq i \leq N$, we have

$$\frac{1}{\mathbf{c}_i^\top \mathbf{M}^{-1} \mathbf{c}_i} \leq \frac{1}{\mathbf{c}_i^\top \mathbf{Q}^{-1} \mathbf{c}_i}.$$

The reasoning for the case with strict inequalities is identical. Then, clearly, P1) follows.

We next turn to the proof of P2) . We first recall that, for every unit vector $\mathbf{c} \in \mathbb{R}^m$, $\|\mathbf{c}\| = 1$ and $\mathbf{M} \in \mathcal{D}$, then

$$\frac{1}{\mathbf{c}^\top \mathbf{M}^{-1} \mathbf{c}} = \inf_{\mathbf{z}^\top \mathbf{c} \neq 0} \frac{\mathbf{z}^\top \mathbf{M} \mathbf{z}}{(\mathbf{c}^\top \mathbf{z})^2} \tag{C.28}$$

and the infimum is reached only on the line generated by $\mathbf{M}^{-1}\mathbf{c}$.

Let $\mathbf{M}$ and $\mathbf{Q} \in \mathcal{D}$. Then, one has

$$\begin{aligned} f(\mathbf{M}+\mathbf{Q}) &= \frac{m}{N} \sum_{i=1}^{N} \min_{\mathbf{z}^\top \mathbf{c}_i \neq 0} \frac{\mathbf{z}^\top (\mathbf{M}+\mathbf{Q}) \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} \\ &= \frac{m}{N} \sum_{i=1}^{N} \min_{\mathbf{z}^\top \mathbf{c}_i \neq 0} \left(\frac{\mathbf{z}^\top \mathbf{M} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} + \frac{\mathbf{z}^\top \mathbf{Q} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} \right). \end{aligned}$$

More generally, the following holds true:

$$\min_{\mathbf{z}\in\mathcal{A}} \left(f_1(\mathbf{z}) + f_2(\mathbf{z})\right) \geq \min_{\mathbf{z}\in\mathcal{A}} f_1(\mathbf{z}) + \min_{\mathbf{z}\in\mathcal{A}} f_2(\mathbf{z}),$$

for every functions f_1 and f_2 and set $\mathcal{A}$ giving a sense to the previous inequality. Then, P2) clearly holds true. It remains to be studied when equality occurs in P2). That happens if and only if, for every $1 \leq i \leq N$, one has

$$\min_{\mathbf{z}^\top \mathbf{c}_i \neq 0} \left(\frac{\mathbf{z}^\top \mathbf{M} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} + \frac{\mathbf{z}^\top \mathbf{Q} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} \right) = \min_{\mathbf{z}^\top \mathbf{c}_i \neq 0} \frac{\mathbf{z}^\top \mathbf{M} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} + \min_{\mathbf{z}^\top \mathbf{c}_i \neq 0} \frac{\mathbf{z}^\top \mathbf{Q} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2}. \tag{C.29}$$

Let us first show that equality occurs in (C.29) if and only if there exists some $\mu_i > 0$ such that

$$\mathbf{M}^{-1}\mathbf{c}_i = \frac{1}{\mu_i} \mathbf{Q}^{-1} \mathbf{c}_i. \tag{C.30}$$

Indeed, for every vector $\mathbf{z} \in \mathbb{R}^m$ with $\mathbf{z}^\top \mathbf{c}_i \neq 0$, we have

$$\frac{\mathbf{z}^\top (\mathbf{M}+\mathbf{Q}) \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2} \geq \frac{1}{\mathbf{c}_i^\top \mathbf{M}^{-1} \mathbf{c}_i} + \frac{\mathbf{z}^\top \mathbf{Q} \mathbf{z}}{\left(\mathbf{c}_i^\top \mathbf{z}\right)^2}.$$

Choosing $\mathbf{z} = (\mathbf{M}+\mathbf{Q})^{-1}\mathbf{c}_i$ yields

$$\begin{aligned} \frac{1}{\mathbf{c}_i^\top \mathbf{M}^{-1} \mathbf{c}_i} + \frac{1}{\mathbf{c}_i^\top \mathbf{Q}^{-1} \mathbf{c}_i} &= \frac{1}{\mathbf{c}_i^\top (\mathbf{M}+\mathbf{Q})^{-1} \mathbf{c}_i} \\ &\geq \frac{1}{\mathbf{c}_i^\top \mathbf{M}^{-1} \mathbf{c}_i} \\ &\quad + \frac{\mathbf{c}_i^\top (\mathbf{M}+\mathbf{Q})^{-1} \mathbf{Q} (\mathbf{M}+\mathbf{Q})^{-1} \mathbf{c}_i}{\left(\mathbf{c}_i^\top (\mathbf{M}+\mathbf{Q})^{-1} \mathbf{c}_i\right)^2}. \end{aligned}$$

Therefore, the function of z given by $\mathbf{z}^\top \mathbf{Q} \mathbf{z} / (\mathbf{c}_i^\top \mathbf{z})^2$ reaches its minimum value $1/(\mathbf{c}_i^\top \mathbf{M}^{-1} \mathbf{c}_i)$ at $\mathbf{z} = (\mathbf{M}+\mathbf{Q})^{-1}\mathbf{c}_i$. Using (C.28), we get that $(\mathbf{M}+\mathbf{Q})^{-1}\mathbf{c}_i$ is colinear to $\mathbf{Q}^{-1}\mathbf{c}_i$. Exchanging $\mathbf{M}$ and $\mathbf{Q}$ and proceeding as previously yields that $(\mathbf{M}+\mathbf{Q})^{-1}\mathbf{c}_i$ is also colinear to $\mathbf{M}^{-1}\mathbf{c}_i$, which finally implies that $\mathbf{M}^{-1}\mathbf{c}_i$ and $\mathbf{Q}^{-1}\mathbf{c}_i$ are themselves colinear. (C.30) is proved.

To finish the proof, one must show that all the (μ_i)'s, $1 \leq i \leq N$, as defined in (C.30), are equal.

Set $\mathbf{D} := \mathrm{diag}(\mu_1, \ldots, \mu_m)$ for the first m indices of $[1, N]$. Since $(\mathbf{c}_1, \ldots, \mathbf{c}_m)$ is a basis of $\mathbb{R}^n$ and $\mathbf{M}^{-1} - \mathbf{D}^{-1}\mathbf{Q}^{-1}$ is equal to $\mathbf{0}$ on that basis, we deduce that $\mathbf{M}^{-1} = \mathbf{D}^{-1}\mathbf{Q}^{-1}$. Consider now another basis of $\mathbb{R}^m$ defined by $(\mathbf{c}_2, \ldots, \mathbf{c}_{m+1})$ and set $\widetilde{\mathbf{D}} = \mathrm{diag}(\mu_2, \ldots, \mu_{m+1})$. Reasoning as previously, we obtain that $\mathbf{M}^{-1} = \widetilde{\mathbf{D}}^{-1}\mathbf{Q}^{-1}$, which first implies that $\widetilde{\mathbf{D}} = \mathbf{D}$ and, second, that $\mu_1 = \mu_2, \mu_2 = \mu_3, \ldots, \mu_m = \mu_{m+1}$. Repeating that reasoning for any pair of m-tuples of distinct indices $(i_1, \cdots, i_m)$ of $[1, N]$, we get that, for every $i \in [1, N]$, $\mu_i = \mu$, yielding $\mathbf{D} = \mu \mathbf{I}_m$. ■

Appendix IV
Proof of Proposition V.3

We first establish the following fact. For every $\mathbf{Q}, \mathbf{P} \in \mathcal{D}$, we have

$$\text{If } \mathbf{Q} \geq \mathbf{P} \text{ and } f(\mathbf{Q}) = f(\mathbf{P}), \qquad \text{then } \mathbf{Q} = \mathbf{P}. \tag{D.31}$$

Indeed, it is clear that $\mathbf{Q} \geq \mathbf{P}$ implies that $\mathbf{P}^{-1} - \mathbf{Q}^{-1} \geq \mathbf{0}$. Therefore, for every $1 \leq i \leq N$, we have

$$\frac{1}{\mathbf{c}_i^\top \mathbf{Q}^{-1} \mathbf{c}_i} \geq \frac{1}{\mathbf{c}_i^\top \mathbf{P}^{-1} \mathbf{c}_i}.$$

Assuming $f(\mathbf{Q}) = f(\mathbf{P})$ implies that, for every $1 \leq i \leq N$, we have $\mathbf{c}_i^\top \mathbf{Q}^{-1} \mathbf{c}_i = \mathbf{c}_i^\top \mathbf{P}^{-1} \mathbf{c}_i$, i.e.,

$$\mathbf{c}_i^\top (\mathbf{P}^{-1} - \mathbf{Q}^{-1}) \mathbf{c}_i = 0.$$

Since $\mathbf{P}^{-1} - \mathbf{Q}^{-1} \geq \mathbf{0}$, the previous equality says that $(\mathbf{P}^{-1} - \mathbf{Q}^{-1})\mathbf{c}_i = \mathbf{0}$, for every $1 \leq i \leq N$. By H2), the claim (D.31) is proved.

We now turn to the proof of Proposition V.3. We consider $\mathbf{Q}$, $\mathbf{P} \in \mathcal{D}$, such that $\mathbf{Q} \geq \mathbf{P}$ and $\mathbf{Q} \neq \mathbf{P}$. From what precedes, we also have that $f(\mathbf{Q}) \geq f(\mathbf{P})$ and $f(\mathbf{Q}) \neq f(\mathbf{P})$. This implies the existence of an index $i_0 \in [1, N]$ such that

$$\xi_{i_0} := \frac{m}{N}\left(\frac{1}{\mathbf{c}_{i_0}^\top \mathbf{Q}^{-1}\mathbf{c}_{i_0}} - \frac{1}{\mathbf{c}_{i_0}^\top \mathbf{P}^{-1}\mathbf{c}_{i_0}}\right) > 0.$$

Up to a relabel, we may assume that $i_0 = 1$. We then have

$$f(\mathbf{Q}) \geq f(\mathbf{P}) + \xi_1 \mathbf{c}_1\mathbf{c}_1^\top. \tag{D.32}$$

Next, we will show by induction on the index $l \leq m$ that there exist l positive real numbers ξ_k, $1 \leq k \leq l$, so that

$$f^l(\mathbf{Q}) \geq f^l(\mathbf{P}) + \sum_{k=1}^{l} \xi_k \mathbf{c}_k\mathbf{c}_k^\top. \tag{D.33}$$

In (D.33), the vectors $(\mathbf{c}_k)_{1\leq k\leq l}$ only need to be 2×2 distinct among all the vectors $(\mathbf{c}_i)_{1\leq i\leq N}$. At each step of the induction, we will have the possibility to relabel the indices in $[l+1, N]$ in such a way to get (D.33). The induction starts for $l = 1$ and, in this case, (D.33) reduces to (D.32). Therefore, the induction is initialized. We then assume that (D.33) holds true for some index $l \leq m-1$ and proceed in showing the same for the index $l+1$. It is clear that it will be a consequence of Lemma D.1.

Lemma D.1: Let $1 \leq l \leq m-1$, $\mathbf{Q}, \mathbf{P} \in \mathcal{D}$, such that

$$\mathbf{Q} \geq \mathbf{P} + \sum_{k=1}^{l} \xi_k \mathbf{c}_k\mathbf{c}_k^\top, \qquad \xi_k > 0. \tag{D.34}$$

Then, there exists a vector of $\{\mathbf{c}_{l+1}, \ldots, \mathbf{c}_N\}$ (to be set equal to $\mathbf{c}_{l+1}$, up to a relabeling of $\{\mathbf{c}_{l+1}, \ldots, \mathbf{c}_N\}$) and a positive real number $\xi_{l+1} > 0$ such that

$$f(\mathbf{Q}) \geq f(\mathbf{P}) + \sum_{k=1}^{l+1} \xi_k \mathbf{c}_k\mathbf{c}_k^\top. \tag{D.35}$$

Proof: Using (D.34), we have for every $j \in [1, N]$

$$\begin{aligned}\frac{1}{\mathbf{c}_j^\top \mathbf{Q}^{-1}\mathbf{c}_j} &= \min_{\mathbf{z}^\top\mathbf{c}_j\neq 0} \frac{\mathbf{z}^\top\mathbf{Q}\mathbf{z}}{(\mathbf{z}^\top\mathbf{c}_j)^2} \\ &\geq \min_{\mathbf{z}^\top\mathbf{c}_j\neq 0}\left(\frac{\mathbf{z}^\top\mathbf{P}\mathbf{z}}{(\mathbf{z}^\top\mathbf{c}_j)^2} + \sum_{k=1}^{l}\xi_k\frac{(\mathbf{z}^\top\mathbf{c}_k)^2}{(\mathbf{z}^\top\mathbf{c}_j)^2}\right).\end{aligned} \tag{D.36}$$

Using the induction hypothesis, we also have for every $1 \leq j \leq l$ that

$$\frac{1}{\mathbf{c}_j^\top \mathbf{Q}^{-1}\mathbf{c}_j} \geq \frac{1}{\mathbf{c}_j^\top \mathbf{P}^{-1}\mathbf{c}_j} + \xi_j.$$

We next show the following claim.

C1) There exists two indices, one index $j_0 \in [l+1, N]$ and another one $k_0 \in [1, l]$, such that $\mathbf{c}_{k_0}^\top \mathbf{Q}^{-1}\mathbf{c}_{j_0} \neq 0$.

Claim C1) is a proved reasoning by contradiction. Therefore, let us assume that $\mathbf{c}_k^\top \mathbf{Q}^{-1}\mathbf{c}_j = 0$, for every $1 \leq k \leq l$ and $l+1 \leq j \leq N$. Since $l < m$ and the vectors $(\mathbf{Q}^{-1}\mathbf{c}_k)$, $1 \leq k \leq l$, generate a vector space $\mathbf{V}_l$ of dimension l, we deduce that, for every $j \in [l+1, N]$, $\mathbf{c}_j$ is orthogonal to $\mathbf{V}_l$ and, therefore, belongs to an $m-l$-dimensional vector space of $\mathbb{R}^m$. However, there are $N-l$ indices j verifying the previous fact. According to H2), these vectors $(\mathbf{c}_j)_{l+1\leq j\leq N}$ generate a vector space of dimension $\min(N-l, m)$ in $\mathbb{R}^m$. We finally get that $\min(N-l, m) \leq m-l$. This is impossible because $N > m$ and claim C1) is proved.

We now finish the proof of Lemma D.1. Choosing in (D.36) $\mathbf{z} = \mathbf{Q}^{-1}\mathbf{c}_{j_0}$, we get

$$\begin{aligned}\frac{1}{\mathbf{c}_{j_0}^\top \mathbf{Q}^{-1}\mathbf{c}_{j_0}} &\geq \frac{\left(\mathbf{c}_{j_0}^\top \mathbf{Q}^{-1}\right)\mathbf{P}\left(\mathbf{Q}^{-1}\mathbf{c}_{j_0}\right)}{\left(\mathbf{c}_{j_0}^\top \mathbf{Q}^{-1}\mathbf{c}_{j_0}\right)^2} + \xi_{k_0}\frac{\left(\mathbf{c}_{j_0}^\top \mathbf{Q}^{-1}\mathbf{c}_{k}\right)^2}{\left(\mathbf{c}_{j_0}^\top \mathbf{Q}^{-1}\mathbf{c}_{j_0}\right)^2} \\ &\geq \frac{1}{\mathbf{c}_{j_0}^\top \mathbf{P}^{-1}\mathbf{c}_{j_0}} + \xi_{j_0}\end{aligned}$$

with $\xi_{j_0} > 0$, thanks to claim C1). It is clear that $\mathbf{c}_{j_0}$ is the vector of $\{\mathbf{c}_{l+1}, \ldots, \mathbf{c}_N\}$ needed with ξ_{j_0} so that, up to relabeling, it yields (D.35). Proofs of Lemma D.1 and Proposition V.3 are now complete. ■

Appendix V
Proof of Proposition V.4

We first need to create a precise definition. An orbit $(\mathbf{M}_k)_{k\geq 0}$ is bounded in $\mathcal{D}$ if it is contained in a compact subset of $\mathcal{D}$, i.e., there exists $\mathbf{M}$, $\mathbf{P} \in \mathcal{D}$ such that, for every $k \geq 0$, $\mathbf{M} \leq \mathbf{M}_k \leq \mathbf{P}$.

We will show the following chain of implications $(A) \Rightarrow (B) \Rightarrow (C) \Rightarrow (A)$.

$\underline{(A) \Rightarrow (B)}$: Trivial (simply $\mathbf{M}_0 = \mathbf{P}$).

$\underline{(B) \Rightarrow (C)}$: Assume that f has a bounded orbit in $\mathcal{D}$, starting at $\mathbf{M}$. Then, there exists $\mu, \mu' > 0$ such that, for every $k \geq 0$, $\mu\mathbf{M} \leq \mathbf{M}_k \leq \mu'\mathbf{M}$, for every $k \geq 0$.

Let $\mathbf{Q}$ be an arbitrary matrix of $\mathcal{D}$. Then, there exists $\lambda, \lambda' > 0$ such that $\lambda\mathbf{M} \leq \mathbf{Q} \leq \lambda', \mathbf{M}$. Using the homogeneity of degree one of f, property P1), and the definition of an orbit of f, we get, after a trivial induction, that $\lambda\mu\mathbf{M} \leq \lambda\mathbf{M}_k \leq \mathbf{Q}_k \leq \lambda'\mathbf{M}_k \leq \lambda'\mu'\mathbf{M}$, for every $k \geq 0$. Then, the orbit associated to $\mathbf{Q}$ is bounded in $\mathcal{D}$.

$\underline{(C) \Rightarrow (A)}$: Consider an orbit $(\mathbf{M}_k)_{k\geq 0}$ of f starting at $\mathbf{M} \in \mathcal{D}$ and bounded in $\mathcal{D}$. It is then contained in a compact $\mathcal{K}$ of $\mathcal{D}$. For $l \geq 1$, set

$$\mathbf{Q}_l := \frac{1}{l}\sum_{i=1}^{l}\mathbf{M}_i.$$

Then, the sequence $(\mathbf{Q}_l)_{l\geq 1}$ is bounded in $\mathcal{D}$ because every point $\mathbf{Q}_l$ belongs to the convex hull of $\mathcal{K}$, which is itself a com-

pact subset of $\mathcal{D}$. For every $l \geq 1$, we have by using Proposition V.2 that

$$\begin{aligned} f(\mathbf{Q}_l) &= \frac{1}{l} f\left(\sum_{i=1}^{l} \mathbf{M}_i\right) \geq \frac{1}{l}\sum_{i=2}^{l+1} \mathbf{M}_i \\ &= \frac{1}{l}\left(\sum_{i=1}^{l+1} \mathbf{M}_i - \mathbf{M}_1\right) \\ &= \mathbf{Q}_l + \frac{\mathbf{M}_{l+1} - \mathbf{M}_1}{l}. \end{aligned}$$

Since $(\mathbf{Q}_l)_{l\geq 1}$ is bounded in $\mathcal{D}$, we have that, up to extracting a subsequence, the sequence $(\mathbf{Q}_l)$ converges to $\overline{\mathbf{Q}}$ with $\overline{\mathbf{Q}} \in \mathcal{D}$, as l tends to $+\infty$. From the last equation, it follows that $f(\overline{\mathbf{Q}}) \geq \overline{\mathbf{Q}}$.

We now consider the orbit of f starting at $\overline{\mathbf{Q}}$. It defines an increasing, bounded in $\mathcal{D}$ sequence. It is, therefore, converging in $\mathcal{D}$ to an FP of f. ■

Appendix VI
Proof of Corollary V.1

The proof of C1) goes by contradiction. Let $\mathbf{P} \in \mathcal{D}$ with $f^l(\mathbf{P}) \geq \mathbf{P}$ and $f^l(\mathbf{P}) \neq \mathbf{P}$ for some positive integer $l \geq 1$. According to Proposition V.3, we have

$$f^n\left(f^l(\mathbf{P})\right) > f^n(\mathbf{P}) \Leftrightarrow f^l\left(f^n(\mathbf{P})\right) > f^n(\mathbf{P}).$$

Set $\mathbf{Q} := f^n(\mathbf{P})$ and $g := f^l$. It is clear that g is a function from $\mathcal{D}$ to $\mathcal{D}$, homogeneous of degree one and it verifies properties P1) and P2) of Proposition V.2. We will show that the orbit of g associated to $\mathbf{Q}$ is not bounded, which will be the desired contradiction.

We have $g(\mathbf{Qq}) > \mathbf{Q}$ which is equivalent to $g(\mathbf{Q}) - \mathbf{Q}$ being positive definite. By a simple continuity argument, there exists $\epsilon_{\mathbf{Q}} > 0$ such that

$$\varepsilon_{\mathbf{Q}}\mathbf{Q} \leq f(\mathbf{Q}) - \mathbf{Q} \Leftrightarrow f(\mathbf{Q}) \geq (1+\varepsilon_{\mathbf{Q}})\mathbf{Q}.$$

By a trivial induction, we have $f^k(\mathbf{Q}) \geq (1+\epsilon_{\mathbf{Q}})^k\mathbf{Q}$, for every $k \geq 0$, with the right-hand side of the previous inequality tending to $+\infty$ as k tends to ∞. Therefore, the orbit of f associated to $\mathbf{M}$ is not bounded.

We now prove statement C2). Let $\widehat{\mathbf{M}}_{\mathrm{FP}}$ and $\mathbf{P}_2$ be two FPs of f. Applying P2), we have

$$f(\widehat{\mathbf{M}}_{\mathrm{FP}} + \mathbf{P}_2) \geq f(\widehat{\mathbf{M}}_{\mathrm{FP}}) + f(\mathbf{P}_2) = \widehat{\mathbf{M}}_{\mathrm{FP}} + \mathbf{P}_2.$$

According to C1), we have that $\widehat{\mathbf{M}}_{\mathrm{FP}} + \mathbf{P}_2$ is also an FP of f and, therefore, we have equality in P2). It implies that $\widehat{\mathbf{M}}_{\mathrm{FP}}$ and $\mathbf{P}_2$ are colinear. The proof of Corollary V.1 is complete and it concludes the argument of Theorem IV.1.

Appendix VII
Proof of Lemma V.1

The argument goes by contradiction. We thus assume that $\omega(\mathbf{M})$ does not contain any periodic orbit. Let $\mathcal{K}$ be a compact subset of $\mathcal{D}$ containing both orbits associated to $\mathbf{M}$ and $\omega(\mathbf{M})$.

Let $\mathbf{Q} \in \omega(\mathbf{M})$. Then, there exists a sequence $(f^{n_j}(\mathbf{M}))_{j\geq 0}$ converging to $\mathbf{Q}$, as j tends to $+\infty$ with $(n_j)_{j\geq 0}$ being a strictly increasing sequence of integers tending to $+\infty$.

Let $\epsilon \in (0,1)$ be small enough and $n_{j_0} \in \mathcal{N}$ such that $\|f^{n_{j_0}}(\mathbf{M}) - \mathbf{Q}\| \leq \epsilon$. It is easy to see that there exists a constant K only depending on $\mathcal{K}$ such that $(1-K\varepsilon)\mathbf{Q} \leq f^{n_{j_0}}(\mathbf{M}) \leq (1+K\varepsilon)\mathbf{Q}$. Using Proposition V.2, we have for every $p \geq 0$

$$(1-K\varepsilon)f^p(\mathbf{Q}) \leq f^{n_0+p}(\mathbf{M}) \leq (1+K\varepsilon)f^p(\mathbf{Q}). \quad \text{(G.37)}$$

Since $\mathbf{Q}$ is a cluster point for the orbit associated to $\mathbf{M}$, there exists $n_{j_1} \geq 0$ such that

$$\left(1-\frac{K\varepsilon}{4}\right)\mathbf{Q} \leq f^{n_1}(\mathbf{M}) \leq \left(1+\frac{K\varepsilon}{4}\right)\mathbf{Q}.$$

Using (G.37) and the previous equation, there exists p large enough such that

$$\left(1-\frac{K\varepsilon}{2}\right)\mathbf{Q} \leq f^p(\mathbf{Q}) \leq \left(1+\frac{K\varepsilon}{2}\right)\mathbf{Q}. \quad \text{(G.38)}$$

We set $\mathbf{Q}_0 := \mathbf{Q}$ and ε_0 "maximal" with respect to (G.38), i.e., ε_0 being the smallest positive real number so that $(1-\varepsilon_0)\mathbf{Q}_0 \leq f^p(\mathbf{Q}_0) \leq (1+\varepsilon_0)\mathbf{Q}_0$ holds true. Then, $\varepsilon_0 \leq (K\varepsilon/2)$ and one of the two previous inequalities is not strict, by maximality of ε_0. Moreover, $\varepsilon_0 > 0$. Indeed, if it were not the case, then $\mathbf{Q}_0$ and $f^p(\mathbf{Q}_0)$ would be comparable and, according to Corollary V.1, the orbit associated to $\mathbf{Q}_0$ would be periodic. We now consider the subset V of $\omega(\mathbf{M})$, made of the matrices $\mathbf{P}$ such that there exists $\varepsilon(\mathbf{P}) > 0$ such that

$$(1-\varepsilon(\mathbf{P}))\,\mathbf{P} \leq f^p(\mathbf{P}) \leq (1+\varepsilon(\mathbf{P}))\,\mathbf{P} \quad \text{(G.39)}$$

and $\varepsilon(\mathbf{P})$ is "maximal" with respect to (G.39).

We showed previously that V is not empty since $\mathbf{Q} \in V$. We next show that $\overline{\varepsilon} = \inf_{\mathbf{P}\in V} \varepsilon(\mathbf{P}) = 0$.

By definition of $\overline{\varepsilon}$, there exists two sequences $(\mathbf{Q}^{(j)})_{j\geq 0}$ and $(\epsilon(\mathbf{Q}^{(j)}))_{j\geq 0}$ such that $(\epsilon(\mathbf{Q}^{(j)}))_{j\geq 0}$ converges to $\overline{\varepsilon}$, as j tends to $+\infty$. Up to considering a subsequence in the compact $\omega(\mathbf{M})$, we may assume that $(\mathbf{Q}^{(j)})_{j\geq 0}$ converges to some $\overline{\mathbf{Q}} \in \omega(\mathbf{M})$. Passing to the limit in (G.39), we get

$$(1-\overline{\varepsilon})\overline{\mathbf{Q}} \leq f^p(\overline{\mathbf{Q}}) \leq (1+\overline{\varepsilon})\overline{\mathbf{Q}}. \quad \text{(G.40)}$$

If $\overline{\varepsilon} > 0$, then necessarily $\overline{\mathbf{Q}} \in V$ and $\overline{\varepsilon}$ is "maximal" with respect to (G.40). Since f is eventually strictly increasing, we get $(1-\overline{\varepsilon})f^n(\overline{\mathbf{Q}}) < f^p(f^n(\overline{\mathbf{Q}})) < (1+\overline{\varepsilon})f^n(\overline{\mathbf{Q}})$. Setting $\widetilde{\mathbf{Q}} := f^n(\overline{\mathbf{Q}})$, then $\widetilde{\mathbf{Q}}$ belongs to $\omega(\mathbf{M})$, since the latter is an invariant set with respect to f. Choosing $\widetilde{\varepsilon}$ "maximal" with respect to

$$(1-\widetilde{\varepsilon})\widetilde{\mathbf{Q}} \leq f^p(\widetilde{\mathbf{Q}}) \leq (1+\widetilde{\varepsilon})\widetilde{\mathbf{Q}}$$

we first have that $\widetilde{\varepsilon} > 0$ (otherwise, we would have a periodic orbit) and $\widetilde{\varepsilon} < \overline{\varepsilon}$. We finally proved that $\widetilde{\mathbf{Q}} \in V$ with $0 < \widetilde{\varepsilon} = \varepsilon(\widetilde{\mathbf{Q}}) < \overline{\varepsilon}$. This is a contradiction with the minimality of $\overline{\varepsilon}$. Therefore, $\overline{\varepsilon} = 0$, which implies that $\overline{\mathbf{Q}} = f^p(\overline{\mathbf{Q}})$, i.e., $\omega(\mathbf{M})$ contains a periodic orbit. Lemma V.2 is proved. ■

APPENDIX VIII
PROOF OF LEMMA V.2

Let $\mathbf{M}_1, \mathbf{M}_2 \in \mathcal{D}$ whose associated orbits are periodic, with respective (positive) periods l_1 and l_2.

We first show that $\mathbf{M}_1$ and $\mathbf{M}_2$ are colinear, which will imply that $l_1 = l_2$.

For $i = 1, 2$, the orbit associated to $\mathbf{M}_i$ is the set $\{\mathbf{M}_i, f(\mathbf{M}_i), \ldots, f^{l_i-1}(\mathbf{M}_i)\}$. Consider $\mathbf{M} := \mathbf{M}_1 + \mathbf{M}_2$ and $l := l_1 l_2$. Then, $f(\mathbf{M}) = f(\mathbf{M}_1 + \mathbf{M}_2) \geq f(\mathbf{M}_1) + f(\mathbf{M}_2)$ and, for every $k \geq 0$, we have

$$f^k(\mathbf{M}) \geq f^k(\mathbf{M}_1) + f^k(\mathbf{M}_2).$$

It implies that $f^l(\mathbf{M}) \geq f^l(\mathbf{M}_1) + f^l(\mathbf{M}_2) = \mathbf{M}_1 + \mathbf{M}_2 = \mathbf{M}$. By Corollary V.1, we get that $f^l(\mathbf{M}) = \mathbf{M}$. It implies that all the previous inequalities must be in fact equalities and, in particular, we have $f(\mathbf{M}) = f(\mathbf{M}_1) + f(\mathbf{M}_2)$. By P2), we deduce that $\mathbf{M}_1$ and $\mathbf{M}_2$ are colinear. It remains to be shown that a periodic orbit reduces to a single point.

Consider $\mathbf{M} \in \mathcal{D}$ such that

$$\begin{cases} l \geq 1, & f^l(\mathbf{M}) = \mathbf{M}, \\ (\text{if } l = 1, \text{ no condition}) & f^{l-1}(\mathbf{M}) \neq \mathbf{M}. \end{cases}$$

We have to prove that $l = 1$.

Since the orbit associated to every $f^j(\mathbf{M}), 0 \leq j \leq l$, is again $\omega(\mathbf{M})$ and thus finite, we deduce that $f^j(\mathbf{M})$ must be colinear to $\mathbf{M}$, according to what precedes. Then, for every $0 \leq j \leq l-1$, we have $f^j(\mathbf{M}) = \lambda_j \mathbf{M}$, for some $\lambda_j > 0$. Obviously, $\lambda_0 = \lambda_l = 1$. In particular, we have $f(\mathbf{M}) = \lambda_1 \mathbf{M}$, implying that, either $f(\mathbf{M}) \leq \mathbf{M}$ or $f(\mathbf{M}) \geq \mathbf{M}$. By C1) of Corollary V.1, we get that $\mathbf{M}$ is an FP of f. The proof of Lemma V.1 is complete. ■

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Annexe I

On a SIRV-CFAR Detector with Radar Experimentations in Impulsive Noise

ON A SIRV-CFAR DETECTOR WITH RADAR EXPERIMENTATIONS IN IMPULSIVE CLUTTER

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ABSTRACT

This paper deals with radar detection in impulsive clutter. Its aim is twofold. Firstly, assuming a Spherically Invariant Random Vectors (SIRV) model for the clutter, the corresponding unknown covariance matrix is estimated by a recently introduced algorithm [1, 2]. A statistical analysis (bias, consistency, asymptotic distribution) of this estimate will be summarized allowing us to give the GLRT properties: the SIRV-CFAR (Constant False Alarm Rate) property, i.e. texture-CFAR and Covariance Matrix-CFAR, and the relationship between the Probability of False Alarm (PFA) and the detection threshold.
Secondly, one of the main contributions of this paper is to give some results obtained with real non-Gaussian data. These results demonstrate the interest of the proposed detection scheme, and show an excellent correspondence between experimental and theoretical false alarm rates.

1. PROBLEM STATEMENT AND BACKGROUND

Non-Gaussian noise characterization has gained many interests since experimental radar clutter measurements, made by organizations like MIT [3], showed that these data are correctly described by non-Gaussian statistical models. One of the most tractable and elegant non-Gaussian model comes from the so-called *Spherically Invariant Random Vector* (SIRV) theory. A SIRV is the product of a Gaussian random process - called *speckle* - with the square root of a non-negative random variable - called *texture*. This model leads to many results [4, 5].

The basic problem of detecting a complex signal corrupted by an additive SIRV clutter $\mathbf{c}$ in a m-dimensional complex vector $\mathbf{y}$ can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1,\ldots,N \\ H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1,\ldots,N \end{cases} \tag{1}$$

where the $\mathbf{c}_i$'s are N signal-free independent measurements, traditionally called the secondary data, used to estimate the clutter covariance matrix.

Under hypothesis H_1, it is assumed that the observed data consists in the sum of a signal $\mathbf{s} = \alpha\,\mathbf{p}$ and clutter $\mathbf{c}$, where $\mathbf{p}$ is a perfectly known complex steering vector and α is the signal complex amplitude.

Let us now recap some SIRV theory results. A noise modelled as a SIRV is a non-homogeneous Gaussian process with random power. More precisely, a SIRV [6] is the product of the square root of a positive random variable τ (*texture*) and a m-dimensional independent complex Gaussian vector $\mathbf{x}$ (*speckle*) with zero mean covariance matrix $\mathbf{M} = \mathbb{E}(\mathbf{x}\mathbf{x}^H)$ with normalization $\mathrm{Tr}(\mathbf{M}) = m$, where H denotes the conjugate transpose operator

$$\mathbf{c} = \sqrt{\tau}\,\mathbf{x}. \tag{2}$$

The SIRV PDF expression is

$$p_m(\mathbf{c}) = \int_0^{+\infty} g_m(\mathbf{c},\tau)\,p(\tau)\,d\tau\,, \tag{3}$$

where

$$g_m(\mathbf{c},\tau) = \frac{1}{(\pi\,\tau)^m\,|\mathbf{M}|}\exp\left(-\frac{\mathbf{c}^H\,\mathbf{M}^{-1}\,\mathbf{c}}{\tau}\right). \tag{4}$$

This model allowed to build several Generalized Likelihood Ratio Tests like the GLRT-Linear Quadratic (GLRT-LQ) in [4, 5] defined by

$$\Lambda(\mathbf{M}) = \frac{|\mathbf{p}^H\mathbf{M}^{-1}\mathbf{y}|^2}{(\mathbf{p}^H\mathbf{M}^{-1}\mathbf{p})(\mathbf{y}^H\mathbf{M}^{-1}\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,, \tag{5}$$

where $\mathbf{p}$ is the steering vector, $\mathbf{y}$ the observed vector and λ the detection threshold associated to this detector.

In many problems, non-Gaussian noise can be characterized by SIRVs but the covariance matrix $\mathbf{M}$ is generally not known and an estimate $\widehat{\mathbf{M}}$ is required. Obviously, it has to satisfy the $\mathbf{M}$-normalization: $\mathrm{Tr}(\widehat{\mathbf{M}}) = m$.

The next section is devoted to an adaptive GLRT built from an Approximate Maximum Likelihood (AML) estimate of the SIRV covariance matrix. Then, Section 3 presents an application of this detector to real data: experimental results perfectly match theoretical analysis.

2. THE FIXED POINT ESTIMATE $\widehat{\mathbf{M}}_{FP}$ AND THE CORRESPONDING ADAPTIVE GLRT

2.1 The AML estimate

Conte and Gini in [1, 2] have shown that an Approximate Maximum Likelihood (AML) estimate $\widehat{\mathbf{M}}$ of $\mathbf{M}$ is a solution of the following equation

$$\widehat{\mathbf{M}} = \frac{m}{N}\sum_{i=1}^{N}\left(\frac{\mathbf{c}_i\mathbf{c}_i^H}{\mathbf{c}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{c}_i}\right). \tag{6}$$

Notice that the ML estimate has been studied in [7]. Existence and uniqueness of the above equation solution, denoted $\widehat{\mathbf{M}}_{FP}$ have already been investigated in [8].

Let the function f be defined as

$$f(\widehat{\mathbf{M}}) = \frac{m}{N}\sum_{i=1}^{N}\left(\frac{\mathbf{c}_i\mathbf{c}_i^H}{\mathbf{c}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{c}_i}\right) = \frac{m}{N}\sum_{i=1}^{N}\left(\frac{\mathbf{x}_i\mathbf{x}_i^H}{\mathbf{x}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{x}_i}\right), \tag{7}$$

where the right hand side of (7) is rewritten in terms of the $\mathbf{x}_i$'s and the τ_i's.

Eqn. (7) obviously implies that $\widehat{\mathbf{M}}_{FP}$ is independent of the τ_i's.

The statistical properties of $\widehat{\mathbf{M}}_{FP}$ have been investigated and published in [11], the main results are recaped here below:

Proposition 2.1
(1) $\widehat{\mathbf{M}}_{FP}$ is a consistent estimate of $\mathbf{M}$;
(2) $\widehat{\mathbf{M}}_{FP}$ is an unbiased estimate of $\mathbf{M}$;
(3) the asymptotic distribution of $\widehat{\mathbf{M}}_{FP}$ is Gaussian and its covariance matrix is fully characterized in [11];
(4) this distribution is the same as the asymptotic distribution of a Wishart matrix with $\left(\frac{m}{m+1}\right)N$ degrees of freedom.

2.2 The studied adaptive GLRT

Let us now present the adaptive GLRT [9, 10], used for detection

$$\hat{\Lambda}(\widehat{\mathbf{M}}) = \frac{|\mathbf{p}^H\widehat{\mathbf{M}}^{-1}\mathbf{y}|^2}{(\mathbf{p}^H\widehat{\mathbf{M}}^{-1}\mathbf{p})(\mathbf{y}^H\widehat{\mathbf{M}}^{-1}\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda. \tag{8}$$

In the next section dealing with applications to real data, we will use $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ as detector. This detector is obviously texture-CFAR (independent of the distribution of τ) and, an original result of this paper is to show the independence of the distribution of $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ with the covariance matrix $\mathbf{M}$: we will say that $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ is Matrix-CFAR (M-CFAR).

Definition 2.1
An adaptive detector $\hat{\Lambda}(\widehat{\mathbf{M}})$ verifies the M-CFAR property if its statistical distribution is independent of the covariance matrix $\mathbf{M}$ estimated by $\widehat{\mathbf{M}}$.

This property is of most interest in a practical work to detect targets when the covariance matrix is unknown.

Theorem 2.1
$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ is M-CFAR.

Theorem 2.1 establishes the M-CFAR property of the adaptive GLRT built with the FP estimate.

Proof 2.1
Let $\mathbf{M}$ be a covariance matrix. Let $\widehat{\mathbf{M}}_{FP}$ be the FP estimate of $\mathbf{M}$. Then, under hypothesis H_0 (no target signal), we will show that

$$\mathscr{L}\left(\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})\right) = \mathscr{L}\left(\hat{\Lambda}(\widehat{\mathbf{M}}_{FP,\mathbf{I}})\right) \tag{9}$$

where $\mathscr{L}(x)$ stands for the statistical distribution of a random variable x and $\widehat{\mathbf{M}}_{FP,\mathbf{I}}$ is the FP estimate of the identity matrix $\mathbf{I}$.

Since the statistics of $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ is independent of the texture τ, we choose $\tau = 1$: secondary data $\mathbf{x}_1,\ldots,\mathbf{x}_N$ are thus Gaussian with covariance matrix $\mathbf{M}$,

$$\mathbf{x}_i \sim \mathscr{N}(\mathbf{0},\mathbf{M}).$$

The FP estimate of $\mathbf{M}$ is defined as the unique solution (up to a scalar factor) of

$$\widehat{\mathbf{M}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{x}_i\mathbf{x}_i^H}{\mathbf{x}_i^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{x}_i}, \tag{10}$$

and the adaptive GLRT detector is

$$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP}) = \frac{|\mathbf{p}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{x}|^2}{(\mathbf{p}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{p})(\mathbf{x}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda,$$

where $\mathbf{x}$ is the observation vector (under hypothesis H_0) such that $\mathbf{x} \sim \mathscr{N}(\mathbf{0},\mathbf{M})$.

The first part of the proof is the whitening of the data. By applying the following change of variable, $\mathbf{y} = \mathbf{M}^{-1/2}\mathbf{x}$ to Eqn. (10), one has

$$\widehat{\mathbf{M}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{M}^{1/2}\mathbf{y}_i\mathbf{y}_i^H\mathbf{M}^{1/2}}{\mathbf{y}_i^H\widehat{\mathbf{T}}^{-1}\mathbf{y}_i}, \tag{11}$$

where

$$\widehat{\mathbf{T}} = \mathbf{M}^{-1/2}\widehat{\mathbf{M}}_{FP}\mathbf{M}^{-1/2}.$$

Therefore,

$$\widehat{\mathbf{T}} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{y}_i\mathbf{y}_i^H}{\mathbf{y}_i^H\widehat{\mathbf{T}}^{-1}\mathbf{y}_i}. \tag{12}$$

$\widehat{\mathbf{T}}$ is thus the unique FP estimate (up to a scalar factor) of the identity matrix. Its statistics is clearly independent of $\mathbf{M}$ since the $\mathbf{y}_i$'s are $\mathscr{N}(\mathbf{0},\mathbf{I})$.

Moreover, for any unitary matrix $\mathbf{U}$, one has

$$\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{z}_i\mathbf{z}_i^H}{\mathbf{z}_i^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{z}_i}, \tag{13}$$

where $\mathbf{z}_i = \mathbf{U}\mathbf{y}_i$ *is also* $\mathcal{N}(\mathbf{0},\mathbf{I})$. *Therefore,* $\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H$ *has the same distribution as* $\widehat{\mathbf{T}}$.

In terms of the adaptive detector, one has

$$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP}) = \frac{|\mathbf{p}_1^H\widehat{\mathbf{T}}^{-1}\mathbf{y}|^2}{(\mathbf{p}_1^H\widehat{\mathbf{T}}^{-1}\mathbf{p}_1)(\mathbf{y}^H\widehat{\mathbf{T}}^{-1}\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda,$$

where $\mathbf{p}_1 = \mathbf{M}^{-1/2}\mathbf{p}$ *and* $\mathbf{y} = \mathbf{M}^{-1/2}\mathbf{x}$ *is* $\mathcal{N}(\mathbf{0},\mathbf{I})$.

Now let $\mathbf{U}$ *be a unitary matrix such that*

$$\mathbf{U}\mathbf{p}_1 = \|\mathbf{p}_1\|\,\mathbf{e} \tag{14}$$

where $\mathbf{e} = (1\,0\ldots0)^\top$, $\top$ *denotes the transpose operator and* $\|\mathbf{p}_1\|$ *denotes the 2-norm of vector* $\mathbf{p}_1$.

Thanks to Eqn. (14), one has

$$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP}) = \frac{|\mathbf{e}^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{z}|^2}{(\mathbf{e}^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{e})(\mathbf{z}^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda,$$

where $\mathbf{z} = \mathbf{U}\mathbf{y}$ *is also* $\mathcal{N}(\mathbf{0},\mathbf{I})$.

By setting $\widehat{\mathbf{M}}_{FP}\mathbf{I} = \mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H$, *we see that the distribution of* $\hat{\Lambda}(\widehat{\mathbf{M}}_F P)$ *does not depend on* $\mathbf{M}$, *which completes the proof of Theorem 2.1. Note also that the distribution of* $\hat{\Lambda}(\widehat{\mathbf{M}}_F P)$ *does not depend on the steering vector* p.

In this section, the statistical performance of the FP estimate has been investigated as well as the SIRV-CFAR (texture-CFAR and M-CFAR) property of the adaptive GLRT, built with $\widehat{\mathbf{M}}_{FP}$. One of the first deduction of previous results is that whatever the SIRV used, for different distributions of the texture and for different covariance matrices, the resulting GLRT $\hat{\Lambda}(\widehat{\mathbf{M}})$ follows the same distribution. This is of a major interest in areas of clutter transition like for example, in coastal areas (ground and sea) or at the edge of forests (fields and trees) because the detector should be insensitive to the different clutter areas. This is the object of the next section.

3. RADAR APPLICATIONS

This section is devoted to the analysis of different radar measurements in which the clutter is strongly impulsive. In a first time, let us give some generalities.

In radar detection, the analysis falls into two independent stages:

- The calculation of the detection threshold λ to ensure a false alarm rate, given by the operator. This part is performed by a learning of the clutter.
- The comparison of the adaptive GLRT $\hat{\Lambda}(\widehat{\mathbf{M}})$ with the detection threshold.

Let us define some notations:

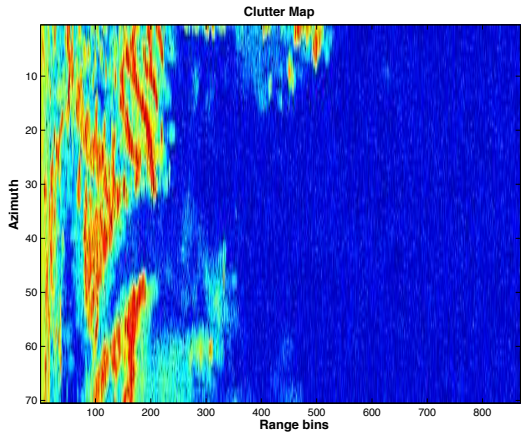

Figure 1: Ground clutter data level (in dB) corresponding to the first pulse echo. Y-coordinates represent 70 azimuth angles and X-coordinates represent $N = 868$ range bins.

- the Probability of False Alarm

$$P_{fa} = \mathbb{P}(\Lambda > \lambda|H_0), \tag{15}$$

- the Probability of Detection (PD)

$$P_d = \mathbb{P}(\Lambda > \lambda|H_1). \tag{16}$$

In [12], a theoretical relationship between the detection threshold λ and the PFA has been established when the covariance matrix $\mathbf{M}$ is estimated by the well known Sample Covariance Matrix (SCM) estimate defined by

$$\widehat{\mathbf{M}}_{SCM} = \frac{m}{N}\sum_{i=1}^{N}\mathbf{c}_i\mathbf{c}_i^H. \tag{17}$$

Now, the expression of PFA-threshold relationship in this specific case (SCM estimate) is

$$P_{fa} = (1-\lambda)^{a-1}\,{}_2F_1(a,a-1;b-1;\lambda), \tag{18}$$

where $a = N-m+2$, $b = N+2$ and ${}_2F_1$ is the hypergeometric function [13] defined as

$${}_2F_1(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{k=0}^{\infty}\frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(c+k)}\frac{x^k}{k!}. \tag{19}$$

Moreover, thanks to point (4) of proposition 2.1 and since the SCM (17) is Wishart distributed [14], expression 18 still holds for large N, when the covariance matrix $\mathbf{M}$ is estimated by the FP estimate:

$$P_{fa} = (1-\lambda)^{a-1}\,{}_2F_1(a,a-1;b-1;\lambda), \tag{20}$$

where $a = \frac{m}{m+1}N - m + 2$ and $b = \frac{m}{m+1}N + 2$. It means that it is the same relationship but with less secondary data ($\frac{m}{m+1}N$ data instead of N in the Wishart case).

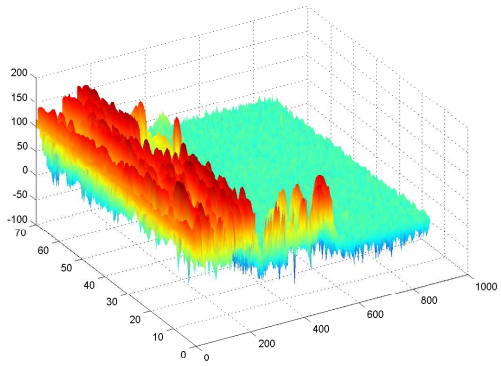

Figure 2: Ground clutter data level (in dB) corresponding to the first pulse echo. Y-coordinates represent 70 azimuth angles and X-coordinates represent $N = 868$ range bins.

This result has never been validated on real measurements: this is one of the purposes of this section.

The ground clutter data presented in this paper were collected by an operational radar at THALES Air Defence[1], placed 13 meters above ground and illuminating the ground at low grazing angle. Ground clutter complex echoes were collected in $N = 868$ range bins for 70 different azimuth angles and for $m = 8$ recurrences, which means that vectors size is $m = 8$. Near the radar, echoes characterize non-Gaussian heterogeneous ground clutter whereas beyond the radioelectric horizon of the radar (around 15 kms) only Gaussian thermal noise (the dark part of the map) is presented (Figure 1). To emphasize the areas of impulsive clutter, Figure 2 represents in 3 dimensions, the same range bin-azimuth map as on Figure 1: the third dimension (vertical) shows the power of the clutter.

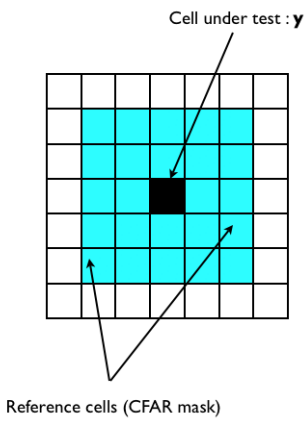

Figure 3: CFAR mask

The analysis of these radar data allows to adjust the detection threshold λ for a given PFA. A common procedure is to set this threshold, which is a system design parameter, based on the designer perception of tradeoffs between false alarms and missed detection. Traditionally, the experimental detection threshold adjustment is determined by counting, by moving a rectangular CFAR-mask of size 5×5. For all central cells of the mask (i.e. the cell under test), the dark cell on Figure 3, corresponding to the studied observation $\mathbf{y}$ (8-vector), a value of $\hat{\Lambda}(\widehat{\mathbf{M}})$ is calculated. The covariance matrix $\widehat{\mathbf{M}}$ has been estimated with the set of $N = 24$ 8-vectors, considered as the secondary data, $\mathbf{y}_1, \ldots, \mathbf{y}_{24}$, and situated around the tested cell. These reference cells are the light blue cells on Figure 3.

3.1 Validation of Eqn.(20) on real data

One purpose of this paper is to validate the theoretical relationship (20) between the detection threshold and the PFA thanks to counting of the real data when the covariance matrix is estimated by the FP estimate.

Moreover, when it is assumed that the covariance matrix $\mathbf{M}$ is known, one has $\lambda = 1 - P_{fa}^{-\frac{1}{1-m}}$ (see for example [15]). Notice that this equation has just a theoretical interest because in practice, $\mathbf{M}$ is always unknown.

Remark 3.1
Note that the counting system on the real data makes sense only thanks to the M-CFAR property of the adaptive detector. Indeed, there is no valid reason why all the sets of 24 data have the same covariance matrix.

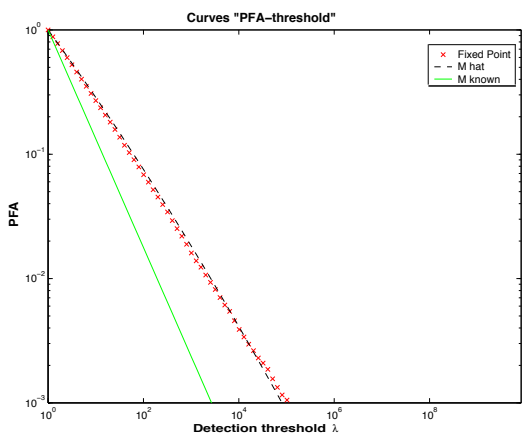

Figure 4: Validation of PFA-threshold relationship

On Figure 4, the solid curve corresponds to the theoretical relationship "PFA-threshold" if $\mathbf{M}$ is known while the dotted curve represents the theoretical relationship "PFA-threshold" when $\mathbf{M}$ and is assumed unknown and estimated by $\widehat{\mathbf{M}}_{FP}$.

The curve made of crosses ($\times$) represents the experimental (made with CFAR masks by counting) relationship "PFA-threshold" when $\mathbf{M}$ is estimated by $\widehat{\mathbf{M}}_{FP}$. It perfectly matches the theoretical relationship. Obtaining this result has

[1] Authors are grateful to Thales Air Defence for the analysis of their data

been possible only because the detector $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ satisfies the M-CFAR property, essential in an heterogeneous clutter.

Thus, this application validates Eqn. (20) and an essential consequence of this result is that thanks to Eqn. (20), the clutter training is not essential any more for the adjustment of the detection threshold.

3.2 Robustness to the clutter transitions

Figure 5 presents, for all the points of the range bin-azimuth map, the GLRT calculated for the FP estimate $\widehat{\mathbf{M}}_{FP}$. This map was made from the 8 available maps of real data.

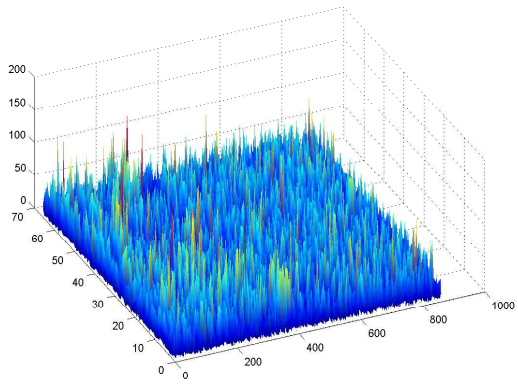

Figure 5: Detector map

We can conclude from Figure 5 that in spite of the clutter heterogeneity, on the right hand side of Figure 1, the use of the FP estimate in the GLRT allows to obtain a completely uniform likelihood ratio map. This experimental result ensures a constant false alarm regulation, even in the transitions areas. Moreover, it is in a good agreement with the theory and is directly provided by the SIRV-CFAR property of $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$.

4. CONCLUSION

In this paper, the M-CFAR property of the adaptive detector GLRT $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ built with the FP estimate of the covariance matrix $\mathbf{M}$ has been established. This result has been used in a radar application on real non-Gaussian data. This property stated the independence between the GLRT distribution and the real covariance matrix $\mathbf{M}$ of the data.
Moreover, a goal of this paper was the analysis of non-Gaussian experimental ground clutter signals. For that purpose, we first validated a theoretical relationship between the detection threshold and the PFA established in [12], thanks to non-Gaussian data. Then, we highlighted the GLRT robustness to the clutter transitions thanks to it SIRV-CFAR property.

Acknowledgement

The authors are grateful to Thales Air Defence for the analysis of their experimental data.

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Annexe J

Persymmetric Adaptive Radar Detectors

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Persymmetric Adaptive Radar Detectors

Guilhem Pailloux, Philippe Forster, Jean-Philippe Ovarlez and Frédéric Pascal

***Abstract*—In the general framework of radar detection, estimation of the Gaussian or non-Gaussian clutter covariance matrix is an important point. This matrix commonly exhibits a particular structure: for instance, this is the case for active systems using a symmetrically spaced linear array with constant pulse repetition interval. In this paper, we propose to use the particular persymmetric structure of the covariance matrix to improve the detection performance.**

In this context, this paper provides two new adaptive detectors for Gaussian additive noise and non-Gaussian additive noise which is modeled by the Spherically Invariant Random Vector. Their statistical properties are then derived and compared to simulations. The vast improvement in their detection performance is demonstrated by way of simulations or experimental ground clutter data. This allows for the analysis of the proposed detectors on both real Gaussian and non-Gaussian data.

***Index Terms*—Adaptive signal detection, Gaussian and non-Gaussian clutter, SIRV, covariance matrix estimation.**

I. Introduction

The problem of adaptive radar detection requires the estimation of the clutter covariance matrix (CCM). In recent years, improvements of the associated estimation schemes have gained tremendous interest in the radar community. For that purpose, the Sample Covariance Matrix (SCM) has been widely used and this non-parametric estimator may be improved by exploiting the CCM structure. Toeplitz structure has been addressed by Burg in [1] while Fuhrmann in [2] used this estimator for radar detection purposes. In radar systems using a symmetrically spaced linear array with constant pulse repetition interval, the CCM has the persymmetric property. This structure information could then be exploited to improve detection performance. In this context, we use a particular linear transformation in order to take into account the persymmetry of the CCM and to study the statistical property of new detectors for both Gaussian and non-Gaussian environments. For Gaussian data, the CCM Maximum Likelihood (ML) estimator has been derived in [3]. The corresponding Generalized Likelihood Ratio Test (GLRT) has been investigated in [4]. For non Gaussian clutter modeled by Spherically Invariant Random Vectors (SIRV), detection schemes have been proposed in [5] and [6]. In [5] the persymmetry is only exploited to build two sets of independent data in order to derive a SIRV-Constant False Alarm Rate (CFAR) detector: the Persymmetric Adaptive Normalized Matched Filter (P-ANMF). In [6] these sets are used to initialize an iterative algorithm simultaneously proposed in [7] and [8]. This allows the derivation of the Recursive P-ANMF (RP-ANMF). Our approach, based on the Fixed Point Adaptive Normalized Matched Filter (FP-ANMF), also called GLRT-FP [7], [8], exploits an original transformation already proposed in [9] for Gaussian case and in [10] for non-Gaussian case. This leads to the Persymmetric Fixed Point Adaptive Normalized Matched Filter (PFP-ANMF) also called GLRT-PFP, i.e. the persymmetry property of the CCM. Its interest is to render the CCM real, leading to a simpler problem. Moreover this approach allows the derivation of the statistical analysis of the proposed detection scheme.

This paper is organized as follows. Section II presents the studied problem in terms of matrix estimation and radar detection and introduces persymmetry tools where it is shown how the persymmetric structure of the CCM can be exploited to provide the new Persymmetric Adaptive Matched Filter (PS-AMF). In section III, we derive the statistical distribution of the PS-AMF under hypothesis H_0, in which only noise is present in order to determine the improvement in terms of Probability of False Alarm (PFA). To confirm this improvement, some Gaussian data are extracted from the experimental data to validate the study. Section III also presents similarly the problem in non-Gaussian noise. The purpose is to derive

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an estimator of the CCM based on the secondary data and to take into account its structure (Persymmetric Fixed Point, $\widehat{\mathbf{M}}_{PFP}$). The statistical properties of $\widehat{\mathbf{M}}_{PFP}$ are also established and enable the investigation of the distribution of the detector $\Lambda_{GLRT-PFP}$, called GLRT-PFP (Generalized Likelihood Ratio Test - Persymmetric Fixed Point). Finally we present in section IV some simulated and experimental results that illustrate the improvement in terms of detection performance of the PS-AMF with respect to the conventional Adaptive Matched Filter (AMF) for Gaussian case. Moreover, results obtained with non-Gaussian real data demonstrate the interest of the proposed detection scheme compared to the existing detectors P-ANMF, RP-ANMF and GLRT-FP.

II. Background and Problem Statement

This paragraph provides a guide to the notation used throughout the remainder of the paper. In general, a boldface, lowercase variable indicates a column vector quantity; a boldface, upper variable indicates a matrix; a variable with a caret ($\widehat{\ }$) is an estimate of an unknown quantity. Superscripts $^\top$ or ^{H} applied to a vector or a matrix denote the transpose or Hermitian (conjugate) transpose operations. The superscript * denotes the conjugate operation. $E[.]$ stands for the statistical expectation operator, $\mathrm{Tr}(.)$ denotes the trace operator, $\mathcal{R}e(.)$ denotes the real part and $|.|$ denotes the determinant. In this paper, $\| \ . \ \|$ stands for the usual $\mathcal{L}^2$-norm. $\mathbf{I}_m$ is the m-th order identity matrix, $Pr(.)$ stands for the probability and the notation $\sim$ means " is distributed as".

In radar detection, the main problem consists in detecting a signal $A\mathbf{p} \in \mathbb{C}^m$ corrupted by an additive clutter $\mathbf{c}$. This problem can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c}, & \mathbf{y_k} = \mathbf{c_k}, \text{ for } \mathbf{1} \leq \mathbf{k} \leq \mathbf{K}, \\ H_1 : \mathbf{y} = \mathbf{A}\,\mathbf{p} + \mathbf{c}, & \mathbf{y_k} = \mathbf{c_k}, \text{ for } \mathbf{1} \leq \mathbf{k} \leq \mathbf{K}, \end{cases} \tag{1}$$

where $\mathbf{y}$ is the complex m-vector of the received signal, A is an unknown complex target amplitude and $\mathbf{p}$ stands for the known "steering vector". Under both hypotheses, it is assumed that K signal-free data $\mathbf{y_k}$ are available for clutter parameters estimation. The $\mathbf{y_k}$'s are the so-called secondary data where they are assumed independent, but their statistical distribution depends on the clutter nature. In this paper two cases will be investigated according to the clutter statistics: the Gaussian clutter and the case of non-Gaussian clutter as modeled by SIRV.

A. Gaussian clutter

In the Gaussian case, $\mathbf{c}$ and $\mathbf{c}_k$ are complex circular zero-mean Gaussian m-vectors sharing the same covariance matrix $\mathbf{M}$, with distribution denoted by $\mathcal{CN}(\mathbf{0}, \mathbf{M})$. When $\mathbf{M}$ is known, the GLRT for A unknown is referred to as the Optimum Gaussian Detector (OGD):

$$\Lambda_{OGD} = \frac{\left|\mathbf{p}^H \mathbf{M}^{-1} \mathbf{y}\right|^2}{\mathbf{p}^H \mathbf{M}^{-1} \mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{OGD}\,, \tag{2}$$

where the detection threshold λ_{OGD} is related to the PFA P_{fa} by $\lambda_{OGD} = -\ln(P_{fa})$. However, in practice, the CCM $\mathbf{M}$ is generally unknown. One solution is to substitute for $\mathbf{M}$ an estimator $\widehat{\mathbf{M}}$ based on the secondary data. When no prior information on the $\mathbf{M}$-structure is available, the SCM is classically used:

$$\widehat{\mathbf{M}}_{SCM} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^H\,. \tag{3}$$

leading to the so-called Adaptive Matched Filter (AMF) test [11]:

$$\Lambda_{AMF} = \frac{\left|\mathbf{p}^H \widehat{\mathbf{M}}_{SCM}^{-1} \mathbf{y}\right|^2}{\mathbf{p}^H \widehat{\mathbf{M}}_{SCM}^{-1} \mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{AMF}\,. \tag{4}$$

The relationship between the PFA P_{fa} and the detection threshold λ_{AMF} is given by [11], [12]:

$$P_{fa} = {}_2F_1\left(K-m+1, K-m+2; K+1; -\frac{\lambda_{AMF}}{K}\right), \tag{5}$$

where ${}_2F_1(.)$ is the hypergeometric function [13] defined by

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 \frac{t^{b-1}\,(1-t)^{c-b-1}}{(1-tz)^a}\,dt\,.$$

B. Non-Gaussian clutter

In recent years, there has been an increasing interest for non-Gaussian clutter models motivated by experimental radar clutter measurements [14], which have shown that the clutter is perfectly modeled by K-distribution or Weibull distribution. More generally, $\mathbf{c}$ can be modeled by a SIRV [15], [16] which is the product of the square root of a positive random variable τ (called the texture) and a m-dimensional independent complex Gaussian vector $\mathbf{g}$ (called the speckle) with zero-mean and covariance matrix $\mathbf{M}$ normalized according to $\mathrm{Tr}(\mathbf{M}) = m$ for identifiability considerations [17]:

$$\mathbf{c} = \sqrt{\tau}\,\mathbf{g}\,. \tag{6}$$

The model when $\mathbf{M}$ is known and texture τ is unknown has been widely studied and this enables the construction of the Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) [18], [17] defined by:

$$\Lambda_{GLRT-LQ} = \frac{\left|\mathbf{p}^H\mathbf{M}^{-1}\mathbf{y}\right|^2}{\left(\mathbf{p}^H\mathbf{M}^{-1}\mathbf{p}\right)\left(\mathbf{y}^H\mathbf{M}^{-1}\mathbf{y}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{GLRT-LQ}\,, \tag{7}$$

where $\lambda_{GLRT-LQ}$ is the corresponding detection threshold.

When $\mathbf{M}$ is unknown, one solution is to substitute a given estimator $\widehat{\mathbf{M}}$ of $\mathbf{M}$ in Eqn. (7) resulting in an adaptive version of the GLRT. When replacing $\mathbf{M}$ by an estimator $\widehat{\mathbf{M}}$, this detector is often called Adaptive Cosine Estimate (ACE) [19] or Adaptive Normalized Matched Filter (ANMF)

In non-Gaussian case, the CCM estimator $\widehat{\mathbf{M}}$ is based on the signal-free secondary data $\mathbf{y}_k = \mathbf{c}_k$ where the clutter samples $\mathbf{c}_k$ are SIRVs sharing the same CCM as $\mathbf{c}$: $\mathbf{c}_k = \sqrt{\tau_k}\,\mathbf{g}_k$, where $E[\mathbf{g}_k\,\mathbf{g}_k^H] = \mathbf{M}$. For the case when no prior information on the $\mathbf{M}$-structure is available, Conte and Gini in [7], [8] proposed an Approximate Maximum Likelihood (AML) estimator $\widehat{\mathbf{M}}_{FP}$ of $\mathbf{M}$, called the Fixed Point (FP) estimator, which is defined as the solution of the following implicit equation:

$$\widehat{\mathbf{M}}_{FP} = \frac{m}{K}\sum_{k=1}^{K}\frac{\mathbf{y}_k\,\mathbf{y}_k^H}{\mathbf{y}_k^H\,\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{y}_k}\,. \tag{8}$$

Existence and uniqueness of the above equation solution have been proven in [20], while the complete statistical properties of $\widehat{\mathbf{M}}_{FP}$ have been derived in [21]. The estimator $\widehat{\mathbf{M}}_{FP}$ does not depend on the texture and allows to obtain the following adaptive detector GLRT-FP:

$$\Lambda_{GLRT-FP} = \frac{\left|\mathbf{p}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{y}\right|^2}{\left(\mathbf{p}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{p}\right)\left(\mathbf{y}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{y}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{GLRT-FP}. \tag{9}$$

The relationship between the PFA P_{fa} and the detection threshold $\lambda_{GLRT-FP}$ is given by [22]:

$$P_{fa} = (1-\lambda_{GLRT-FP})^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda_{GLRT-FP}). \tag{10}$$

where $K' = \frac{m}{m+1}K$, $a = K' - m + 2$ and $b = K' + 2$.

C. Persymmetry considerations and problem formulation

It is clear that the estimation accuracy of $\widehat{\mathbf{M}}$ has an important impact on the adaptive detection performance in both Gaussian and non-Gaussian clutter. $\widehat{\mathbf{M}}_{SCM}$ and $\widehat{\mathbf{M}}_{FP}$ defined by Eqn. (3) and Eqn. (8) do not take into account any prior information on the CCM structure. However many applications lead to a CCM which exhibits some particular structure, and considering this structure may lead to a improvement in both estimation and detection performance. Such a situation is frequently met in radar systems using a symmetrically spaced linear array and a symmetrically spaced pulse train for temporal domain processing [4], [5], [1]. In these systems, the CCM $\mathbf{M}$ has the persymmetric property, defined as follows:

$$\mathbf{M} = \mathbf{J}_m\,\mathbf{M}^*\,\mathbf{J}_m\,, \tag{11}$$

where $\mathbf{J}_m$ is the m-dimensional antidiagonal matrix having 1 as non-zero elements. The steering vector of the problem is also persymmetric, i.e. it satisfies:

$$\mathbf{p} = \mathbf{J}_m\,\mathbf{p}^*\,. \tag{12}$$

The persymmetric structure of $\mathbf{M}$ will be exploited in this paper in order to improve its estimation accuracy compared to unstructured estimators. This will be done by means of the transformation matrix $\mathbf{T}$ introduced in [23] and whose properties are recalled in the following proposition.

Proposition II.1 ([23]) *Let $\boldsymbol{T}$ be the unitary matrix defined as:*

$$\boldsymbol{T} = \begin{cases} \dfrac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{I}_{m/2} & \boldsymbol{J}_{m/2} \\ i\,\boldsymbol{I}_{m/2} & -i\,\boldsymbol{J}_{m/2} \end{pmatrix} & \textit{for } m \textit{ even} \\[2ex] \dfrac{1}{\sqrt{2}}\begin{pmatrix} \boldsymbol{I}_{(m-1)/2} & 0 & \boldsymbol{J}_{(m-1)/2} \\ 0 & \sqrt{2} & 0 \\ i\,\boldsymbol{I}_{(m-1)/2} & 0 & -i\,\boldsymbol{J}_{(m-1)/2} \end{pmatrix} & \textit{for } m \textit{ odd.} \end{cases} \tag{13}$$

Persymmetric vectors and Hermitian matrices are characterized by the following properties:

- *$\boldsymbol{p} \in \mathbb{C}^m$ is a persymmetric vector if and only if $\boldsymbol{T}\,\boldsymbol{p}$ is a real vector.*
- *$\boldsymbol{M}$ is a persymmetric Hermitian matrix if and only if $\boldsymbol{T}\,\boldsymbol{M}\,\boldsymbol{T}^H$ is a real symmetric matrix.*

Using proposition II.1, the original problem (1) can be equivalently reformulated. Let us introduce the transformed primary data $\mathbf{x}$, the transformed secondary data $\mathbf{x}_k$, the transformed clutter vector $\mathbf{n}$ and the transformed signal steering vector $\mathbf{s}$ defined as: $\mathbf{x} = \mathbf{T}\,\mathbf{y}$, $\mathbf{x}_k = \mathbf{T}\,\mathbf{y}_k$, $\mathbf{s} = \mathbf{T}\,\mathbf{p}$, $\mathbf{n} = \mathbf{T}\,\mathbf{c}$, $\mathbf{n}_k = \mathbf{T}\,\mathbf{c}_k$.

It follows that the transformed signal steering vector $\mathbf{s}$ and the transformed CCM are both real. Then, the

original problem (1) is equivalent to:

$$\begin{cases} H_0 : \mathbf{x} = \mathbf{n} & \mathbf{x_k} = \mathbf{n_k}, \text{ for } \mathbf{1} \leq \mathbf{k} \leq \mathbf{K}, \\ H_1 : \mathbf{x} = \mathbf{A}\,\mathbf{s} + \mathbf{n} & \mathbf{x_k} = \mathbf{n_k}, \text{ for } \mathbf{1} \leq \mathbf{k} \leq \mathbf{K}, \end{cases} \tag{14}$$

where $\mathbf{x} \in \mathbb{C}^m$, $\mathbf{s}$ is a known real vector.

In the Gaussian case, under hypothesis H_0, $\mathbf{n}$ and the K transformed secondary data $\mathbf{x}_k$ are i.i.d and share the same $\mathcal{CN}(\mathbf{0}, \mathbf{R})$ distribution where $\mathbf{R} = \mathbf{T}\,\mathbf{M}\,\mathbf{T}^H$ is a real symmetric matrix according to proposition II.1.
In the Non-Gaussian case, one has:

$$\mathbf{n} = \sqrt{\tau}\,\mathbf{h} \tag{15}$$

$$\mathbf{n}_k = \sqrt{\tau_k}\,\mathbf{h}_k \tag{16}$$

where $\mathbf{h} = \mathbf{T}\,\mathbf{g}$ and $\mathbf{h}_k = \mathbf{T}\,\mathbf{g}_k$ denote the transformed speckle vector with the same real covariance matrix $\mathbf{R} = \mathbf{T}\,\mathbf{M}\,\mathbf{T}^H$. $\mathbf{n}$ and $\mathbf{n}_k$ are still SIRV's with the same texture and CCM $\mathbf{R} = \mathbf{T}\,\mathbf{M}\,\mathbf{T}^H$. From now on, the problem under study is the problem defined by (14).

III. Detection Scheme

In this section the detection problem (14) is investigated in Gaussian and non-Gaussian frameworks. More precisely, the CCM estimation problem is addressed and adaptive detection schemes are proposed. In both cases, the adaptive detector properties are studied.

A. Detection in circular Gaussian noise

Let us first investigate the ML estimator of the real covariance matrix $\mathbf{R}$ from the K secondary data $\mathbf{x}_k$. The main motivation for introducing the transformed data is that the resulting distribution of the ML estimator of $\mathbf{R}$ is very simple. This was not the case in [3] when dealing with the original secondary data $\mathbf{y}_k$ with persymmetric covariance matrix.

Proposition III.1 *The ML estimator $\widehat{\boldsymbol{R}}_P$ of the real matrix $\boldsymbol{R}$ is unbiased and is given by:*

$$\widehat{\boldsymbol{R}}_P = \mathcal{R}e(\widehat{\boldsymbol{R}}_{SCM})\,, \tag{17}$$

where:

$$\widehat{\boldsymbol{R}}_{SCM} = \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{x}_k\,\boldsymbol{x}_k^H = \boldsymbol{T}\,\widehat{\boldsymbol{M}}_{SCM}\,\boldsymbol{T}^H\,. \tag{18}$$

$\widehat{\boldsymbol{R}}_P$ is an unbiased estimator and $K\,\widehat{\boldsymbol{R}}_P$ is real Wishart distributed with parameter matrix $\frac{1}{2}\boldsymbol{R}$ and $2K$ degrees of freedom.

Proof: It is easy to show that the MLE of the real covariance matrix is provided by equation (17).

Let us now investigate its statistical properties. Let $\mathbf{a}_k$ and $\mathbf{b}_k$ be the real and imaginary parts of the secondary data:

$$\mathbf{x}_k = \mathbf{a}_k + i\,\mathbf{b}_k \tag{19}$$

and

$$K\,\widehat{\mathbf{R}}_P = \sum_{k=1}^{K} \mathbf{a}_k\,\mathbf{a}_k^\top + \sum_{k=1}^{K} \mathbf{b}_k\,\mathbf{b}_k^\top. \tag{20}$$

$\mathbf{x}_k$ is circular, i.e. $E[\mathbf{x}_k\,\mathbf{x}_k^\top] = \mathbf{0}$ which leads to:

$$\begin{aligned} E[\mathbf{a}_k\,\mathbf{a}_k^\top] - E[\mathbf{b}_k\,\mathbf{b}_k^\top] &= \mathbf{0} \\ E[\mathbf{a}_k\,\mathbf{b}_k^\top] + E[\mathbf{b}_k\,\mathbf{a}_k^\top] &= \mathbf{0} \end{aligned} \tag{21}$$

Moreover, $\mathbf{x}_k$ has a real covariance matrix $\mathbf{R}$ which implies:

$$\begin{aligned} E[\mathbf{a}_k\,\mathbf{a}_k^\top] + E[\mathbf{b}_k\,\mathbf{b}_k^\top] &= \mathbf{R} \\ E[\mathbf{a}_k\,\mathbf{b}_k^\top] - E[\mathbf{b}_k\,\mathbf{a}_k^\top] &= \mathbf{0} \end{aligned} \tag{22}$$

Equations (21) and (22) yield:

$$\begin{aligned} E[\mathbf{a}_k\,\mathbf{a}_k^\top] &= E[\mathbf{b}_k\,\mathbf{b}_k^\top] = \frac{1}{2}\mathbf{R} \\ E[\mathbf{a}_k\mathbf{b}_k^\top] &= \mathbf{0} \end{aligned} \tag{23}$$

showing that the $\mathbf{a}_k$'s and the $\mathbf{b}_k$'s are independent and share the same covariance matrix $\frac{1}{2}\mathbf{R}$.

From Eqn. (20), $K\widehat{\mathbf{R}}_P$ has a real Wishart distribution with $2K$ degrees of freedom and parameter matrix $\frac{1}{2}\mathbf{R}$. Moreover,

$$E[K\,\widehat{\mathbf{R}}_P] = 2\,K\,\frac{1}{2}\,\mathbf{R}\,, \tag{24}$$

resulting in $E[\widehat{\mathbf{R}}_P] = \mathbf{R}$.

■

Actually, taking into account the real structure of $\mathbf{R}$ (or equivalently the persymmetric structure of $\mathbf{M}$) in the ML estimation procedure allows to virtually double the number of secondary data. Let us now consider the AMF for the detection problem (14) based on the estimator $\widehat{\mathbf{R}}_P$ defined by (17). This leads to the following detection test, called the PS-AMF,

$$\Lambda_{PS-AMF} = \frac{\left|\mathbf{s}^\top\widehat{\mathbf{R}}_P^{-1}\mathbf{x}\right|^2}{\mathbf{s}^\top\widehat{\mathbf{R}}_P^{-1}\mathbf{s}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{PS-AMF}\,, \tag{25}$$

or equivalently, in terms of the original data,

$$\Lambda_{PS-AMF} = \frac{\left|\mathbf{p}^H\mathbf{T}^H[\mathcal{R}e(\mathbf{T}\widehat{\mathbf{M}}_{SCM}\mathbf{T}^H)]^{-1}\mathbf{T}\mathbf{y}\right|^2}{\mathbf{p}^H\mathbf{T}^H[\mathcal{R}e(\mathbf{T}\widehat{\mathbf{M}}_{SCM}\mathbf{T}^H)]^{-1}\mathbf{T}\mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{PS-AMF}\,. \tag{26}$$

The distribution of Eqn. (25) is well-known when $K\,\widehat{\mathbf{R}}_P$ is complex Wishart distributed with parameter matrix $K\,\mathbf{R}$ and K degrees of freedom: this is the classical AMF distribution [11], [?]. However, in our problem, $K\,\widehat{\mathbf{R}}_P$ is real Wishart distributed with parameter matrix $\frac{1}{2}\mathbf{R}$ and $2K$ degrees of freedom while $\mathbf{x}$ is complex. The following proposition establishes the statistical distribution of the PS-AMF and the relationship between the PFA P_{fa} and the detection threshold λ_{PS-AMF}.

Proposition III.2 • *Under H_0, the PDF $p(z)$ of Λ_{PS-AMF}, defined by Eqn. (25), is:*

$$p(z) = \frac{(2K-m+1)\,(2K-m+2)}{2\,K\,(2K+1)}$$
$${}_2F_1\left(\frac{2K-m+3}{2}, \frac{2K-m+4}{2}; \frac{2K+3}{2}; -\frac{z}{K}\right), \tag{27}$$

• *The relationship between P_{fa} and the detection threshold λ is:*

$$P_{fa} = {}_2F_1\left(\frac{2K-m+1}{2}, \frac{2K-m+2}{2}; \frac{2K+1}{2}; -\frac{\lambda_{PS-AMF}}{K}\right). \tag{28}$$

Proof: The proof is given on Appendix VII-A ■

As will be seen in section IV for both simulated and experimental data, the PS-AMF outperforms the AMF, especially for a small number of secondary data.

B. Detection in non-Gaussian noise

The purpose of this section is to address the non-Gaussian case for the detection problem (14). Let us first recall some notations. The additive SIRV noise $\mathbf{n}$ is defined by:

$$\mathbf{n} = \sqrt{\tau}\,\mathbf{h} \tag{29}$$

where τ is a positive random variable, and $\mathbf{h}$ is a zero-mean circular complex Gaussian vector with real CCM $\mathbf{R}$. The K secondary data $\mathbf{n}_k = \sqrt{\tau_k}\,\mathbf{h}_k$ are i.i.d. and share the same distribution as $\mathbf{n}$.

Since the transformed CCM $\mathbf{R}$ is real, its structure may be taken into account in the estimation procedure by retaining only the real part of the FP estimator. This leads to the proposed covariance estimator called the Persymmetric Fixed-Point since it results from the persymmetric structure of the original speckle covariance matrix:

$$\widehat{\mathbf{R}}_{PFP} = \mathcal{R}e(\widehat{\mathbf{R}}_{FP}), \tag{30}$$

with

$$\widehat{\mathbf{R}}_{FP} = \mathbf{T}\,\widehat{\mathbf{M}}_{FP}\,\mathbf{T}^H. \tag{31}$$

In this section, the statistical properties of the detector $\Lambda_{GLRT-PFP}$ are investigated under the null hypothesis H_0. Let us recall some basic definitions:

- A test statistic is said to be texture-CFAR when its distribution is independent of the texture distribution,
- A test statistic is said to be matrix-CFAR when its distribution is independent of $\mathbf{R}$,
- A test statistic is said to be SIRV-CFAR when it is both texture-CFAR and matrix-CFAR.

The statistical properties of $\widehat{\mathbf{R}}_{PFP}$ are provided by the following proposition.

Proposition III.3 (Statistical performance of $\widehat{\mathbf{R}}_{PFP}$)

- *The distribution of $\widehat{\boldsymbol{R}}_{PFP}$ does not depend on the texture.*
- *$\widehat{\boldsymbol{R}}_{PFP}$ is a consistent estimator of $\boldsymbol{R}$.*
- *$\widehat{\boldsymbol{R}}_{PFP}$ is an unbiased estimator of $\boldsymbol{R}$.*
- *$\dfrac{\widehat{\boldsymbol{R}}_{PFP}}{Tr\left(\boldsymbol{R}^{-1}\,\widehat{\boldsymbol{R}}_{PFP}\right)}$ has the same asymptotic distribution as $\dfrac{\widehat{\boldsymbol{R}}}{Tr\left(\boldsymbol{R}^{-1}\,\widehat{\boldsymbol{R}}\right)}$, where $\widehat{\boldsymbol{R}}$ is real Wishart distributed with $\dfrac{m}{m+1}\,2K$ degrees of freedom and parameter matrix $\boldsymbol{R}$.*

Proof: In [21], unbiasedness and consistency of $\widehat{\mathbf{R}}_{FP}$ are proved. Taking the real part of this estimator does not change these two statistical properties.

It has been shown in [21] that $\dfrac{\widehat{\mathbf{R}}_{FP}}{\mathrm{Tr}\left(\mathbf{R}^{-1}\,\widehat{\mathbf{R}}_{FP}\right)}$ has the same asymptotic distribution as $\dfrac{\widehat{\mathbf{R}}_c}{\mathrm{Tr}\left(\mathbf{R}^{-1}\,\widehat{\mathbf{R}}_c\right)}$ with $\widehat{\mathbf{R}}_c$ complex Wishart distributed with $\frac{m}{m+1}K$ degrees of freedom and parameter matrix $\mathbf{R}$. Therefore $\dfrac{\widehat{\mathbf{R}}_{PFP}}{\mathrm{Tr}\left(\mathbf{R}^{-1}\,\widehat{\mathbf{R}}_{PFP}\right)}$ has the same asymptotic distribution as $\dfrac{\mathcal{R}e(\widehat{\mathbf{R}}_{FP})}{\mathrm{Tr}\left(\mathbf{R}^{-1}\,\mathcal{R}e(\widehat{\mathbf{R}}_{FP})\right)}$. By noting that $\mathrm{Tr}\,(\mathbf{AB}) = \mathrm{Tr}\,(\mathbf{A}\,\mathcal{R}e(\mathbf{B}))$ when $\mathbf{A}$ is real symmetric and $\mathbf{B}$ is Hermitian, it follows that $\dfrac{\widehat{\mathbf{R}}_{PFP}}{\mathrm{Tr}\left(\mathbf{R}^{-1}\,\widehat{\mathbf{R}}_{PFP}\right)}$ has the same

asymptotic distribution as $\dfrac{\widehat{\mathbf{R}}}{\mathrm{Tr}\left(\mathbf{R}^{-1}\,\widehat{\mathbf{R}}\right)}$, where $\widehat{\mathbf{R}} = 2\,\mathcal{R}e(\mathbf{R}_c)$ is real Wishart distributed with parameter matrix $\mathbf{R}$ and $K' = \dfrac{m}{m+1}2K$ degrees of freedom. ■

Definition III.1 *The adaptive GLRT, for the transformed problem (14), based on Eqn. (7) and on the PFP estimator is:*

$$\begin{aligned}\Lambda_{GLRT-PFP} &= \frac{\left|\boldsymbol{s}^\top\,\widehat{\boldsymbol{R}}_{PFP}^{-1}\boldsymbol{x}\right|^2}{\left(\boldsymbol{s}^\top\,\widehat{\boldsymbol{R}}_{PFP}^{-1}\boldsymbol{s}\right)\left(\boldsymbol{x}^H\,\widehat{\boldsymbol{R}}_{PFP}^{-1}\boldsymbol{x}\right)} \\ &\underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{GLRT-PFP}. \end{aligned} \tag{32}$$

Proposition III.4 *$\Lambda_{GLRT-PFP}$ is SIRV-CFAR. For large K, under hypothesis H_0, $\Lambda_{GLRT-PFP}$ has the same distribution as $\Lambda = \dfrac{\left|\boldsymbol{e}_1^\top\,\widehat{\boldsymbol{W}}^{-1}\boldsymbol{w}\right|^2}{\left(\boldsymbol{e}_1^\top\,\widehat{\boldsymbol{W}}^{-1}\boldsymbol{e}_1\right)\left(\boldsymbol{w}^H\,\widehat{\boldsymbol{W}}^{-1}\boldsymbol{w}\right)}$ where $\boldsymbol{w} \sim \mathcal{CN}(0,\boldsymbol{I})$, $\boldsymbol{e}_1 = (1,0,...,0)^\top$ and where $\widehat{\boldsymbol{W}}$ is real Wishart distributed with parameter matrix $\boldsymbol{I}$ and $K' = \dfrac{m}{m+1}2K$ degrees of freedom.*

Proof: Since the FP estimator $\widehat{\mathbf{M}}_{FP}$ does not depend on the texture, it follows from Eqn. (30) and (31) that $\widehat{\mathbf{R}}_{PFP}$ is itself texture independent. Moreover, under hypothesis H_0, $\Lambda_{GLRT-PFP}$ is homogeneous in terms of τ. Therefore, $\Lambda_{GLRT-PFP}$ is texture-CFAR.

Let us now investigate the matrix-CFAR property. Let $\mathbf{R}^{\frac{1}{2}}\,\mathbf{R}^{\frac{\top}{2}}$ be a real factorization of $\mathbf{R}$, and let $\mathbf{Q}$ be a real unitary matrix such that:

$$\mathbf{Q}\,\mathbf{R}^{-1/2}\,\mathbf{s} = (\mathbf{s}^\top\,\mathbf{R}^{-1}\,\mathbf{s})^{1/2}\,\mathbf{e}_1. \tag{33}$$

Note that the last equation is possible with $\mathbf{Q}$ real since $\mathbf{s}$ is itself real.
The test statistic $\Lambda_{GLRT-PFP}$ may then be rewritten

$$\Lambda_{GLRT-PFP} = \frac{\left|\mathbf{e}_1^\top\widehat{\mathbf{A}}^{-1}\mathbf{w}\right|^2}{\left(\mathbf{e}_1^\top\widehat{\mathbf{A}}^{-1}\mathbf{e}_1\right)\left(\mathbf{w}^H\widehat{\mathbf{A}}^{-1}\mathbf{w}\right)}, \tag{34}$$

where

$$\mathbf{w} = \mathbf{Q}\,\mathbf{R}^{-\frac{1}{2}}\,\mathbf{h} \sim \mathcal{CN}(\mathbf{0},\mathbf{I}) \tag{35}$$

and where

$$\begin{aligned}\widehat{\mathbf{A}} &= \mathbf{Q}\,\mathbf{R}^{-\frac{1}{2}}\,\widehat{\mathbf{R}}_{PFP}\,\mathbf{R}^{-\frac{\top}{2}}\,\mathbf{Q}^\top \\ &= \mathcal{R}e\left(\mathbf{Q}\,\mathbf{R}^{-\frac{1}{2}}\,\widehat{\mathbf{R}}_{FP}\,\mathbf{R}^{-\frac{\top}{2}}\,\mathbf{Q}^\top\right). \end{aligned} \tag{36}$$

It has been shown in [24] that $\mathbf{Q}\,\mathbf{R}^{-\frac{1}{2}}\,\widehat{\mathbf{R}}_{FP}\,\mathbf{R}^{-\frac{\top}{2}}\,\mathbf{Q}^\top$ in Eqn. (36) is a FP estimator of the identity matrix and that its distribution is therefore independent of $\mathbf{R}$. Thus, the same conclusion holds for its real part $\widehat{\mathbf{A}}$ defined by Eqn. (36) and the matrix-CFAR property is proved.

From the fourth point of proposition III.3, $\Lambda_{GLRT-PFP}$ has the same distribution as

$$\Lambda = \frac{\left|\mathbf{s}^\top\,\widehat{\mathbf{R}}^{-1}\,\mathbf{x}\right|^2}{\left(\mathbf{s}^\top\,\widehat{\mathbf{R}}^{-1}\,\mathbf{s}\right)\left(\mathbf{x}^H\,\widehat{\mathbf{R}}^{-1}\,\mathbf{x}\right)}, \tag{37}$$

where $\widehat{\mathbf{R}}$ is real Wishart distributed with K' degrees of freedom and parameter matrix $\mathbf{R}$.

Let $\mathbf{Q}$ be the real unitary matrix defined by Eqn. (33) and let us define $\widehat{\mathbf{W}} = \mathbf{Q}\mathbf{R}^{-\frac{1}{2}}\widehat{\mathbf{R}}\mathbf{R}^{-\frac{\top}{2}}\mathbf{Q}^\top$. The matrix $\widehat{\mathbf{W}}$ is real Wishart distributed with K' degrees of freedom and parameter matrix $\mathbf{I}$. Then Λ defined by Eqn. (37) can be rewritten as

$$\Lambda = \frac{\left|\mathbf{e}_1^\top\widehat{\mathbf{W}}^{-1}\mathbf{w}\right|^2}{\left(\mathbf{e}_1^\top\widehat{\mathbf{W}}^{-1}\mathbf{e}_1\right)\left(\mathbf{w}^H\widehat{\mathbf{W}}^{-1}\mathbf{w}\right)}, \tag{38}$$

which concludes the proof.

■

Moreover, in the sequel, all the statistical properties can be analyzed by choosing $\mathbf{n}$ and $\mathbf{n}_k$ to be Gaussian distributed because of these texture-CFAR properties. The analytical expression for the PDF of the detector $\Lambda_{GLRT-PFP}$ has not been derived but the following theorem gives some insight about its distribution. This derivation is important from an operational point of view in order to regulate the false alarm.

Proposition III.5 *For large K, $\Lambda_{GLRT-PFP}$ has the same distribution as $\dfrac{F}{F+1}$ where:*

$$F = \frac{(\alpha_1\,l_{22} - \alpha_2\,l_{21})^2 + \left(1 + \left(\dfrac{\beta_3}{l_{33}}\right)^2\right)(a\,l_{22} - b\,l_{21})^2}{(\alpha_2\,l_{11})^2 + \left(l_{11}\,l_{22}\dfrac{\beta_3}{l_{33}}\right)^2 + l_{11}^2\left(1 + \left(\dfrac{\beta_3}{l_{33}}\right)^2\right)b^2} \tag{39}$$

and where all the following random variables are independent and distributed according to:

$$a, b, \alpha_1, l_{21} \sim \mathcal{N}(0,1), \alpha_2^2 \sim \chi^2_{m-1}, \beta_3^2 \sim \chi^2_{m-2},$$

and

$$l_{11}^2 \sim \chi^2_{K'-m+1}, l_{22}^2 \sim \chi^2_{K'-m+2}, l_{33}^2 \sim \chi^2_{K'-m+3},$$

with $K' = \dfrac{m}{m+1}2K$.

Proof: The proof of this proposition is given on Appendix VII-B. ■

Proposition III.5 may be used to obtain, through Monte-Carlo simulations, the relation between the PFA and the detection threshold λ for the GRLT-PFP (32).

IV. Validation on experimental data

After the statistical study of these detectors, this section presents some results obtained on some experimental and simulated data in Gaussian and non-Gaussian case.

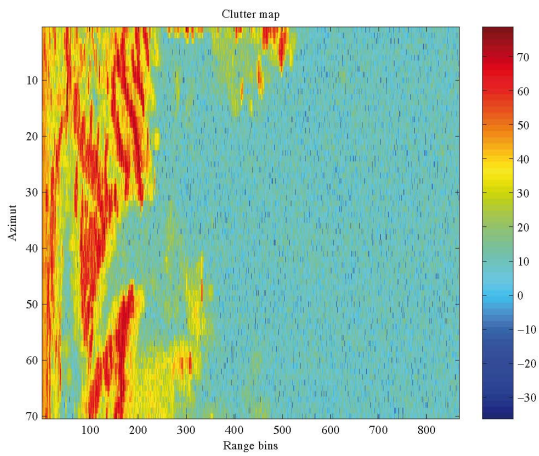


Fig. 1. Ground clutter data level (in dB) corresponding to the first pulse.

A. Experimental Gaussian data extraction and validation

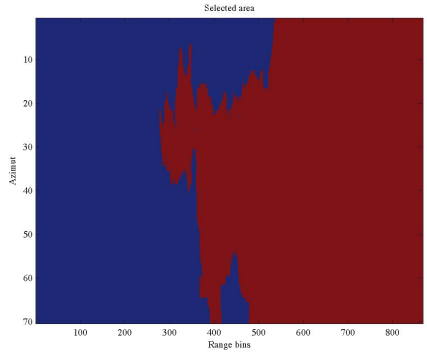
The ground clutter data used in this paper were collected by an operational radar at Thales Air System. The radar is 13 meters above the ground and illuminating the ground at low grazing angle. Ground clutter complex echoes were collected for $N = 868$ range bins, for 70 different azimuth angles and for $m = 8$ pulses. Fig. 1 displays the ground clutter data level (in dB) corresponding to the first pulse echo. Near the radar, echoes characterize heterogeneous ground clutter whereas beyond the radioelectric horizon of the radar (around 15 km), only heterogeneous thermal noise (the blue part of the map) is present.

In order to test the PS-AMF and given the non-Gaussian nature of experimental data, it is necessary to select particular data on the radar map. In addition, on some operational map, further parts of the data present a low amplitude. Beyond the electromagnetic horizon of the radar, the absence of reflectors gives an homogeneous area of data, characterized as Gaussian thermal noise. A statistical selection of these data allows us to obtain experimental Gaussian noise to test our detection algorithm.

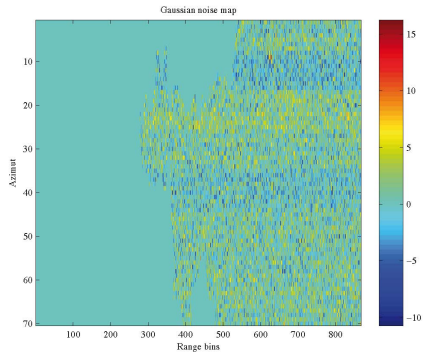
For that purpose, the well-known goodness-of-fit test of Kolmogorov-Smirnov (KS) is widely used [25]. This test verifies the adequacy of a given data sample to a Gaussian distributed sample $\mathcal{N}(\mu;\sigma)$. In order to obtain an homogeneous area of data, the KS test is applied successively on little blocks of data. With this test, the non-Gaussian data are rejected but the obtained map is lacunar. Indeed, the original clutter map presents a particular structure where there are horizontal band corresponding to constant azimuths of the radar. The transitions between these bands show a significant difference of the mean of the data and a consequence is that the KS test is then inefficient. This is also the case for another goodness-of-fit test like Anderson-Darling [25].

In order to get round this problem, a robust algorithm using connectedness of the data is developed. The idea is to include in the Gaussian area all the little blocks of supposed non-Gaussian data when they are not connected with the main non-Gaussian area. All the little blocks of data which are Gaussian but not characterized as Gaussian because of the non homogeneity of the KS test are now included in the main Gaussian area. We obtain with this method a wide area of Gaussian homogeneous data. In Fig. 2(a), we present the Gaussian area selected by the statistical study of the entire map. Gaussian area is colored in red and non-Gaussian area is in blue. This map confirms that only Gaussian thermal noise is present on the radioelectric horizon of the radar. In terms of experimental data, we present on Fig. 2(b) the clutter map with only Gaussian data extracted. The similar color map as in Fig. 1 allows to verify the adequacy between original and extracted data.

Once Gaussian data have been extracted from entire clutter map, we present the classical performance of this detector. In Fig. 3(a) and Fig. 4(a), the PFA with respect to detection threshold are presented. The theoretical relations for the OGD, the AMF and the PS-AMF are compared to the experimental relation (AMF and PS-AMF) in order to confirm the validity of the statistical study. Experimental curves are determined by Monte-Carlo counting, moving a (5×3) and (5×5) CFAR mask with different number of simulations ($nsimu$). The theoretical relation is then validated. In Fig. 3(b)

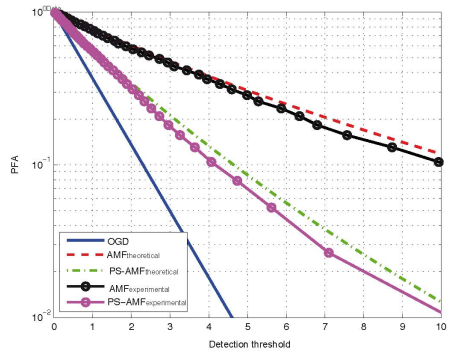


(a) Binary area selected for Gaussian data extraction (Gaussian area in red and non-Gaussian in blue).

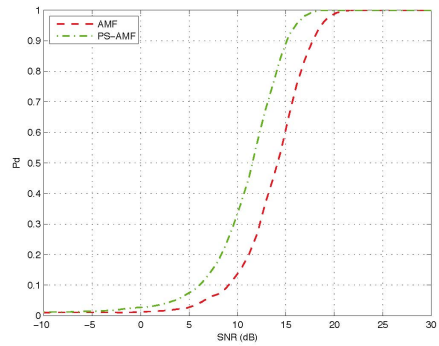


(b) Gaussian map (in dB) corresponding to the Gaussian selection and extracted from entire map

Fig. 2. Selection of the Gaussian area in the data presented in Fig. 1



(a) Theoretical and experimental PFA-threshold curves for various Gaussian detectors.



(b) Probability of Detection versus SNR for the AMF and PS-AMF

Fig. 3. Comparison between Performance of detectors for $P_{fa} = 10^{-2}, m = 8$ and $K = 14$.

and 4(b), we present the Probability of Detection versus SNR in order to verify and to quantify the improvement in terms of detection for the PS-AMF compared to the AMF. However, these figures show the benefit of taking into account the persymmetric structure of the CCM in the Gaussian case.

B. Non-Gaussian experiments

In the context of non-Gaussian clutter, Conte and De Maio in [5] and [6] have proposed two detectors derived respectively from the GLRT with some different estimators: the P-ANMF and the RP-ANMF. In [5], the persymmetry property is only used to separate their original set of secondary data $\mathbf{n}_k$ into two new uncorrelated and then independent sets of data $\mathbf{r}_{ek}$ and $\mathbf{r}_{ok}$, in order to render the detector matrix-CFAR and improve the performance in terms of detection. These new vectors have the same size as the original, and share the same texture. Their speckle components are i.i.d. and zero-mean complex Gaussian vectors. These news sets of secondary data allow the introduction of their new estimator for the CCM:

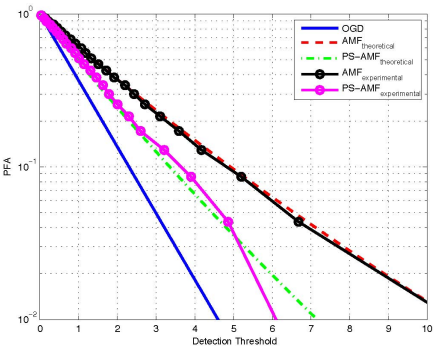
$$\widehat{\mathbf{\Sigma}} = \frac{1}{K}\sum_{k=1}^{K} \frac{\mathbf{r}_{ek}\,\mathbf{r}_{ek}^H}{\left(\left|\mathbf{r}_{ok}\mathbf{r}_{ok}^H\right|\right)_{i,i}}. \tag{40}$$

where $(\mathbf{A})_{i,i}$ stands for any (i,i)th element of the matrix $\mathbf{A}$.

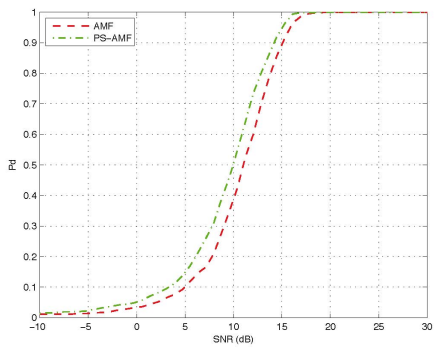
The previous estimator is then replaced in the classical GLRT-LQ given by (7) which leads to the P-ANMF detector defined as:

$$\Lambda_{P-ANMF} = \frac{|\mathbf{p}^H\,\widehat{\mathbf{\Sigma}}^{-1}\,\mathbf{x}|^2}{(\mathbf{p}^H\,\widehat{\mathbf{\Sigma}}^{-1}\,\mathbf{p})(\mathbf{x}^H\,\widehat{\mathbf{\Sigma}}^{-1}\,\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{P-ANMF}. \tag{41}$$

In [6], the same method is used to define two set of secondary data $\mathbf{r}_{ek}$ and $\mathbf{r}_{ok}$ and the FP matrix estimator



(a) Theoretical and experimental PFA-threshold curves for various Gaussian detectors.



(b) Probability of Detection versus SNR for the AMF and PS-AMF

Fig. 4. Comparison between Performance of detectors for $P_{fa} = 10^{-2}, m = 8$ and $K = 24$.

$\widehat{\mathbf{\Sigma}}_{(\text{inf})}$ is found by using the recursive procedure:

$$\widehat{\mathbf{\Sigma}}_{(t+1)} = \frac{N}{K} \sum_{k=1}^{K} \frac{\mathbf{r}_{ek}\, \mathbf{r}_{ek}^H}{\mathbf{r}_{ek}^H \left(\widehat{\mathbf{\Sigma}}_{(t)}\right)^{-1} \mathbf{r}_{ek}}, \tag{42}$$

with starting point:

$$\widehat{\mathbf{\Sigma}}_{(0)} = \frac{1}{K} \sum_{1}^{K} \frac{\mathbf{r}_{ek}\, \mathbf{r}_{ek}^H}{\left(\left|\mathbf{T}\, \mathbf{r}_{ok} \mathbf{r}_{ok}^H\, \mathbf{T}^H\right|\right)_{i,i}}. \tag{43}$$

This estimator is next replaced in the GLRT-LQ (7) to provide the RP-ANMF:

$$\Lambda_{RP-ANMF} = \frac{\left|\mathbf{p}^H\, \widehat{\mathbf{\Sigma}}_{(\text{inf})}^{-1}\, \mathbf{x}\right|^2}{\left(\mathbf{p}^H\, \widehat{\mathbf{\Sigma}}_{(\text{inf})}^{-1}\, \mathbf{p}\right)\left(\mathbf{x}^H\, \widehat{\mathbf{\Sigma}}_{(\text{inf})}^{-1}\, \mathbf{x}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{RP-ANMF}. \tag{44}$$

Please note that, as stated in [20], the solution $\widehat{\mathbf{\Sigma}}^{(\text{inf})}$ of the implicit FP matrix equation is unique and does not depend on the starting point.

In order to compare all these detectors (GLRT-LQ with $\mathbf{M}$ known, GLRT-SCM with the classical SCM, GLRT-FP, GLRT-PFP, P-ANMF and RP-ANMF), we present in Fig. 5(a) the PFA versus the detection threshold for all these detectors and in Fig. 5(b), the Probability of Detection versus the Signal to Noise Ratio (SNR). The simulated impulsive clutter is in this case chosen to be K-distributed [26]:

$$f_x(x) = \frac{2}{a\Gamma(\nu+1)} \left(\frac{x}{2a}\right)^{\nu+1} K_\nu\left(\frac{x}{a}\right),$$

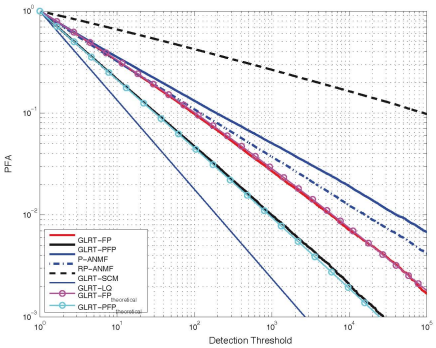
where $\Gamma(.)$ is the standard Gamma function [13] and K_ν is the modified Bessel function of order ν [13] and where a and ν are constant positive parameters.

These figures show the improvement in terms of detection of the RP-ANMF on the conventional GLRT-SCM (which is not efficient on non-Gaussian data) but also the improvement of the GLRT-PFP on all the other detectors. Moreover, theoretical results based on the asymptotic Wishart distributions of $\widehat{\mathbf{R}}_{FP}$ and $\widehat{\mathbf{R}}_{PFP}$ (circle lines) are displayed. It can be noticed that the simulated results are in very good agreement with the theory.

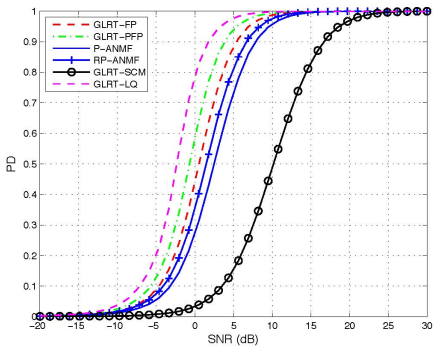
Similar analyses performed on experimental sea-clutter data give the same conclusion. Figure 6(a) and figure 6(b) show the sea clutter signal (range bins versus Pulse Repetition Interval and Range-Doppler) collected by the operational Over The Horizon radar from the French Aerospace Lab (ONERA) illumining the Atlantic ocean, and its associated range-Doppler image. In this context, we use a set of $m = 8$ pulses of the signal on the entire range bins group and $K = 16$ reference range bins to estimate the CCM. Fig. 7(a) and Fig. 7(b) show the improvement in detection performance on these data and the agreement between theoretical (circle line) and practical (solid line) results.

V. Conclusion

In the radar detection framework, estimation of the CCM is a major procedure in the detection process. In many applications, since this matrix commonly exhibits a particular structure, we therefore introduced in this paper two adaptive detection tests which take into account the widespread persymmetric structure of the CCM. In both contexts of Gaussian and non-Gaussian environments, we have presented and analyzed new

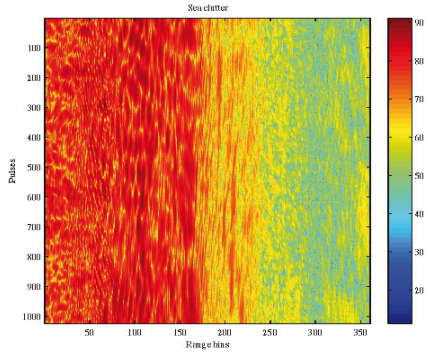


(a) Theoretical and experimental PFA-threshold curves for various non-Gaussian detectors.

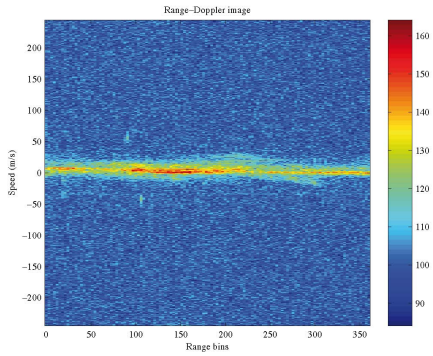


(b) Probability of Detection versus SNR for various non-Gaussian detectors

Fig. 5. Comparison between radar performance for various non-Gaussian detectors in simulated K-distributed clutter characterized by its parameter ν with $P_{fa} = 10^{-3}, m = 8, K = 16$ and $\nu = 0.2$.



(a) Sea clutter data from an over the horizon radar.



(b) Range-Doppler image of Atlantic Ocean sea clutter.

Fig. 6. Atlantic Ocean sea clutter data collected by the ONERA Over The Horizon Radar Nostradamus

detectors based on a modified estimator of the CCM.

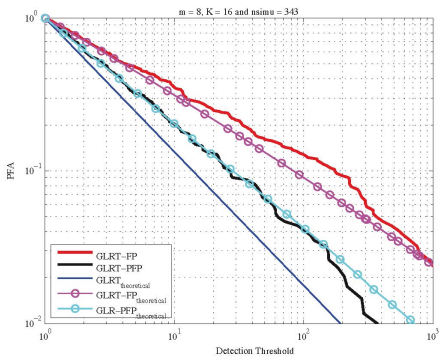
Under Gaussian assumption, the CCM estimator is developed based on the Maximum Likelihood procedure. The analytical distribution and some statistical properties of the corresponding detector, called the PS-AMF, have been established. These results are important since they enable a theoretical regulation of the false alarm, which is essential in the radar detection process.

The second detector is an extended version of the GLRT-LQ. It has been derived in the case of persymmetric non-Gaussian clutter modeled by SIRVs. After a transformation of the detection scheme, we have proposed an improved covariance matrix estimator, the PFP estimator. Its complete statistical analyses have exhibited good statistical performance. Moreover the corresponding GLRT-PFP detector has shown a wide improvement in terms of detection performance, as compared to the classical detectors.

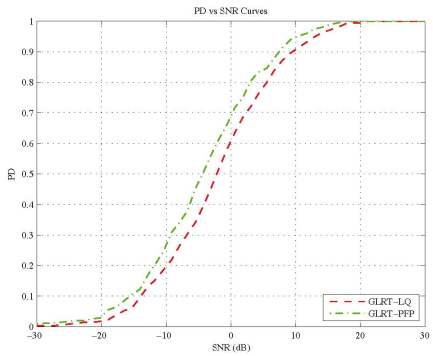
Finally, all these theoretical results have been validated on simulations for both Gaussian and non-Gaussian environments. Moreover, we have shown the validity and the good agreement between the theoretical and the experimental results, on both real ground and sea clutter data. Furthermore, the performance of the proposed detectors and the classical ones have been compared on these real data which highlight the improved performance of the former. These analysis have demonstrated the relevance and the advantage of exploiting the CCM structure.

VI. Acknowledgment

Authors would like to thank Thales Air System and DEMR/RBF unit of the ”French Aerospace lab” for providing their experimental data.



(a) Theoretical and experimental PFA-threshold curves for various non-Gaussian detectors



(b) Probability of Detection versus SNR for the GLRT-FP and GLRT-PFP

Fig. 7. Comparison between radar performance for various non-Gaussian detectors in sea clutter data for $P_{fa} = 10^{-2}, m = 8$ and $K = 16$.

VII. Appendix

A. Appendix A : Proof of the Proposition III.2

We will use in this derivation the Bartlett matrix decomposition [27]. Let us set $\mathbf{e}_1 = (1, 0, ..., 0)^\top$, $\mathbf{R} = \mathbf{R}^{\mathbf{1/2}}\,\mathbf{R}^{\top\mathbf{/2}}$ a factorization of $\mathbf{R}$ and $\mathbf{Q}$ a real unitary matrix such that $\mathbf{Q}\,\mathbf{R}^{-\mathbf{1/2}}\,\mathbf{s} = (\mathbf{s}^\top\,\mathbf{R}^{-\mathbf{1}}\,\mathbf{s})^{\mathbf{1/2}}\,\mathbf{e}_\mathbf{1}$. Note that the last equation is possible with $\mathbf{Q}$ real since $\mathbf{s}$ is itself real. Let us set

$$\widehat{\mathbf{W}} = 2K\,\mathbf{Q}\,\mathbf{R}^{-\mathbf{1/2}}\,\widehat{\mathbf{R}}_\mathbf{P}\,\mathbf{R}^{-\top/2}\,\mathbf{Q}^\top \text{ and } \mathbf{z} = \mathbf{Q}\mathbf{R}^{-\mathbf{1/2}}\mathbf{x}\,.$$

$\widehat{\mathbf{W}}$ is real Wishart distributed with $2K$ degrees of freedom and parameter matrix $\mathbf{I_m}$, $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I_m})$. Then, the test statistic Λ_{PS-AMF} (25) is equal to:

$$\Lambda_{PS-AMF} = \frac{\left|\mathbf{s}^\top\widehat{\mathbf{R}}_P^{-1}\mathbf{x}\right|^2}{\mathbf{s}^\top\widehat{\mathbf{R}}_P^{-1}\mathbf{s}}$$

$$= \frac{\left|\mathbf{s}^\top\mathbf{R}^{-\top/2}\mathbf{Q}^\top(\mathbf{Q}\mathbf{R}^{-1/2}\widehat{\mathbf{R}}_P\mathbf{R}^{-\top/2}\mathbf{Q}^\top)^{-1}\mathbf{Q}\mathbf{R}^{-1/2}\mathbf{x}\right|^2}{\mathbf{s}^\top\mathbf{R}^{-\top/2}\mathbf{Q}^\top(\mathbf{Q}\mathbf{R}^{-1/2}\widehat{\mathbf{R}}_P\mathbf{R}^{-\top/2}\mathbf{Q}^\top)^{-1}\mathbf{Q}\mathbf{R}^{-1/2}\mathbf{s}}$$

$$= 2K\frac{\left|\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-1}\,\mathbf{z}\right|^2}{\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-1}\,\mathbf{e}_1} \tag{45}$$

which may be rewritten, for our statistical analysis, as:

$$\Lambda_{PS-AMF} = 2K\,b\,a, \tag{46}$$

where:

$$a = \frac{\left|\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-1}\,\mathbf{x}\right|^2}{\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-2}\,\mathbf{e}_1}, b = \left(\frac{\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-2}\,\mathbf{e}_1}{\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-1}\,\mathbf{e}_1}\right).$$

We will show that a and b are independent and we will derive their statistical distribution. Let us first investigate the distribution of a. By introducing the unitary vector $\mathbf{v}$, defined by:

$$\mathbf{v} = \frac{1}{\left(\mathbf{e}_1^\top\widehat{\mathbf{W}}^{-2}\mathbf{e}_1\right)^{1/2}}\,\widehat{\mathbf{W}}^{-1}\mathbf{e}_1,$$

a may be rewritten as:

$$a = |\mathbf{v}^\top\mathbf{x}|^2.$$

It follows that the conditional distribution of $2a$ given $\widehat{\mathbf{W}}$ is a Chi square distribution with 2 degrees of freedom denoted by χ_2^2. This distribution does not involve $\widehat{\mathbf{W}}$, and

$$a \sim \frac{1}{2}\chi_2^2 \tag{47}$$

is therefore independent of $\widehat{\mathbf{W}}$ and consequently of b.

Now, to derive the PDF of b, we use the Bartlett matrix decomposition $\widehat{\mathbf{W}} = \mathbf{U}.\mathbf{U}^\top$ where $\mathbf{U} = (u_{ij})_{1\le i\le j\le m}$ is an upper triangular matrix whose random elements are independent and distributed as:

$$u_{i,i}^2 \sim \chi_{2K+i-m}^2 \text{ and } u_{i,j} \sim \mathcal{N}(0,1) \text{ for } i < j.$$

Let $u'_{i,j}$ be the elements of the matrix $\mathbf{U}^{-1}$ which is itself upper-triangular. By noticing that $\mathbf{U}^{-1}\,\mathbf{e}_1 = u'_{1,1}\,\mathbf{e}_1$, we have :

$$\begin{aligned} b &= \frac{\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-2}\,\mathbf{e}_1}{\mathbf{e}_1^\top\,\widehat{\mathbf{W}}^{-1}\,\mathbf{e}_1}. \\ &= \frac{\mathbf{e}_1^\top\,\mathbf{U}^{-\top}\,\mathbf{U}^{-1}\,\mathbf{U}^{-\top}\,\mathbf{U}^{-1}\mathbf{e}_1}{\mathbf{e}_1^\top\mathbf{U}^{-\top}\mathbf{U}^{-1}\mathbf{e}_1} \\ &= \|\mathbf{e}^\top\,\mathbf{U}^{-1}\|^2 \\ &= \sum_{j=1}^{m} u'^2_{1,j} \end{aligned} \tag{48}$$

We are thus lead to investigate the distribution of the squared norm of the first row of $\mathbf{U}^{-1}$. Let $\mathbf{u}'_{1,k}$ be the k-th order vector whose components are the k first elements of the first row of $\mathbf{U}^{-1}$ and $\mathbf{u}_{i,j}$ the i-th order vector the components of which are the i first elements of the j-th column of $\mathbf{U}$. The definition of $\mathbf{U}^{-1}$, i.e. $\mathbf{U}^{-1}\,\mathbf{U} = \mathbf{I}_m$, allows to determine recursively its elements according to:

$$u'_{1,1} = u_{1,1}^{-1}$$

and

$$u'_{1,k+1} = \frac{-\mathbf{u}'^{\top}_{1,k}\mathbf{u}_{k,k+1}}{u_{k+1,k+1}} \text{ for } 1 \leqslant k < m.$$

It follows that $\mathbf{u}'_{1,k}$ is independent of the $u_{i,j}$'s for $j > k$. Now we have from the above equation:

$$\begin{aligned} u'^2_{1,k+1} &= \frac{|\mathbf{u}'^{\top}_{1,k}\,\mathbf{u}_{k,k+1}|^2}{u^2_{k+1,k+1}} \\ &= \frac{|\mathbf{u}'^{\top}_{1,k}\,\mathbf{u}_{k,k+1}|^2}{\parallel \mathbf{u}'_{1,k} \parallel^2}\,\frac{\parallel \mathbf{u}'_{1,k} \parallel^2}{u^2_{k+1,k+1}} \\ &= = \alpha_k\,\frac{\parallel \mathbf{u}'_{1,k} \parallel^2}{u^2_{k+1,k+1}}. \end{aligned} \tag{49}$$

The conditional distribution of α_k given $\mathbf{u}'_{1,k}$ is a Chi square distribution with 1 degree of freedom denoted by χ^2_1. This distribution does not involve $\mathbf{u}'_{1,k}$ and $\alpha_k \sim \chi^2_1$ is therefore independent of $\mathbf{u}'_{1,k}$. Moreover, α_k is independent of $u_{i,j}$ for i and $j > k$. Now, notice from Eqn. (49) that $b = \|\mathbf{u}'_{1,m}\|^2$. Since $\parallel \mathbf{u}'_{1,m} \parallel^2 = \parallel \mathbf{u}'_{1,m-1} \parallel^2 + u'^2_{1,m}$, one has therefore:

$$\begin{aligned} b &= \parallel \mathbf{u}'_{m-1} \parallel^2 \left(1 + \frac{\alpha_{m-1}}{u^2_{m,m}}\right) \\ &= \frac{1}{u^2_{1,1}} \prod_{k=2}^{m} \left(1 + \frac{\alpha_{k-1}}{u^2_{k,k}}\right), \end{aligned} \tag{50}$$

where the α_k's are independent, independent of the $u^2_{k,k}$'s and χ^2_1-distributed. Since $\left(1 + \frac{\alpha_{k-1}}{u^2_{k,k}}\right)^{-1} \sim \beta_1\left(\frac{2K-m+k}{2}, \frac{1}{2}\right)$, we have from [28]:

$$\begin{aligned} b &= \frac{1}{u^2_{1,1}} \frac{1}{\prod_{k=2}^{m} \beta_1\left(\frac{2K-m+k}{2}, \frac{1}{2}\right)} \\ &\sim \frac{1}{\chi^2_{2K-m+1}} \frac{1}{\beta_1\left(\frac{2K-m+2}{2}, \frac{m-1}{2}\right)}, \end{aligned} \tag{51}$$

where the β_1's are independent (first kind) Beta distributed random variables. Finally we obtain from Eqn. (46), Eqn. (47) and Eqn. (50):

$$\Lambda_{PS-AMF} \sim \frac{\chi^2_2}{\chi^2_{2K-m+1}} \frac{K}{\beta_1\left(\frac{2K-m+2}{2}, \frac{m-1}{2}\right)} \tag{52}$$

which can be rewritten in terms of an F-distributed random variable:

$$\Lambda_{PS-AMF} \sim \frac{2\,K\,F(2, 2K-m+1)}{(2K-m+1)\,\beta_1\left(\frac{2K-m+2}{2}, \frac{m-1}{2}\right)} \tag{53}$$

Let us now derive the PFA-threshold relation. From [13], page 946:

$$\begin{aligned} PFA &= Pr(\Lambda_{PS-AMF} > \lambda) \\ &= \int_0^1 \frac{f_{\nu 1,\nu 2}(x)}{(1+\lambda\,x/K)^{\frac{2K-m+1}{2}}}\,dx \end{aligned} \tag{54}$$

where $f_{\nu 1,\nu 2}$ is the PDF of a first kind beta random variable with parameters $\nu 1 = \frac{2K-m+2}{2}$ and $\nu 2 = \frac{m-1}{2}$. We finally obtain:

$$PFA = \frac{\int_0^1 \frac{x^{\frac{2K-m+2}{2}-1}(1-x)^{\frac{m-1}{2}-1}}{(1+\lambda\,x/K)^{\frac{2K-m+1}{2}}}\,dx,}{B\left(\frac{2K-m+2}{2}, \frac{m-1}{2}\right)} \tag{55}$$

which may be expressed in terms of the hypergeometric function [13] page 558:

$$P_{fa} = {}_2F_1\left(\frac{2K-m+1}{2}, \frac{2K-m+2}{2}; \frac{2K+1}{2}; -\frac{\lambda}{K}\right). \tag{56}$$

The negative derivative of P_{fa} with respect to λ yields the PDF of Λ_{PS-AMF}. By using the expression of the derivative of hypergeometric functions given in [13] page 557, we obtain Eqn. (27) which concludes the proof.

B. Appendix B : Proof of the proposition III.5

From theorem III.4, $\Lambda_{GLRT-PFP}$ has the same asymptotic distribution as

$$\begin{aligned} \Lambda &= \frac{\left|\mathbf{e}_1^{\top}\widehat{\mathbf{W}}^{-1}\mathbf{w}\right|^2}{\left(\mathbf{e}_1^{\top}\widehat{\mathbf{W}}^{-1}\mathbf{e}_1\right)\left(\mathbf{w}^H\widehat{\mathbf{W}}^{-1}\mathbf{w}\right)} \\ &= \frac{\left|\mathbf{e}_1^{\top}\,\widehat{\mathbf{W}}^{-1}\,(\sqrt{2}\,\mathbf{w})\right|^2}{\left(\mathbf{e}_1^{\top}\,\widehat{\mathbf{W}}^{-1}\,\mathbf{e}_1\right)\left(\sqrt{2}\,\mathbf{w}^H\,\widehat{\mathbf{W}}^{-1}\,\sqrt{2}\,\mathbf{w}\right)}, \end{aligned} \tag{57}$$

where $(\sqrt{2}\,\mathbf{w}) = \mathbf{w}_1 + i\,\mathbf{w}_2$ with $\mathbf{w}_1$ and $\mathbf{w}_2$ uncorrelated and $\mathcal{N}(\mathbf{0}, \mathbf{I})$ distributed.

Thus

$$\Lambda = \frac{\left|\mathbf{e}_1^\top \widehat{\mathbf{W}}^{-1}\mathbf{w}_1\right|^2 + \left|\mathbf{e}_1^\top \widehat{\mathbf{W}}^{-1}\mathbf{w}_2\right|^2}{\left(\mathbf{e}_1^\top \widehat{\mathbf{W}}^{-1}\mathbf{e}_1\right)\left(\mathbf{w}_1^\top \widehat{\mathbf{W}}^{-1}\mathbf{w}_1 + \mathbf{w}_2^\top \widehat{\mathbf{W}}^{-1}\mathbf{w}_2\right)}.$$

For large K, $\widehat{\mathbf{W}}$ is real Wishart distributed with $K' = \frac{m}{m+1}2K$ degrees of freedom. The vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ can be decomposed on an orthonormal vectors triplet $(\mathbf{e}_1, \mathbf{f}_2, \mathbf{f}_3)$:

$$\begin{aligned}\mathbf{w}_1 &= \alpha_1\,\mathbf{e}_1 + \alpha_2\,\mathbf{f}_2\\ \mathbf{w}_2 &= \beta_1\,\mathbf{e}_1 + \beta_2\,\mathbf{f}_2 + \beta_3\,\mathbf{f}_3.\end{aligned}$$

where α_1, β_1 and β_2 are $\mathcal{N}(0,1)$ distributed, α_2^2 is χ^2_{m-1} distributed and β_3^2 is χ^2_{m-2} distributed. Moreover α_1, α_2, β_1, β_2, β_3 are independent and independent of $(\mathbf{f}_2, \mathbf{f}_3)$.

Let $(\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_m)$ be the canonical basis. Using an appropriate rotation $\mathbf{G}$ such that $\mathbf{G}\,(\mathbf{e}_1, \mathbf{f}_2, \mathbf{f}_3) = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, we have

$$\begin{aligned}\mathbf{G}\mathbf{w}_1 &= \alpha_1\,\mathbf{e}_1 + \alpha_2\,\mathbf{e}_2\\ &= \mathbf{v}_1 \qquad (58)\end{aligned}$$

$$\begin{aligned}\mathbf{G}\mathbf{w}_2 &= \beta_1\,\mathbf{e}_1 + \beta_2\,\mathbf{e}_2 + \beta_3\,\mathbf{e}_3\\ &= \mathbf{v}_2 \qquad (59)\end{aligned}$$

and Λ can be rewritten as

$$\Lambda = \frac{\left|\mathbf{e}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_1\right|^2 + \left|\mathbf{e}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_2\right|^2}{\left(\mathbf{e}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{e}_1\right)\left(\mathbf{v}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_1 + \mathbf{v}_2^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_2\right)}, \qquad (60)$$

where $\widehat{\mathbf{Z}} = \mathbf{G}\,\widehat{\mathbf{W}}\,\mathbf{G}^\top$.

Conditionally (and unconditionally) to $\mathbf{G}$, $\widehat{\mathbf{Z}}$ is Wishart distributed with K' degrees of freedom and parameter matrix $\mathbf{I}$. Let $\widehat{\mathbf{Z}} = \mathbf{L}^\top\,\mathbf{L}$ be the Bartlett's decomposition of $\widehat{\mathbf{Z}}$ [27] where $\mathbf{L} = (l_{i,j})_{1\le i\le j\le m}$ is a lower triangular matrix whose non-zeros random elements are independent and distributed as:

$$l_{i,i}^2 \sim \chi^2_{K'+i-m} \text{ and } l_{i,j} \sim \mathcal{N}(0,1) \text{ for } i > j.$$

Let $l'_{i,j}$ be the elements of the matrix $\mathbf{L}^{-1}$ which is lower-triangular itself. The following elements of $\mathbf{L}^{-1}$ are involved in Eqn. (60):

$$\begin{aligned}l'_{11} &= \frac{1}{l_{11}}, l'_{22} = \frac{1}{l_{22}},\\ l'_{21} &= -\frac{l_{21}}{l_{11}\,l_{22}}, l'_{33} = \frac{1}{l_{33}},\\ l'_{32} &= -\frac{l_{32}}{l_{22}\,l_{33}}, l'_{31} = -\frac{1}{l_{11}}\left(\frac{l_{31}}{l_{33}} - \frac{l_{32}\,l_{21}}{l_{22}\,l_{33}}\right). \qquad (61)\end{aligned}$$

From Eqn. (60), we define:

$$\alpha = \frac{\left|\mathbf{e}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_1\right|^2 + \left|\mathbf{e}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_2\right|^2}{\mathbf{e}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{e}_1},$$

which can be rewritten as:

$$\begin{aligned}\alpha &= \frac{\left|\mathbf{e}_1^\top \mathbf{L}^{-1}\mathbf{L}^{-\top}\mathbf{G}\,\mathbf{w}_1\right|^2 + \left|\mathbf{e}_1^\top \mathbf{L}^{-1}\mathbf{L}^{-\top}\mathbf{G}\,\mathbf{w}_2\right|^2}{\mathbf{e}_1^\top \mathbf{L}^{-1}\mathbf{L}^{-\top}\mathbf{e}_1},\\ &= (\alpha_1\,l'_{11} + \alpha_2\,l'_{21})^2 + (\beta_1\,l'_{11} + \beta_2\,l'_{21} + \beta_3\,l'_{31})^2,\end{aligned}$$

and

$$\beta = \mathbf{v}_1^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_1 + \mathbf{v}_2^\top \widehat{\mathbf{Z}}^{-1}\mathbf{v}_2,$$

which can be rewritten as:

$$\begin{aligned}\beta &= \mathbf{v}_1^\top \mathbf{L}^{-1}\mathbf{L}^{-\top}\mathbf{v}_1 + \mathbf{v}_2^\top \mathbf{L}^{-1}\mathbf{L}^{-\top}\mathbf{v}_2,\\ &= \alpha + (l'_{22}\,\alpha_2)^2 + (l'_{22}\,\beta_2 + l'_{32}\,\beta_3)^2 + (l'_{33}\,\beta_3)^2.\end{aligned}$$

We deduce that $\Lambda = \dfrac{\alpha}{\beta} = \dfrac{F}{1+F}$ with

$$\begin{aligned}F &= \frac{(\alpha_1\,l'_{11} + \alpha_2\,l'_{21})^2 + (\beta_1\,l'_{11} + \beta_2\,l'_{21} + \beta_3\,l'_{31})^2}{(l'_{22}\,\alpha_2)^2 + (l'_{22}\,\beta_2 + l'_{32}\,\beta_3)^2 + (l'_{33}\,\beta_3)^2}\\ &= \frac{A^2 + B^2}{C^2 + D^2 + E^2}. \qquad (62)\end{aligned}$$

where $A = (\alpha_1\,l_{22} - \alpha_2\,l_{21})$, $B = \beta_1\,l_{22} - \beta_2\,l_{21} - \beta_3\,\dfrac{l_{31}}{l_{33}}l_{22} + \dfrac{l_{32}l_{21}}{l_{33}}\,\beta_3$, $C = l_{11}\,\alpha_2$, $D = \dfrac{l_{11}l_{22}}{l_{33}}\,\beta_3$ and $E = l_{11}\left(\beta_2 - \dfrac{\beta_3}{l_{33}}\,l_{32}\right)$. In this equation, we have

$$\begin{aligned}&\left(\beta_1\,l_{22} - \beta_2\,l_{21} - \beta_3\,\frac{l_{31}}{l_{33}}l_{32} + \frac{l_{32}l_{21}}{l_{33}}\,\beta_3\right)^2\\ &= \left(l_{22}\left(\beta_1 - \frac{\beta_3}{l_{33}}l_{31}\right) - l_{21}\left(\beta_2 - \frac{\beta_3}{l_{33}}l_{32})\right)\right)^2\\ &= \left(1 + \left(\frac{\beta_3}{l_{33}}\right)^2\right)(l_{22}a - l_{21}b)^2 \qquad (63)\end{aligned}$$

with $a = \dfrac{1}{\sqrt{1 + (\beta_3/l_{33})^2}}\left(\beta_1 - \dfrac{\beta_3}{l_{33}}l_{31}\right)$ and $b = \dfrac{1}{\sqrt{1 + (\beta_3/l_{33})^2}}\left(\beta_2 - \dfrac{\beta_3}{l_{33}}l_{32}\right)$. Conditionally to β_3 and l_{33}, a and b are independent and $\mathcal{N}(0,1)$ distributed. Since their distribution does not involve β_3 and l_{33}, a and b are also independent of β_3 and u_{33}.

By replacing it in Eqn. (62), we finally obtain

$$F = \frac{(\alpha_1\,l_{22} - \alpha_2\,l_{21})^2 + \left(1 + \left(\frac{\beta_3}{l_{33}}\right)^2\right)(a\,l_{22} - b\,l_{21})^2}{(\alpha_2\,l_{11})^2 + \left(l_{11}\,l_{22}\frac{\beta_3}{l_{33}}\right)^2 + l_{11}^2\left(1 + \left(\frac{\beta_3}{l_{33}}\right)^2\right)b^2} \qquad (64)$$

where all the following random variables are independent and distributed according to:

$$a, b, \alpha_1, l_{21} \sim \mathcal{N}(0,1), \alpha_2^2 \sim \chi^2_{m-1}, \beta_3^2 \sim \chi^2_{m-2},$$

and

$$l_{11}^2 \sim \chi^2_{K'-m+1}, l_{22}^2 \sim \chi^2_{K'-m+2}, l_{33}^2 \sim \chi^2_{K'-m+3},$$

with $K' = \frac{m}{m+1}2K$ which concludes the proof.

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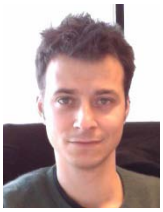

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Annexe K

Spatio-Temporal Adaptive Detector in Non-Homogeneous and Low-Rank Clutter

SPATIO-TEMPORAL ADAPTIVE DETECTOR IN NON-HOMOGENEOUS AND LOW-RANK CLUTTER

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ABSTRACT

Reducing the number of secondary data used to estimate the Clutter Covariance Matrix (CCM) for Space Time Adaptive Processing (STAP) techniques is still an active research topic. Low rank CCM estimates have already been proposed but only for homogeneous and Gaussian clutter. We propose in this paper to extend the low-rank CCM methods for heterogeneous and/or non-Gaussian clutter. We derive a new detector based on low-rank techniques and exploiting properties of the Normalized Sample Covariance Matrix (NSCM). This detector is shown to exhibit a smaller SNR loss than classical STAP detectors. Moreover, the new detector has a texture-CFAR property with respect to non-Gaussian SIRV model and has more robust behavior when some targets are present in the secondary data. We also give experimental comparison results between the classical STAP detectors and the new one for STAP data.

Index Terms— STAP, non-homogeneous and low rank clutter, CFAR detector, Normalized Sample Covariance Matrix.

1. INTRODUCTION

Space Time Adaptive Processing (STAP) is a recent technique [1] used in airborne phased array radar to detect moving target embedded in an interference background such as jamming or strong clutter. While conventional radars are capable of detecting targets both in the time domain related to target range and in the frequency domain related to target velocity, STAP uses an additional domain (space) related to the target angular localization. The consequence is a two-dimensional adaptive filtering technique which uses jointly temporal and spatial dimensions to suppress interference and to improve target detection. Recently, STAP can be jointly used with High Resolution Synthetic Aperture Radar (SAR) imaging waveforms for a better classification of the moving target. But in this case, the widely used hypothesis of a Gaussian noise is not valid anymore and detection performance significantly decreases. For that purpose, non-Gaussian models for the clutter have to be considered. In the literature of radar detection and estimation, the Spherically Invariant Random Vectors (SIRV) [2] are generally used for their statistical properties and for their good fitting with real data [3]. This modeling includes classical distributions as for example the Gaussian distribution, the K-distribution or the Weibull distribution.

One of the main challenging problem in STAP detection is to estimate the Clutter Covariance Matrix (CCM); better the accuracy of the estimate, better the detection performance. The CCM is estimated from signal-free and independent data, called the secondary data. Under non-Gaussian clutter assumptions, this raises several difficulties. First, the heterogeneity of the clutter deteriorates the CCM estimation accuracy. Indeed, if the secondary data do not result from the same parametrized distribution, i.e. the same covariance matrix, this estimation process makes no sense. On the other hand, the secondary data may contain parts of the target (e.g. sidelobes) present in the range cell under study, even if this problem can be partly avoided with guard cells. These two problems show the necessity of a robust CCM estimator and, as a consequence, the robustness of the STAP detector. This is the purpose of this paper.

In this paper, to overcome the problem of clutter heterogeneity, we propose to use the Normalized Sample Covariance Matrix (NSCM) [4] which is invariant to the clutter power variations (texture), while for the problem of target contamination, we propose a Low Rank (LR) [5] approach which requires fewer secondary data for the CCM estimation. The combination of these two techniques leads to a new improved detector, called the Low Rank-Normalized Sample Covariance Matrix Test (LR-NSCMT). Then, the statistical analysis of this detector provides the Constant False Alarm Rate (CFAR) properties with respect to the texture and, asymptotically, in term of the CCM. Moreover, the LR-NSCMT is applied to realistic STAP data (built from a true Very High Resolution SAR image) and its robustness to target contamination is validated.

The paper is organized as follows: Section 2 gives the

The authors thank the CELAR Agency for the STAP data.

problem statement while Sections 3 and 4 contain the main contributions. First, we derive the LR-NSCMT and we analyze its statistical properties and then, the detector is applied to real STAP data. Finally, Section 5 concludes this work.

2. PROBLEM STATEMENT

The problem of STAP [1] is considered when an airborne radar is used to detect a moving target. We focus here without loss of generality on GMTI (Ground Target Moving Indicator) applications for which illumination is made across the flying path. Typically, the radar receiver consists of an array of N antenna elements collecting M pulses in a coherent processing interval. We are interested in detecting a complex signal $\mathbf{s}$ corrupted by an additive SIRV clutter $\mathbf{c}$ in a NM-dimensional complex vector $\mathbf{y}$. The complex signal is parameterized by the target characteristics: the target amplitude α_0, the normalized Doppler frequency f_0 and the azimuthal angle θ_0 of the target. For a given target range bin under test, the detection scheme can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \ldots, N_s \\ H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \ldots, N_s \end{cases}, \tag{1}$$

where $\mathbf{y}_i$ are the N_s signal-free independent measurements, i.e. the secondary data, used to estimate the CCM.

Under the hypothesis H_1, it is assumed that the observed data consists in the sum of a signal $\mathbf{s} = \alpha_0\, \mathbf{p}(\theta_0, f_0)$ and clutter $\mathbf{c}$, where $\mathbf{p}(\theta_0, f_0)$ is the classical complex steering vector and α_0 is the signal complex amplitude. The parameters f_0, θ_0 and α_0 are unknown.

The clutter is modeled in this paper as a SIRV, a non-homogeneous Gaussian process with random power: its randomness is induced by spatial variation in the radar backscattering. A SIRV [2] is the product of the square root of a positive random variable τ (called the texture) with unknown Probability Density Function (PDF) $p(\tau)$, and a NM-dimensional independent complex Gaussian vector $\mathbf{x}$ (called the *speckle*), with zero-mean and covariance matrix $\mathbf{M} = E(\mathbf{x}\mathbf{x}^H)$ (^{H} denotes the conjugate transpose operator):

$$\mathbf{c} = \sqrt{\tau}\mathbf{x} \tag{2}$$

Note that the covariance matrix is normalized according to $\mathrm{Tr}(\mathbf{M}) = NM$ [6] for identifiability considerations.

Under hypotheses H_0 and H_1, the observations PDF depends on several unknown quantities: the amplitude α_0, the target parameters θ_0 and f_0, the texture PDF $p(\tau)$ and the covariance matrix $\mathbf{M}$. Therefore, a Generalized Likelihood Ratio Test (GLRT) is usually developed. The major difficulty comes from the estimation of the texture Probability Density Function (PDF). When $\mathbf{M}$ is known, this problem was solved in a different ways in [7, 8] (BORD) and [9] (GLRT - Linear Quadratic):

$$\Lambda(\mathbf{M}^{-1}, \theta, f) = \max_{\theta, f} \frac{|\mathbf{p}(\theta, f)^H \mathbf{M}^{-1} \mathbf{y}|^2}{(\mathbf{p}(\theta, f)^H \mathbf{M}^{-1} \mathbf{p}(\theta, f))(\mathbf{y}^H \mathbf{M}^{-1} \mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{3}$$

This test has interesting properties: it is CFAR with regards to the texture distribution and a closed-form relationship between the Probability of False Alarm (PFA) and the threshold λ has been derived.

But, in practice, $\mathbf{M}$ is generally unknown and has to be estimated from the secondary data $\mathbf{y}_i$. The CCM estimate is denoted $\hat{\mathbf{M}}$. Then, test (3) may be rewritten in its adaptive version by replacing $\mathbf{M}$ by its estimate $\hat{\mathbf{M}}$. In this case, and for Gaussian clutter, it is well known that the Reed-Mallett-Brennan's rule ensures a SNR Loss equal to 3 dB for $N_s = 2NM$.

Let λ_n with $1 \leq n \leq NM$ be the CCM eigenvalues. The CCM is well known to exhibit a low rank $r \ll NM$ provided by Brennan's rule [10]. This low rank assumption will be made in the following:

$$\lambda_1, \ldots, \lambda_r \gg \lambda_{r+1} = \ldots = \lambda_{NM} \tag{4}$$

In this low-rank clutter STAP context [5], let us denote by $\hat{\mathbf{\Pi}}_c$ the estimated projector onto the clutter subspace, i.e. the subspace spanned by the eigenvectors associated to the r greatest eigenvalues. The inverse CCM may then be approximated as follows (up to a scal factor):

$$\hat{\mathbf{M}}^{-1} \propto \mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_c \tag{5}$$

By substituting $\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_c$ for $\hat{\mathbf{M}}^{-1}$ in Eq. (3), we obtain a new test $\hat{\Lambda}(\hat{\mathbf{\Pi}}_c, \theta, f) = \Lambda(\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_c, \theta, f) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda'$.

This new test has several advantages: the first one is that the SNR loss has been shown to be roughly equal to 3 dB when $N_s = 2r$ in the case of homogeneous Gaussian secondary data [11, 12]. The second one concerns the possibility for the data $\mathbf{y}_i$ to be corrupted by the signal without a strong decrease of performance when the signal to clutter ratio is low [13]. This allows to have firstly a significant reduction of the required number of secondary data and, secondly, robustness to data corrupted by a target.

This paper proposes a new method to estimate the projector $\mathbf{\Pi}_c$ in the case of a non-homogeneous clutter.

3. ESTIMATION OF $\mathbf{\Pi}_C$

Let us first recall the common way to estimate the projector $\mathbf{\Pi}_c$ in the case of homogeneous clutter. The Sample Covariance Matrix (SCM) is computed from secondary data $\mathbf{y}_i$:

$$\hat{\mathbf{M}}_{SCM} = \frac{1}{N_s} \sum_{i=1}^{N_s} \mathbf{y}_i\, \mathbf{y}_i^H \tag{6}$$

The Eigenvalue Value Decomposition (EVD) of $\hat{\mathbf{M}}_{SCM}$ is next performed:

$$\hat{\mathbf{M}}_{SCM} = (\mathbf{U}_r \;\; \mathbf{U}_0) \begin{pmatrix} \mathbf{\Sigma}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_0 \end{pmatrix} (\mathbf{U}_r \;\; \mathbf{U}_0)^H, \tag{7}$$

where $\mathbf{U}_r$ and $\mathbf{U}_0$ are respectively two $(NM \times r)$ and $NM \times (NM - r)$ unitary matrices, $\mathbf{\Sigma}_r = \text{diag}\{\hat{\lambda}_1, \ldots, \hat{\lambda}_r\}$ and $\mathbf{\Sigma}_0 = \text{diag}\{\hat{\lambda}_{r+1}, \ldots, \hat{\lambda}_{NM}\}$ are the matrices of the estimated eigenvalues. From the low rank clutter assumption, we have the following eigenvalue property: $\hat{\lambda}_1, \ldots, \hat{\lambda}_r \gg \hat{\lambda}_{r+1}, \ldots, \hat{\lambda}_{NM}$. Next, we obtain the estimated projector onto the clutter subspace [13]:

$$\hat{\mathbf{\Pi}}_{cSCM} = \mathbf{U}_r \, \mathbf{U}_r^H \tag{8}$$

The resulting Low Rank Sample Covariance Matrix Test (LR-SCMT) becomes:

$$\hat{\Lambda}_{LR-SCMT}(\hat{\mathbf{\Pi}}_{cSCM}, \theta, f) = \Lambda(\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_{cSCM}, \theta, f) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda'' \tag{9}$$

In the case of non-homogeneous clutter, it is well-known that the SCM is not a good estimate of the clutter covariance matrix. As a consequence, the same conclusion holds for the estimate $\hat{\mathbf{\Pi}}_{cSCM}$ of $\mathbf{\Pi}_c$. In heterogeneous clutter modeled by SIRV processes, the NSCM is a very interesting alternative to $\hat{\mathbf{M}}_{SCM}$:

$$\hat{\mathbf{M}}_{NSCM} = \frac{NM}{N_s} \sum_{i=1}^{N_s} \frac{\mathbf{y}_i \, \mathbf{y}_i^H}{\mathbf{y}_i^H \, \mathbf{y}_i} \tag{10}$$

The statistical properties of $\hat{\mathbf{M}}_{NSCM}$ have been studied in detail in [14]. In this paper it was shown that, despite its bias, $E(\hat{\mathbf{M}}_{NSCM})$ has the same eigenvectors as the CCM $\mathbf{M}_c$. The associated eigenvalues are different, but their ordering and their multiplicity are identical. Therefore, the projector $\hat{\mathbf{\Pi}}_{cNSCM}$ built from the NSCM (10) using steps (7) and (8) is a good candidate for estimating $\mathbf{\Pi}_c$ in the case of non-homogeneous clutter. Moreover, this estimate is easily shown to be consistent in our SIRV framework:

$$\hat{\mathbf{\Pi}}_{cNSCM} \xrightarrow{P} \mathbf{\Pi}_c, \tag{11}$$

when $N_s \to \infty$. This new estimate of the projector leads to a detection test that we call the Low Rank Normalized Sample Covariance Matrix Test (LR-NSCMT):

$$\hat{\Lambda}_{LR-NSCMT}(\hat{\mathbf{\Pi}}_{cNSCM}, \theta, f) = \Lambda(\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_{cNSCM}, \theta, f) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda''' \tag{12}$$

This new detector is texture-CFAR and also asymptotically $\mathbf{M}$-CFAR thanks to the consistency of $\hat{\mathbf{\Pi}}_{cNSCM}$ (Eq. (11)).

In the next section, the performance of this detector are compared on experimental STAP data to those of classical detection schemes.

4. RESULTS ON STAP DATA

For non-homogeneous clutter, performance of $\hat{\Lambda}_{LR-NSCMT}$ would be better than $\hat{\Lambda}_{SCMT}$ and $\hat{\Lambda}_{LR-SCMT}$ ones, but another interested advantage of this new detector can be pointed out: its excellent behavior when the secondary data are contaminated by the target.

The STAP data are provided by the french DGA/CELAR's simulator that allows to synthesize, in side looking configuration, STAP datacubes from very high resolution RAMSES Synthetic Aperture Radar (SAR) [15]. The number of sensors is $N = 4$ and the number of coherent pulses can be up to $M = 64$. The center frequency and the bandwidth are respectively equal to $f_0 = 10$ GHz and $B = 5$ MHz. The radar velocity is given by $V = 100$ m/s. The inter-element spacing is $d = 0.3$ m and the pulse repetition frequency is $f_r = 1kHz$. The value of parameter β is therefore equal to $2/3$ and the estimated clutter rank is here equal to $r = 46$ (for $M = 64$) in comparison to the full size of clutter covariance matrix, $MN = 256$. For this particular STAP datacube, clutter statistic is closer to Gaussian's one than SIRV's one. In this scenario, three targets are present: (4 m/s, $0\,\text{deg}$, bin 216), (4 m/s, $0\,\text{deg}$, bin 256) and (-4 m/s, $0\,\text{deg}$, bin 296). This allows to compare performance of the three detectors $\hat{\Lambda}_{SCMT}$, $\hat{\Lambda}_{LR-SCMT}$ and $\hat{\Lambda}_{LR-NSCMT}$ for the range bin 256 under test when the secondary data are contaminated by the targets located in range bins 216 and 296.

Figure 1 shows the results of $\hat{\Lambda}_{SCMT}$ (result of $\hat{\Lambda}_{NSCMT}$ is quite the same), $\hat{\Lambda}_{LR-SCMT}$ and $\hat{\Lambda}_{LR-NSCMT}$ when the targets in range bins 216 and 296 are removed from secondary data. As the number of secondary data is $N_s = 410 < 2NM$, performance of $\hat{\Lambda}_{SCMT}$ detector is poor but the target can still be detected. Results of both low rank detectors are much better. As clutter is homogeneous, their results are the same. This is the first advantage of low rank detectors which need less secondary data for an equivalent performance ($2r \ll 2NM$).

Figure 2 shows the results of $\hat{\Lambda}_{SCMT}$, $\hat{\Lambda}_{LR-SCMT}$ and $\hat{\Lambda}_{LR-NSCMT}$ when all secondary data are considered. First, it can be noticed that the target response is not detected anymore by $\hat{\Lambda}_{SCMT}$ due to the presence of the target contamination in the secondary data. Performance of $\hat{\Lambda}_{LR-SCMT}$ is degraded: in comparison to the previous $\hat{\Lambda}_{LR-SCMT}$ performance without contamination, there is a loss of 6 dB but the target of interest is still detected. Actually, a part of the target in secondary data is estimated in the orthogonal clutter subspace. Finally, performance of $\hat{\Lambda}_{LR-NSCMT}$ is better (no dB loss) and close to the previous one when secondary data are not suffering from target contamination. This is the second advantage of the combination of low rank and the NSCM.

5. CONCLUSION

In this paper, we developed a new low-rank STAP detector for non-homogeneous clutter. This detector is based on statistical properties of the Normalized Sample Covariance Matrix. We showed that this detector has CFAR properties in term of texture and, asymptotically, in term of the CCM. When tested on experimental STAP data obtained from a real clutter, this detector has been shown to provide increased performance compared to conventional full rank detectors. It also exhibits

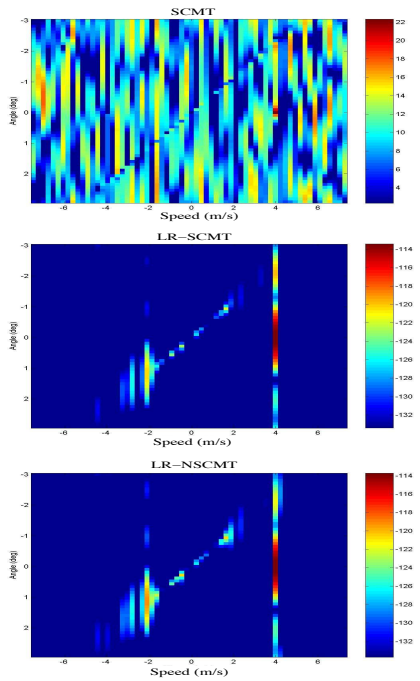

Fig. 1. Comparison of the three detectors $\hat{\Lambda}_{SCMT}$ (top), $\hat{\Lambda}_{LR-SCMT}$ (middle) and $\hat{\Lambda}_{LR-NSCMT}$ (bottom) without target contamination.

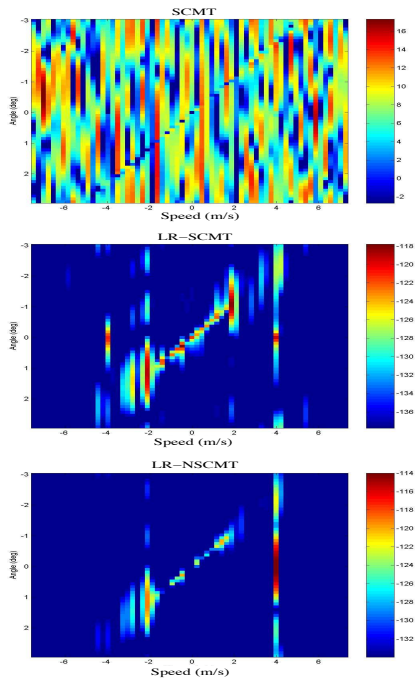

Fig. 2. Comparison of the three detectors $\hat{\Lambda}_{SCMT}$ (top), $\hat{\Lambda}_{LR-SCMT}$ (middle) and $\hat{\Lambda}_{LR-NSCMT}$ (bottom) with target contamination.

robustness to target contamination compared to existing low rank detection schemes.

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Annexe L

MIMO Radar Detection in Non-Gaussian and Heterogeneous Clutter

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MIMO Radar Detection in Non-Gaussian and Heterogeneous Clutter

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***Abstract*—In this paper, the generalized likelihood ratio test-linear quadratic (GLRT-LQ) has been extended to the multiple-input multiple-output (MIMO) case where all transmit–receive subarrays are considered jointly as a system such that only one detection threshold is used. The GLRT-LQ detector has been derived based on the Spherically Invariant Random Vector (SIRV) model and is constant false alarm rate (CFAR) with respect to the clutter power fluctuations (also known as the texture). The new MIMO detector is then shown to be texture-CFAR as well. The theoretical performance of this new detector is first analytically derived and then validated using Monte Carlo simulations. Its detection performance is then compared to that of the well-known Optimum Gaussian Detector (OGD) under Gaussian and non-Gaussian clutter. Next, the adaptive version of the detector is investigated. The covariance matrix is estimated using the Fixed Point (FP) algorithm which enables the detector to remain texture- and matrix-CFAR. The effects of the estimation of the covariance matrix on the detection performance are also investigated.**

***Index Terms*—Detection performance, generalized likelihood ratio test-linear quadratic (GLRT-LQ), multiple-input multiple-output (MIMO) radar, non-Gaussian clutter, Spherically Invariant Random Vector (SIRV).**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) is a technique used in communications to increase data throughput and link range without additional bandwidth or transmit power. This is achieved by higher spectral efficiency and link reliability or diversity. Recently, this concept has been used for radar applications [1]. In the context of radar, a (statistical) MIMO radar is one where both the transmit and receive elements are sufficiently separated so as to provide spatial diversity. This reduces the fluctuations of the target radar cross section (RCS) due to the different target aspects seen by each pair of transmit–receive elements [2]. It can also be used to improve the probability of detection [3] and resolutions [4]. Several applications including target classification and high-resolution imaging can be enhanced by a MIMO radar. On top of that, each transmit element sends a different (orthogonal) waveform which can be separated at the receive end. This provides waveform diversity which in turn increases the separation between clutter and target returns [5]. According to [6], it also improves the identifiability of target parameters, enables the direct application of adaptive arrays for target detection, and enhances flexibility for transmit beampattern design.

MIMO procedures for radar have been widely studied for the case of additive Gaussian noise. In [3], [7], and [8], the authors consider the detection performance for widely separated antennas while in [6] and [9] the authors consider colocated antennas. The combined case of having widely separated antenna subarrays which contain colocated antenna elements is considered in [10]. However, these detectors may exhibit poor performance when the additive noise is no longer Gaussian.

In the case of MIMO radar, it is even more important to consider non-Gaussian models. First, as mentioned earlier, one advantage of a MIMO imaging radar is improved resolution. Usually, in each resolution cell, there is a large number of scatterers. According to the Central Limit Theorem (CLT), the clutter power in each cell is almost constant and the clutter is considered to follow a Gaussian distribution. However, as the resolution cell becomes smaller, there are fewer scatterers in it and the CLT will no longer apply. Non-Gaussian models which take into account the variation in clutter power therefore have to be used. Moreover, as the resolution cell becomes smaller, it is more likely for the illuminated area to be nonhomogeneous.

Second, for configurations with widely separated transmit–receive subsystems, the target returns received by each receive subsystem are different due to different aspect angles, thus reducing target RCS fluctuations. Similarly, the clutter returns vary from subsystem to subsystem. Hence, it is important to use non-Gaussian models which better reflect the clutter power fluctuations. Indeed, experimental radar clutter measurements [11]–[14] have been found to fit non-Gaussian statistical models such as the well-known Spherically Invariant Random Vector (SIRV) model [15]–[17] which has been widely studied, particularly in terms of detection [18]–[20].

In this paper, we consider MIMO radar detection under non-Gaussian heterogeneous and impulsive clutter. The non-Gaussian statistical model we use is the SIRV model. This model writes clutter returns as the product of a Gaussian random process (*speckle*) and the square-root of a positive

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random variable τ (*texture*). Speckle models temporal fluctuations of clutter while texture models spatial fluctuations of clutter power. The SIRV can model different non-Gaussian clutter depending on the chosen texture, with Gaussian clutter as the special case where the texture is a constant. It also has a Gaussian kernel which means that certain classical results can still be applied. For example, the maximum-likelihood (ML) estimates of target parameters are given by the maximization of the traditional matched filter.

In radar applications, the clutter covariance matrix is usually unknown and has to be estimated from target-free secondary data. Under non-Gaussian clutter, the classical Sample Covariance Matrix (SCM) is no longer the ML estimate. Hence, we consider here the fixed point estimate (FPE) which was first introduced in [21] and [22] and then fully analyzed in [23] and [24]. FPE is the exact ML estimate when the texture is assumed to be deterministic and unknown. On the other hand, when τ is a positive random variable, FPE is an approximate ML estimate.

The main contribution of this paper is the derivation of a MIMO non-Gaussian detector for constant false alarm rate (CFAR) detection and estimation. Previously, several generalized likelihood ratio tests (GLRTs) like the GLRT-linear quadratic (GLRT-LQ) detector in [18] and [19] and asymptotic Bayesian optimum radar detector (aBORD) in [20] have been derived based on the SIRV model and have been shown to be CFAR with respect to the texture (texture-CFAR). We generalized the GLRT-LQ detector to the MIMO case where all subarrays are considered jointly as a system such that only one detection threshold is used to regulate the false alarm rate. The resulting MIMO detector is also texture-CFAR. The Gaussian case is a special case of SIRV where the probability density function (pdf) of the texture is given by $p(\tau) = \sigma^2\delta(\tau - 1)$ and σ^2 is the clutter power. Hence, this new detector is expected to give similar results as the well-known optimum Gaussian detector (OGD) under Gaussian clutter and superior results under non-Gaussian clutter.

The adaptive version of this new MIMO detector is then considered. Using the FPE to estimate the covariance matrix, this detector is shown to be texture-CFAR. Moreover, it is matrix-CFAR as it does not depend on the unknown covariance matrix. The detection performance depends on an additional parameter, L_r: the number of secondary data containing only clutter returns which are used to obtain the FPE.

This paper is organized as follows. First, we consider a general signal model for MIMO radar (Section II). Instead of considering a single element at each location, we assume that there is a subarray containing one or more elements. Section III-A shows the derivation of a MIMO non-Gaussian detector. In Section III-B, the theoretical performance of this new detector is derived and verified using Monte Carlo simulations. The detection performance is then analyzed through Monte Carlo simulations and compared to that of the OGD (extended to the MIMO case in [7], [25]) under both Gaussian and non-Gaussian clutter (Section III-C). Next, in Section IV-A, the adaptive version of this new detector is considered, using the FPE as the covariance matrix estimate. The effects of the covariance matrix estimation on the detection performance are investigated using Monte Carlo simulations in Section IV-B. Finally, conclusions are presented in Section V.

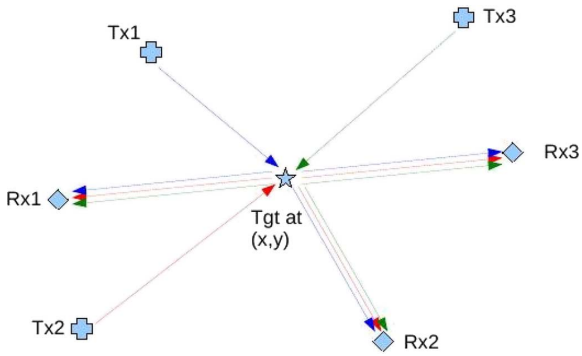

Fig. 1. Configuration where $\tilde{N} = \tilde{M} = 3$.

II. Signal Model

In this paper, vectors and matrices are denoted by lowercase and uppercase bold, respectively. The superscript "†" denotes the Hermitian operator.

In this section, we consider a target located at (x, y). Let there be $\tilde{N}$ transmit subarrays and $\tilde{M}$ receive subarrays. The nth transmit and mth receive subarray contain N_n and M_m elements, respectively, for $n = 1, \ldots, \tilde{N}$ and $m = 1, \ldots, \tilde{M}$. The configuration can be seen in Fig. 1. The RCS of the target seen by each receive–transmit pair is assumed to be different but deterministic. The elements within each subarray are assumed to be closely spaced such that the RCS is the same.

Let $\mathbf{v}(\theta_{t,n})$ and $\mathbf{a}(\theta_{r,m})$ be the steering vectors and $\theta_{t,n}$ and $\theta_{r,m}$ the angular location for the target for the nth transmit and mth receive subarray, respectively.

Different orthogonal waveforms are assumed to be transmitted from every transmit element, even those belonging to the same subarray. This means that while there is a closely spaced configuration within the subarray, the configuration is still MIMO due to the diversity of waveforms within the subarray. This enables the received signal from each transmit element to be separated such that transmit angular information can be obtained.

The received signal after matched filtering can be expressed as

$$\begin{aligned}\mathbf{y}_{m,n}(t_{m,n}) &= \mathrm{B}(m,n)\mathbf{a}(\theta_{r,m}) \otimes \mathbf{v}(\theta_{t,n}) + \mathbf{z}_{m,n}, \\ &= \mathrm{B}(m,n)\mathbf{p}_{m,n} + \mathbf{z}_{m,n} \end{aligned} \tag{1}$$

where $t_{m,n}$ is the corresponding time-delay for $m - n$ receive–transmit pair and $\mathbf{B}$ is the $\tilde{M} \times \tilde{N}$ matrix containing the RCS of the target seen by each receive–transmit pair. The vector $\mathbf{p}_{m,n}$ is the $M_m N_n \times 1$ bistatic angular steering vector which is equal to $\mathbf{a}(\theta_{r,m}) \otimes \mathbf{v}(\theta_{t,n})$ and $\otimes$ stands for the Kronecker product.

The vector $\mathbf{z}_{m,n}$ is a $M_m N_n \times 1$ vector containing the clutter returns and it is modeled by SIRV which is essentially a non-homogeneous and non-Gaussian process with random power. The variation in the power arises from the spatial variation in

the backscattering of the clutter. According to [16], a SIRV is the product of the square root of a positive random variable τ (texture) and a L-dimensional independent complex circular Gaussian vector $\mathbf{x}$

$$\mathbf{z} = \sqrt{\tau}\mathbf{x}.$$

The vector $\mathbf{x}$ has zero mean and covariance matrix $\mathbf{M}$, denoted by $\mathcal{CN}(\mathbf{0}, \mathbf{M})$. The matrix $\mathbf{M} = \mathbb{E}[\mathbf{x}\mathbf{x}^\dagger]$ is assumed to be normalized such that $tr(\mathbf{M}) = L$ for identifiability considerations [21]. $\mathbb{E}[\cdot]$ denotes the expectation and $tr(\cdot)$ the trace. The pdf of a SIRV is then given by

$$p_{\mathbf{z}}(\mathbf{z}) = \int_0^\infty g_{\mathbf{z}}(\mathbf{z}, \tau) p(\tau) d\tau$$

where $p(\tau)$ is the texture pdf and

$$g_{\mathbf{z}}(\mathbf{z}, \tau) = \frac{1}{(\pi\tau)^L |\mathbf{M}|} \exp\left(-\frac{\mathbf{z}^\dagger \mathbf{M}^{-1} \mathbf{z}}{\tau}\right).$$

For each radar parameter to be determined, a steering vector is required. The resulting steering vector is simply the Kronecker product of all the steering vectors. In the bistatic case, as the transmit and receive angles are no longer the same, a separate steering vector is required for transmit $(\mathbf{v}(\theta_{t,n}))$ and receive $(\mathbf{a}(\theta_{r,m}))$.

For simplicity of notation, $\mathbf{y}_{m,n}(t_{m,n})$ will be written simply as $\mathbf{y}_{m,n}$.

III. MIMO Non-Gaussian Detector

A. Derivation

Let us now consider the detection problem as the following binary hypothesis test

$$\begin{cases} \mathrm{H}_0 : \mathbf{y}_{m,n} = \mathbf{z}_{m,n} \\ \mathrm{H}_1 : \mathbf{y}_{m,n} = \mathrm{B}(m,n)\mathbf{p}_{m,n} + \mathbf{z}_{m,n} \end{cases} \quad n = 1, \ldots, \tilde{N},\ m = 1, \ldots, \tilde{M}$$

Under the hypothesis H_0, it is assumed that the received signal contains only clutter returns and hence there is no target. Under the hypothesis H_1, it is assumed that the received signal contains a deterministic signal on top of the clutter returns and hence a target is present at the location (x, y).

The dimension of each received signal, $M_m N_n$, is assumed to be greater than one since the steering vector does not give any information if its length is one.

1) $\tilde{M} = 1$ and $\tilde{N} = 1$: We begin with the case where $\tilde{M} = 1$ and $\tilde{N} = 1$. The dimension of the received signal is $L = M_1 N_1$.

As there is only one received signal and steering vector, let them be denoted simply by $\mathbf{y} = \mathbf{y}_{1,1}$ and $\mathbf{p} = \mathbf{p}_{1,1}$. The classical likelihood ratio test is given by

$$\Lambda(\mathbf{y}) = \frac{p(\mathbf{y}|\mathrm{H}_1)}{p(\mathbf{y}|\mathrm{H}_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta.$$

If we assume that the covariance matrix $\mathbf{M}$ is known and according to [20], $p(\mathbf{y}|\mathrm{H}_1)$ and $p(\mathbf{y}|\mathrm{H}_0)$ are replaced by their Bayesian estimates and asymptotically, we obtain

$$\Lambda_1(\mathbf{y})^L = \left[\frac{1}{1 - \frac{|\mathbf{p}^\dagger \mathbf{M}^{-1} \mathbf{y}|^2}{(\mathbf{p}^\dagger \mathbf{M}^{-1} \mathbf{p})(\mathbf{y}^\dagger \mathbf{M}^{-1} \mathbf{y})}}\right]^L \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta \tag{2}$$

which is equivalent to

$$1 - \frac{1}{\Lambda_1(\mathbf{y})} = \frac{|\mathbf{p}^\dagger \mathbf{M}^{-1} \mathbf{y}|^2}{(\mathbf{p}^\dagger \mathbf{M}^{-1} \mathbf{p})(\mathbf{y}^\dagger \mathbf{M}^{-1} \mathbf{y})} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \frac{\sqrt[L]{\eta} - 1}{\sqrt[L]{\eta}} \tag{3}$$

and leads to the GLRT-LQ test. The same test is derived in [18] through an asymptotic development of the test statistic in the GLRT, and in [19] it is based on the ML estimation of the clutter power. Note that no explicit form on the steering vector is assumed in the derivation of the GLRT-LQ. Thus, these detectors can also be applied to our case where $\mathbf{p} = \mathbf{a}(\theta_{r,1}) \otimes \mathbf{v}(\theta_{t,1})$.

2) $\tilde{M} \geq 1$ and $\tilde{N} \geq 1$: We consider now the general case where $\tilde{M} \geq 1$ and $\tilde{N} \geq 1$. As the transmit–receive subarrays are widely separated, the clutter returns can be considered to be independent, hence $\mathbf{z}_{m,n}$ are independent and the likelihood ratio test becomes

$$\begin{aligned} \Lambda(\mathbf{Y}) &= \frac{p(\mathbf{Y}|\mathrm{H}_1)}{p(\mathbf{Y}|\mathrm{H}_0)}, \\ &= \frac{\prod_{m,n} p(\mathbf{y}_{m,n}|\mathrm{H}_1)}{\prod_{m,n} p(\mathbf{y}_{m,n}|\mathrm{H}_0)}, \\ &= \prod_{m,n} \frac{p(\mathbf{y}_{m,n}|\mathrm{H}_1)}{p(\mathbf{y}_{m,n}|\mathrm{H}_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta \end{aligned}$$

where $\mathbf{Y}$ is the matrix containing all the received signals $\mathbf{Y} = [\mathbf{y}_{1,1}, \cdots, \mathbf{y}_{\tilde{M},\tilde{N}}]$.

Using (2), the GLRT-LQ, extended to the MIMO case, is given by

$$\Lambda(\mathbf{Y}) = \prod_{m,n} \left[\frac{1}{1 - \frac{|\mathbf{p}_{m,n}^\dagger \mathbf{M}_{m,n}^{-1} \mathbf{y}_{m,n}|^2}{(\mathbf{p}_{m,n}^\dagger \mathbf{M}_{m,n}^{-1} \mathbf{p}_{m,n})(\mathbf{y}_{m,n}^\dagger \mathbf{M}_{m,n}^{-1} \mathbf{y}_{m,n})}}\right]^{M_m N_n} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta \tag{4}$$

where $\mathbf{M}_{m,n}$ is the covariance matrix for the $m - n$ receive–transmit pair.

From (4), we see that when the number of elements in each subarray $M_m N_n$ is not a constant, the expression for MIMO GLRT-LQ is very complicated and it is difficult for its detection performance to be analyzed theoretically. Monte Carlo simulations have to be run in such cases to obtain the detection performance. Instead, in the next section, we will consider the particular case where $M_m N_n = L$ is a constant.

3) $\tilde{M} \geq 1, \tilde{N} \geq 1$ and $M_m N_n = L$: If we consider now that the number of elements in each transmit and receive subarray is the same, i.e.,

$$\begin{cases} M_m = M_{m'} & m \neq m' \\ N_n = N_{n'} & n \neq n' \end{cases}$$

then the total number of elements in each subarray is the same

$$M_m N_n = L \quad \forall\, m = 1, \ldots, \tilde{M},\ n = 1, \ldots, \tilde{N}.$$

Moreover, as the returns from each transmit–receive pair are independent, having $\tilde{N}$ transmit subarrays and $\tilde{M}$ receive subarrays is equivalent to having one transmit subarray and $K = \tilde{N}\tilde{M}$ receive subarrays. Equation (4) can thus be simplified to be

$$\prod_{i=1}^{K} \Lambda_i(\mathbf{y}_i) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \sqrt[L]{\eta} = \lambda \tag{5}$$

where

$$\Lambda_i(\mathbf{y}_i) = \left[\frac{1}{1 - \frac{\left|\mathbf{p}_i^\dagger \mathbf{M}_i^{-1} \mathbf{y}_i\right|^2}{\left(\mathbf{p}_i^\dagger \mathbf{M}_i^{-1} \mathbf{p}_i\right)\left(\mathbf{y}_i^\dagger \mathbf{M}_i^{-1} \mathbf{y}_i\right)}} \right] \tag{6}$$

and $\mathbf{M}_i$ is the covariance matrix of $\mathbf{y}_i$.

B. Theoretical Performance

The probability of false alarm P_{fa} is the probability of choosing H_1 when the target is absent (H_0)

$$P_{fa} = P\left(\prod_{i=1}^{K} \Lambda_i(\mathbf{y}_i) > \lambda | \mathrm{H}_0\right).$$

Theorem III.1: Given a MIMO radar system containing K subsystems and L elements $(L > 1)$ in each subsystem and using the detector given in (5), the probability of false alarm is given by

$$P_{fa} = \lambda^{-L+1} \sum_{k=0}^{K-1} \frac{(L-1)^k}{k!} (\ln \lambda)^k. \tag{7}$$

Remark III.1:

- When there is only one subsystem $(K = 1)$, the probability of false alarm is given by $P_{fa} = \lambda^{-L+1}$. This is equivalent to the expression derived in [18]. When there is only one element in the subsystem $(L = 1)$, this gives us $P_{fa} = 1$. This implies that the test breaks down as more than one element is required to estimate the clutter power.
- P_{fa} depends only on K and L and not on the clutter parameters, hence showing the texture-CFAR property of the test given by (5). Moreover, it is clear that the detector does not depend on the covariance matrices which can be different for each transmit–receive subarray.
- The closed-form expression is useful for the analysis of detection performance. Using (7), the threshold to be used in the detection test is theoretically set to ensure a given P_{fa}.

Proof: Under H_0, the received signal contains only clutter returns

$$\mathbf{y}_i = \mathbf{z}_i = \sqrt{\tau_i}\mathbf{x}_i$$

where $\mathbf{x}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{M}_i)$, and $\sim$ means to be distributed. As τ_i is canceled out in (6), $\Lambda_i(\mathbf{y}_i) = \Lambda_i(\mathbf{x}_i)$.

According to [26], the GLRT-LQ detector can be expressed in terms of an F-statistics

$$\frac{\left|\mathbf{p}_i^\dagger \mathbf{M}_i^{-1} \mathbf{y}_i\right|^2}{\left(\mathbf{p}_i^\dagger \mathbf{M}_i^{-1} \mathbf{p}_i\right)\left(\mathbf{y}_i^\dagger \mathbf{M}_i^{-1} \mathbf{y}_i\right)} = \frac{t}{t+1}$$

where $(L-1)t$ is a centralized F-distributed random variable with parameters $v_1 = 2$ and $v_2 = 2L - 2$. The pdf of $t' = (L-1)t$ can be expressed as

$$f(t') = \frac{\Gamma\left(\frac{v_1+v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} \frac{1}{t'} \left(\frac{v_1 t'}{v_1 t' + v_2}\right)^{\frac{v_1}{2}} \left(1 - \frac{v_1 t'}{v_1 t' + v_2}\right)^{\frac{v_2}{2}} \tag{8}$$

where Γ is the Gamma function as defined in [27]. As v_1 and v_2 are both positive integers, (8) can be simplied to

$$f(t') = \left(\frac{L-1}{t' + L - 1}\right)^L.$$

Each $\Lambda_i(\mathbf{x}_i)$ can also be expressed in terms of t'

$$\Lambda_i(\mathbf{x}_i) = \frac{1}{1 - \frac{t}{t+1}} = 1 + t = 1 + \frac{t'}{L-1}.$$

Let $u_i = \Lambda_i(\mathbf{x}_i)$ such that $t' = (u_i - 1)(L-1)$

$$\begin{aligned} f(u_i) &= f(t') \cdot \left|\frac{dt'}{du_i}\right| \\ &= (L-1)u_i^{-L}. \end{aligned}$$

To find the pdf for $u = \prod_{i=1}^{K} u_i$, we consider the bijective function from $(u_1, \ldots, u_K)$ to $(u, v_1, \ldots, v_{K-1})$ where

$$\begin{aligned} u &= \prod_{i=1}^{K} u_i \in [1, +\infty), \\ v_1 &= u_1 \in [1, u], \\ v_i &= u_i \in [1, v_{i-1}] \quad i = 2, \ldots, K-1. \end{aligned}$$

To obtain the pdf of u, we perform a change of variable (J denotes Jacobian)

$$\begin{aligned}
&f_1(u_1,\ldots,u_K)\\
&\quad\equiv \prod_i^K f(u_i) = (L-1)^K u^{-L},\\
&f_2(u,v_1,\ldots,v_{K-1})\\
&\quad\equiv \mathbb{1}_{u\geq v_1\geq\ldots\geq u_{K-1}\geq 1} f_1(u_1,\ldots,u_K)|J|^{-1}\\
&\quad= (L-1)^K u^{-L}\frac{1}{\prod_{i=1}^{K-1} v_i},\\
&f_3(u,v_1,\ldots,v_{K-2})\\
&\quad\equiv \mathbb{1}_{u\geq v_1\geq\ldots\geq u_{K-2}\geq 1}\int_1^{v_{K-2}} f_2(u,v_1,\ldots,v_{K-1})dv_{K-1}\\
&\quad= (L-1)^K u^{-L}\frac{\ln v_{K-2}}{\prod_{i=1}^{K-2} v_i},\\
&f_4(u,v_1,\ldots,v_{K-3})\\
&\quad\equiv \mathbb{1}_{u\geq v_1\geq\ldots\geq u_{K-3}\geq 1}\int_1^{v_{K-3}} f_3(u,v_1,\ldots,v_{K-2})dv_{K-2}\\
&\quad= (L-1)^K u^{-L}\frac{1}{2}\frac{(\ln v_{K-3})^2}{\prod_{i=1}^{K-3} v_i},\\
&\quad\vdots\\
&f_K(u)\\
&\quad\equiv \int_1^u f_{K-1}(u,v_1)dv_1\\
&\quad= (L-1)^K u^{-L}\frac{1}{(K-1)!}(\ln u)^{K-1}.
\end{aligned}$$

To obtain the P_{fa}, we integrate $f_K(u)$ from the threshold λ to infinity

$$\begin{aligned}
P_{fa} &= \int_\lambda^\infty f_K(u)du,\\
&= \lambda^{-L+1}\sum_{k=0}^{K-1}\frac{(L-1)^k}{k!}(\ln\lambda)^k.
\end{aligned}$$

■

To verify Theorem III.1, Monte Carlo simulations ($N_r = 10^6$) are carried out. The parameters used can be seen in Table I.

Due to Remark III.1, the covariance matrix $\mathbf{M}_i$ of each $\mathbf{y}_i$, without loss of generalities, is chosen identically and equal to $\mathbf{M}$. $\mathbf{M}$ is spatially colored and its elements are given by

$$\mathrm{M}(p,q) = \rho^{|p-q|}e^{j\frac{\pi}{2}(p-q)}.$$

ρ is the correlation coefficient and it is chosen to be 0.2 such that there is a slight correlation between different elements of the subarray.

TABLE I
PARAMETERS USED FOR MONTE CARLO SIMULATIONS

$\tilde{M}$	$\tilde{N}$	M_m	N_n	$K=\tilde{M}\tilde{N}$	$L=M_mN_n$
3	2	4	3	6	12

TABLE II
TEXTURE PARAMETERS USED FOR MONTE CARLO SIMULATIONS

	Texture distribution	a	b
1	Gamma	0.5	$\frac{\sigma^2}{a}=2$
2	Gamma	2	$\frac{\sigma^2}{a}=0.5$
1	Weibull	$\frac{\sigma^2}{\Gamma(1+\frac{1}{b})}=0.7418$	0.658
2	Weibull	$\frac{\sigma^2}{\Gamma(1+\frac{1}{b})}=1.1233$	1.763

Experimental radar clutter measurements have shown that the texture can be distributed according to a Gamma distribution [12], [13] or a Weibull one [28], [29]. When the texture is Gamma-distributed, the resulting clutter corresponds to one that is K-distributed. This has been widely studied in the literature, see, e.g., [19], [30], and [31].

The pdf and the statistical mean of the Gamma distribution with parameters a and b are given by

$$p(\tau) = \frac{1}{b^a\Gamma(a)}\tau^{a-1}e^{-\tau/b},\quad \Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt \tag{9}$$

$$\mathbb{E}[\tau] = ab. \tag{10}$$

The pdf and the statistical mean of the Weibull distribution on non-negative reals with parameters a and b are given by

$$p(\tau) = \frac{b}{a}\left(\frac{\tau}{a}\right)^{b-1}e^{-\left(\frac{\tau}{a}\right)^b} \tag{11}$$

$$\mathbb{E}[\tau] = a\Gamma\left(1+\frac{1}{b}\right). \tag{12}$$

The clutter power for each element σ^2 is given by

$$\sigma^2 = \mathbb{E}[\tau].$$

In order to keep σ^2 constant, the two parameters of the distributions are set such that the statistical mean of the texture, given in (10) and (12), remains the same. In this simulation, σ^2 is chosen to be one. The parameters used to simulate the texture are shown in Table II. The parameters are chosen such that for each texture, the first case is an instance of impulsive clutter while the second one is more similar to the Gaussian case.

For comparison, the Gaussian case where $p(\tau) = \sigma^2\delta(\tau-1)$, is also simulated. In this case, σ^2 is also equal to one.

On Fig. 2, we have plotted the "P_{fa}-threshold" curves under different hypotheses of the clutter distribution: Gaussian, K-distributed and Weibull-distributed texture. We see that there is perfect agreement between the theory given by Theorem III.1 and the simulation. The texture-CFAR property can also be seen clearly since the curves do not depend on the distribution of the texture.

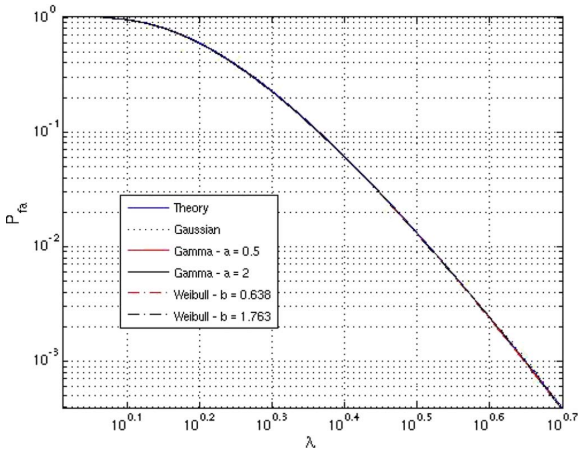

Fig. 2. P_{fa} against detection threshold λ for theoretical calculations and Monte Carlo simulations under Gaussian clutter and Gamma and Weibull texture.

C. Simulation Results

The probability of detection P_d is the probability of correctly choosing H_1 when the target is present

$$P_d = P\left(\prod_{i=1}^{K} \Lambda_i(\mathbf{y}_i) > \lambda | \mathrm{H}_1\right).$$

Under H_1, the received signal contains both a deterministic signal and clutter returns

$$\mathbf{y}_{m,n} = \mathrm{B}(m,n)\mathbf{p}_{m,n} + \mathbf{z}_{m,n}.$$

For a given signal-to-clutter ratio (SCR), denoted by $\mathrm{SCR}_{m,n}$ the amplitude of $\mathrm{B}(m,n)$ is given by

$$|\mathrm{B}(m,n)| = \sqrt{\mathrm{SCR}_{m,n} \cdot \sigma^2}.$$

If $|\mathrm{B}(m,n)|$ is considered to be different for each subarray, a multidimensional graph would be necessary to represent the detection performance and it would be very difficult to interpret the results. Hence, in this paper, we consider that $|\mathrm{B}(m,n)|$ is the same for all m and n.

For comparison, the OGD detector which is optimum under Gaussian clutter is considered. According to [7] and [25], the OGD detector extended to MIMO case is given by

$$\Lambda(\mathbf{Y}) = \sum_{i=1}^{K} \frac{\left|\mathbf{p}_i^{\dagger}\mathbf{M}_i^{-1}\mathbf{y}_i\right|^2}{\mathbf{p}_i^{\dagger}\mathbf{M}_i^{-1}\mathbf{p}_i} \tag{13}$$

where $\mathbf{Y} = [\mathbf{y}_1, \cdots, \mathbf{y}_K]$ and $\mathbf{M}_i$ which is the covariance matrix of $\mathbf{y}_i$, is assumed to be known. Under Gaussian clutter, this detector has a Chi-square distribution with $2K$ degrees of freedom, denoted by χ^2_{2K}.

The same parameters as before are used (see Tables I and II). The P_{fa} is set to be 0.001 and $N_r = 10^4$. The detection performance for MIMO GLRT-LQ and MIMO OGD under K-distributed clutter is shown in Fig. 3. The SCR values in the y-axis are for only one single subarray. The clutter is more impulsive for smaller values of a. We see that when $a = 0.5$, MIMO GLRT-LQ performs much better than MIMO OGD. This is because for MIMO OGD, the big variation of clutter power results in a high detection threshold to maintain the same P_{fa}. The increase in threshold results in a drop in detection performance. On the other hand, there is a normalizing term $\mathbf{y}^{\dagger}\mathbf{M}^{-1}\mathbf{y}$ in MIMO GLRT-LQ to take into account this variation. When the clutter is less impulsive, it becomes more similar to Gaussian clutter and the performance of both detectors tends to be similar.

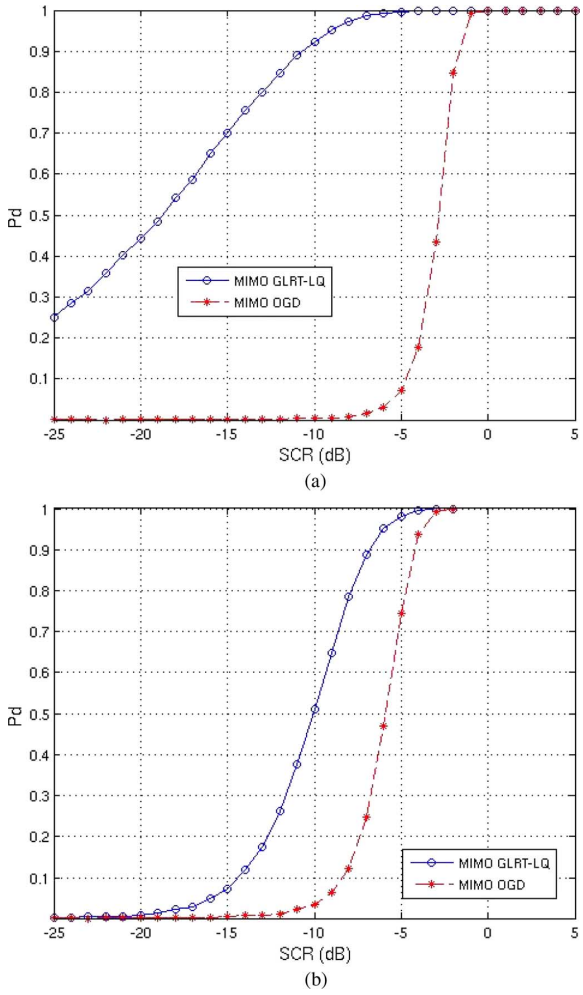

Fig. 3. P_d against SCR for Monte Carlo simulations where the texture has a Gamma distribution with different parameters, with $P_{fa} = 0.001$. (a) $a = 0.5$. (b) $a = 2$.

The performance results for clutter with Weibull-distributed texture can be seen in Fig. 4. For this case, the clutter is more impulsive for smaller values of b. As expected, MIMO GLRT-LQ performs much better than MIMO OGD when the clutter is impulsive and almost the same when the clutter is less impulsive.

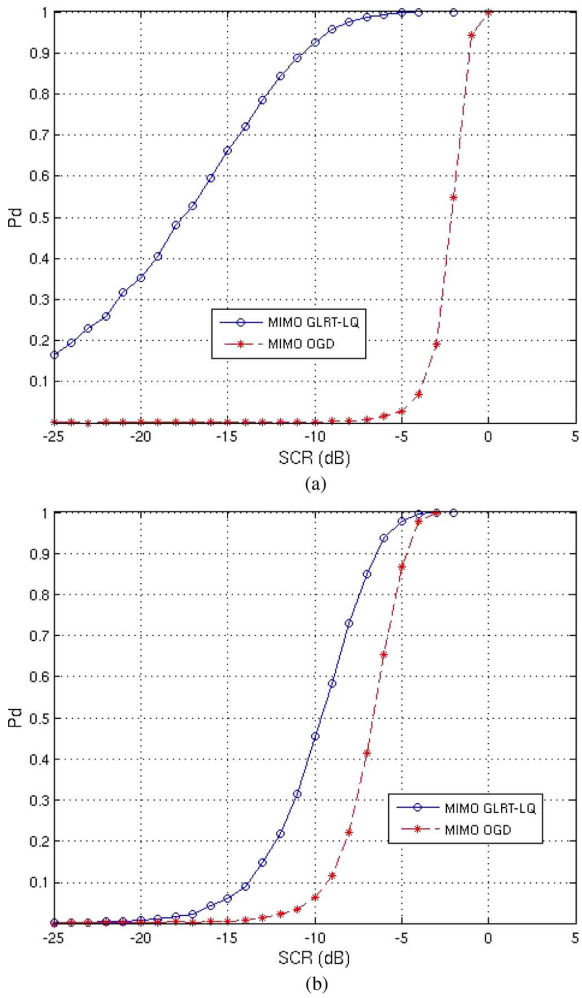

Fig. 4. P_d against SCR for Monte Carlo simulations where the texture has a Weibull distribution with different parameters, with $P_{fa} = 0.001$. (a) $b = 0.658$. (b) $a = 1.763$.

In Fig. 5, we see that the new detector works slightly worse than MIMO OGD under Gaussian clutter. This is expected since MIMO OGD is the optimal detector under Gaussian clutter. However, MIMO GLRT-LQ is much more robust as it maintains good detection performance for different types of clutter.

IV. Adaptive MIMO Non-Gaussian Detector

A. Derivation

In the previous sections, we assumed that the covariance matrix was known. However, it is usually unknown in reality.

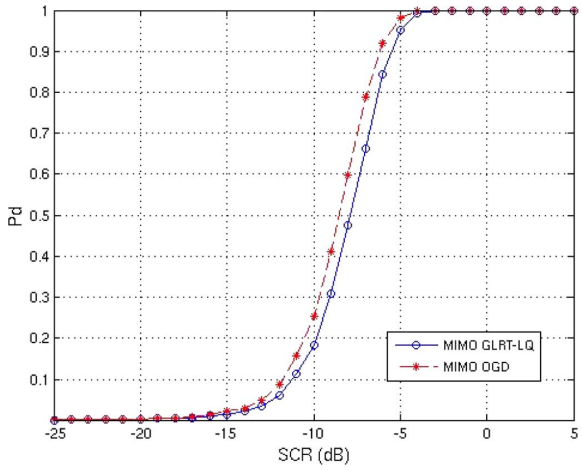

Fig. 5. P_d against SCR for Monte Carlo simulations where the clutter is Gaussian, with $P_{fa} = 0.001$.

Hence, we consider the adaptive version of the detector; i.e., the covariance matrix $\mathbf{M}_{m,n}$ is replaced by its estimate $\hat{\mathbf{M}}_{m,n}$

$$\hat{\Lambda}(\mathbf{Y}) = \prod_{m,n} \left[\frac{1}{1 - \frac{\left|\mathbf{p}_{m,n}^{\dagger} \hat{\mathbf{M}}_{m,n}^{-1} \mathbf{y}_{m,n}\right|^2}{\left(\mathbf{p}_{m,n}^{\dagger} \hat{\mathbf{M}}_{m,n}^{-1} \mathbf{p}_{m,n}\right)\left(\mathbf{y}_{m,n}^{\dagger} \hat{\mathbf{M}}_{m,n}^{-1} \mathbf{y}_{m,n}\right)}} \right]^{M_m N_n} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta. \tag{14}$$

Under Gaussian clutter, the classical SCM is the ML estimate, given by

$$\hat{\mathbf{M}}_{m,n} = \frac{1}{L_r} \sum_{l=1}^{L_r} \mathbf{x}_{m,n}(l) \mathbf{x}_{m,n}^{\dagger}(l) \tag{15}$$

where $\mathbf{x}_{m,n}(l) \sim \mathcal{CN}(\mathbf{0}, \mathbf{M}_{m,n})$ are the secondary data which are independent and identically distributed. L_r is the number of secondary data used to estimate the SCM. SCM follows the complex Wishart distribution, denoted as $\mathcal{CW}(L_r, L; \mathbf{M}_{m,n})$.

However, under non-Gaussian clutter, $\mathbf{y}_{m,n}(l) = \sqrt{\tau_{m,n}} \mathbf{x}_{m,n}(l)$

$$\begin{aligned} \hat{\mathbf{M}}_{m,n} &= \frac{1}{L_r} \sum_{l=1}^{L_r} \mathbf{y}_{m,n}(l) \mathbf{y}_{m,n}^{\dagger}(l), \\ &= \frac{1}{L_r} \sum_{l=1}^{L_r} \tau_{m,n} \mathbf{x}_{m,n}(l) \mathbf{x}_{m,n}^{\dagger}(l) \end{aligned}$$

we see that the SCM is no longer the ML estimate. Instead, we consider the FPE [21]–[23]. In [23], it has been shown that the FPE is unique up to a scalar factor. Here, due to the matrix

normalization, the resulting FPE is unique and it is defined as the unique solution of the equation

$$\hat{\mathbf{M}}_{m,n} = \frac{L}{L_r}\sum_{l=1}^{L_r}\frac{\mathbf{y}_{m,n}(l)\mathbf{y}_{m,n}^{\dagger}(l)}{\mathbf{y}_{m,n}^{\dagger}(l)\hat{\mathbf{M}}_{m,n}^{-1}\mathbf{y}_{m,n}(l)}. \quad (16)$$

Due to the normalizing term in the denominator, the FPE does not depend on the texture of the clutter. Moreover, the FPE is the ML estimate when the texture is deterministic but unknown [23]. In the case where the texture is a random variable, the FPE is the approximate ML estimate [21], [22].

The FPE is computed using the following iterative algorithm:

$$\hat{\mathbf{M}}_{m,n}(k+1) = \frac{L}{L_r}\sum_{l=1}^{L_r}\frac{\mathbf{y}_{m,n}(l)\mathbf{y}_{m,n}^{\dagger}(l)}{\mathbf{y}_{m,n}^{\dagger}(l)\hat{\mathbf{M}}_{m,n}^{-1}(k)\mathbf{y}_{m,n}(l)}. \quad (17)$$

The solution converges towards the FPE regardless of the choice of the initial matrix $\hat{\mathbf{M}}_{m,n}(0)$ as shown in [23]. One obvious choice is $\mathbf{I}$ which will give the normalized SCM after the first iteration. Consequently, the detector is also matrix-CFAR when the FPE is used [32].

More importantly, it has been shown in [24] that the asymptotic distribution of $\hat{\mathbf{M}}_{FP}$ is the same as that of the SCM with $(L/(L+1))L_r$ secondary data under *Gaussian* clutter.

1) $\tilde{M} = 1$ and $\tilde{N} = 1$ (Adaptive): As before, we begin with the case where $\tilde{M} = 1$ and $\tilde{N} = 1$. The dimension of the received signal is $L = M_1N_1$. As there is only one received signal and steering vector, let them be denoted simply by $\mathbf{y} = \mathbf{y}_{1,1}$ and $\mathbf{p} = \mathbf{p}_{1,1}$. The adaptive detector is given by

$$\hat{\Lambda}_1(\mathbf{y}) = \left[\frac{1}{1-\frac{|\mathbf{p}^{\dagger}\hat{\mathbf{M}}^{-1}\mathbf{y}|^2}{(\mathbf{p}^{\dagger}\hat{\mathbf{M}}^{-1}\mathbf{p})(\mathbf{y}^{\dagger}\hat{\mathbf{M}}^{-1}\mathbf{y})}}\right] \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \sqrt[L]{\eta} = \lambda.$$

According to [26], under H_0, considering Gaussian clutter and using the SCM as the estimated covariance matrix, $\hat{\Lambda}_1(\mathbf{y})$ can be expressed with a random variable $\hat{F}$ which in turn depends on another random variable B

$$\hat{\Lambda}_1(\mathbf{y}) = 1 + \hat{F}.$$

The distributions of the two variables are as follows: $\hat{F}|B \sim \beta^1_{1,L_r-L+1}$ and $B \sim \beta^1_{L_r-L+2,L-1}$. The pdf of $\beta^1_{a,b}$ and $\beta^2_{a,b}$ are defined in [27] as

$$\beta^1_{a,b}(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}\mathbb{1}_{[0,1]}(x) \quad (18)$$

$$\beta^2_{a,b}(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\frac{x^{a-1}}{(1+x)^{a+b}}, \quad x > 0. \quad (19)$$

Here, we consider instead non-Gaussian clutter modeled as a SIRV. Using the FPE, $\hat{\Lambda}_1(\mathbf{y})$ can still be expressed with a conditional random variable $\hat{F}|B$ but with L_r replaced by $(L/(L+1))L_r$. Hence, the distribution becomes $\hat{F}|B \sim \beta^1_{1,a-1}$ with $B \sim \beta^1_{a,b-a-1}$ where $a = (L/(L+1))L_r - L + 2$ and $b = (L/(L+1))L_r + 2$.

Using (18) and (19), we obtain the pdf of $\hat{F}$

$$\begin{aligned} f(\hat{F}) &= \int_0^1 \frac{\Gamma(a)}{\Gamma(a-1)}\frac{1-B}{\left(1+\hat{F}(1-B)\right)^a}\frac{\Gamma(b-1)}{\Gamma(a)\Gamma(b-a-1)} \\ &\quad \times B^{a-1}(1-B)^{b-a-2}dB, \\ &= \frac{\Gamma(b-1)}{\Gamma(a-1)\Gamma(b-a-1)}\frac{1}{(1+\hat{F})^a} \\ &\quad \times \int_0^1 \frac{B^{a-1}(1-B)^{b-a-1}}{\left(1-\frac{\hat{F}}{1+\hat{F}}B\right)^a}dB, \\ &= \frac{\Gamma(b-1)}{\Gamma(a-1)\Gamma(b-a-1)}\frac{\Gamma(a)\Gamma(b-a)}{\Gamma(b)}\frac{1}{(1+\hat{F})^a} \\ &\quad \times {}_2\mathrm{F}_1\left(a,a,b,\frac{\hat{F}}{1+\hat{F}}\right), \\ &= \frac{(a-1)(b-a-1)}{(b-1)}\frac{1}{(1+\hat{F})^a}{}_2\mathrm{F}_1\left(a,a,b,\frac{\hat{F}}{1+\hat{F}}\right) \end{aligned}$$

where ${}_2\mathrm{F}_1(a,b,c,x)$ is the hypergeometric function as defined in [27]

$${}_2\mathrm{F}_1(a,b,c,x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-tx)^a}dt. \quad (20)$$

Letting $\hat{\Lambda}_1(\mathbf{y}) = u_1 = 1 + \hat{F}$, the pdf becomes

$$f(u_1) = \frac{(a-1)(b-a-1)}{(b-1)}u_1^{-a}{}_2\mathrm{F}_1\left(a,a,b,1-\frac{1}{u_1}\right). \quad (21)$$

To obtain the P_{fa}, we integrate the pdf from λ to ∞

$$P_{fa} = \int_{\lambda}^{\infty} f(u_1)du_1.$$

Through a change of variable $x = 1 - (1/u_1)$ and the Euler Transformation for hypergeometric functions [27, Eq. (15.3.4]), we get

$$\begin{aligned} P_{fa} &= \int_{1-1/\lambda}^{1} \frac{(a-1)(b-a-1)}{(b-1)}(1-x)^a{}_2\mathrm{F}_1(a,a,b,x) \\ &\quad \times \frac{1}{(1-x)^2}dx, \\ &= \int_{1-1/\lambda}^{1} \frac{(a-1)(b-a-1)}{(b-1)}{}_2\mathrm{F}_1\left(a,b-a,b,\frac{x}{x-1}\right) \\ &\quad \times \frac{1}{(1-x)^2}dx. \end{aligned}$$

Consider the hypergeometric function $F = {}_2\mathrm{F}_1(a-1, b-a-1, b-1, (x/(x-1))$, the derivative of F is then

$$\frac{dF}{dx} = \frac{(a-1)(b-a-1)}{(b-1)} {}_2\mathrm{F}_1\left(a, b-a, b, \frac{x}{x-1}\right) \frac{-1}{(1-x)^2}.$$

Using this derivative, we can obtain the final expression for P_{fa}

$$\begin{aligned} P_{fa} &= \int_{1-1/\lambda}^{1} -\frac{dF}{dx} dx \\ &= {}_2\mathrm{F}_1(a-1, b-a-1, b-1, 1-\lambda). \end{aligned} \tag{22}$$

From (21) and (22), we see that the distribution of the detector and the P_{fa}, respectively, depends only on L_r and L. This means that the adaptive detector is also CFAR.

As $L_r \to \infty$, $a-1 \approx b-1$. Using the identity given by [27, Eq. (15.1.8)], ${}_2\mathrm{F}_1(a,b,b,z) = {}_2\mathrm{F}_1(b,a,b,z) = (1-z)^{-a}$, we see that, asymptotically, P_{fa} tends to

$$P_{fa} \to \lambda^{-(b-a-1)} = \lambda^{-L+1}$$

which is the same expression as that for the case where the covariance matrix is known and $K = 1$.

2) $\hat{M} \geq 1, \hat{N} \geq 1$ and $M_m N_n = L$ (Adaptive): As before, we consider the product of all the individual detection tests for each subsystem $u = \prod_{i=1}^{K} u_i$. The joint density function is given by

$$f(u_1, \ldots, u_K) = \left(\frac{(a-1)(b-a-1)}{(b-1)}\right)^K \times \prod_{i=1}^{K} u_i^{-a} {}_2\mathrm{F}_1\left(a, a, b, 1-\frac{1}{u_i}\right).$$

As before, we consider the bijective function from $(u_1, \ldots, u_K)$ to $(u, v_1, \ldots, v_{K-1})$

$$\begin{aligned} f(u, v_1, \ldots, v_{K-1}) = &\left(\frac{(a-1)(b-a-1)}{(b-1)}\right)^K u^{-a} \\ &\times \prod_{i=1}^{K-1} {}_2\mathrm{F}_1\left(a, a, b, 1-\frac{1}{u_i}\right) \\ &\times {}_2\mathrm{F}_1\left(a, a, b, 1-\frac{\prod_{j=1}^{K-1} u_j}{u}\right) \\ &\times \frac{1}{\prod_{i=1}^{K-1} v_i}. \end{aligned}$$

Due to the complexity of the above expression, the pdf of $f(u)$ and consequently P_{fa} has not been obtained analytically. Instead, P_{fa} is computed empirically using Monte Carlo simulations.

B. Simulation Results (Adaptive)

The same parameters as before (Tables I and II) are used. To study the effects of the estimation of the covariance matrix on the detection performance, we consider two cases: $L_r = 2L$ and $L_r = 20L$ to compute $\hat{\mathbf{M}}_i$ using the Fixed Point algorithm, and compare them to the case where $\mathbf{M}_i$ is known.

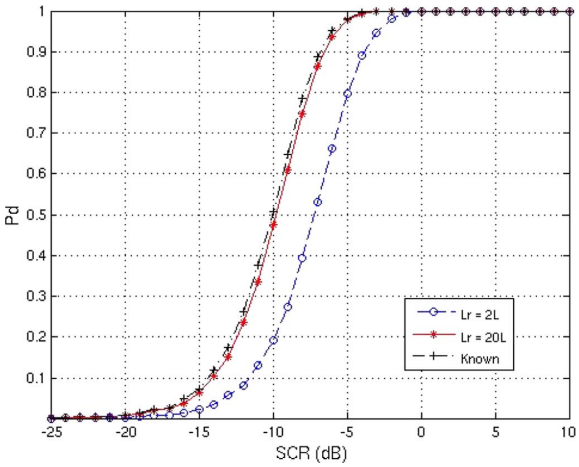

Fig. 6. P_d against SCR for Monte Carlo simulations with the adaptive MIMO GLRT-LQ using FPEs and clutter with Gamma-distributed texture ($a = 2$).

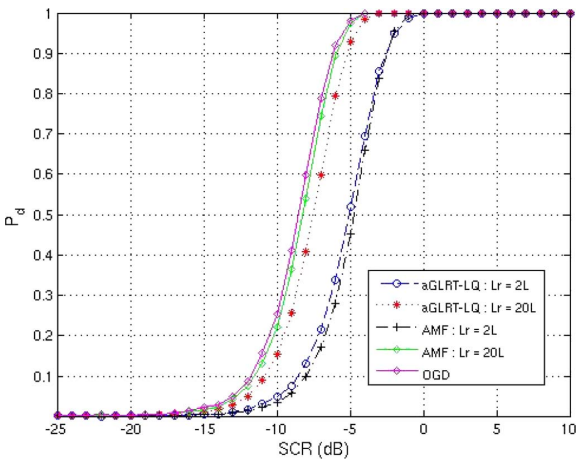

Fig. 7. P_d against SCR for Monte Carlo simulations for both the adaptive MIMO GLRT-LQ using FPEs and the MIMO AMF using SCM with Gaussian clutter. P_d against SCR for MIMO OGD is given as reference. $P_{fa} = 0.001$.

As a rule of thumb, it is considered that L_r has to be at least $2L$ and this will give a loss of approximately 3 dB in detection performance [33]. On the other hand, $L_r = 20L$ has been chosen to approach the asymptotic case where $\mathbf{M}_i$ is known.

The FPE is obtained using (17) with ten iterations. However, from an operational point of view, it suffices to compute the estimate with four or five iterations to achieve a relative error of 10^{-2} [23].

As expected, when L_r is large, the detection performance of the adaptive detector tends towards that of the detector where the covariance matrices are known (Fig. 6).

1) Under Gaussian Clutter (Adaptive): For comparison, the MIMO Adaptive Matched Filter (MIMO AMF), the adaptive version of MIMO-OGD, has been considered. It is obtained

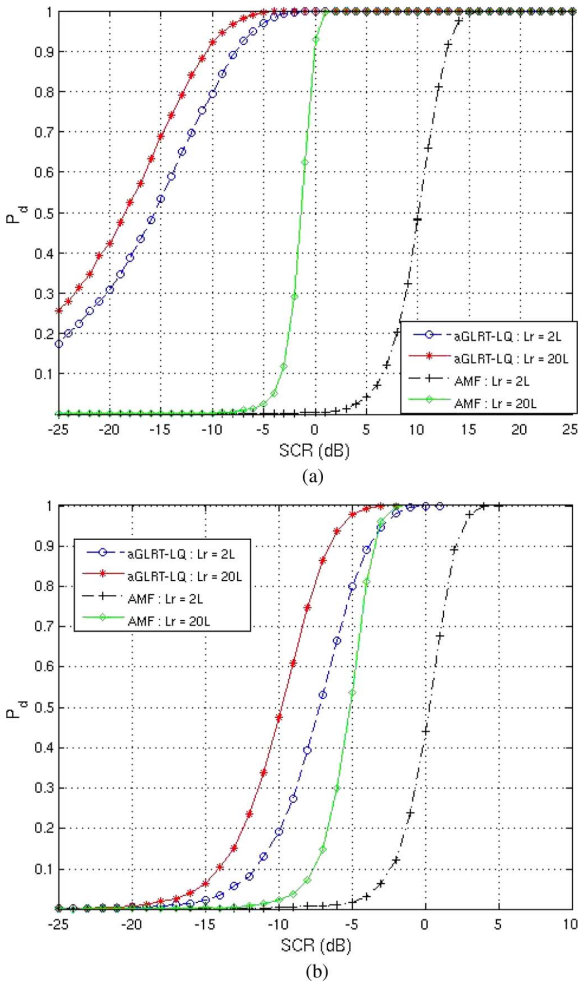

Fig. 8. P_d against SCR for Monte Carlo simulations for both the adaptive MIMO GLRT-LQ using FPEs and the MIMO AMF using SCM with K-distributed clutter. $P_{fa} = 0.001$. (a) $a = 0.5$. (b) $a = 2$.

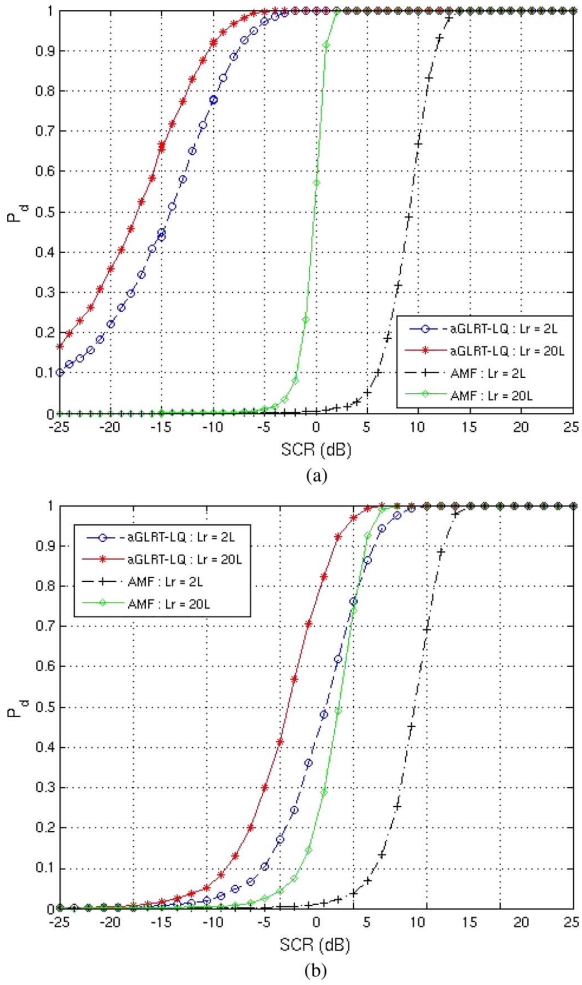

Fig. 9. P_d against SCR for Monte Carlo simulations for both the adaptive MIMO GLRT-LQ using FPEs and the MIMO AMF using SCM and clutter with Weibull-distributed texture. $P_{fa} = 0.001$. (a) $b = 0.658$. (b) $b = 1.763$.

simply by replacing the covariance matrix in (13) by its estimate

$$\hat{\Lambda}(\mathbf{Y}) = \sum_{i=1}^{K} \frac{\left|\mathbf{p}_i^\dagger \hat{\mathbf{M}}_i^{-1} \mathbf{y}_i\right|^2}{\mathbf{p}_i^\dagger \hat{\mathbf{M}}_i^{-1} \mathbf{p}_i}. \tag{23}$$

The SCM given in (15) is used in the MIMO AMF since it is the ML estimate under Gaussian clutter. In Fig. 7, we have the detection performance, under Gaussian clutter, of both the adaptive MIMO GLRT-LQ using FPE and the MIMO AMF using SCM. Even in this case, the estimation of the covariance matrix does not affect the performance of the adaptive MIMO GLRT-LQ much and it remains comparable to that of the MIMO AMF.

According to [34], under Gaussian clutter, the AMF is expected to perform worse than Kelly's Test [35] as the signal vector $\mathbf{y}_i$ is not used in the estimation of the covariance matrix. Kelly's Test, for the case where $K = 1$, is given by

$$\hat{\Lambda}_1(\mathbf{y}_1) = \frac{\left|\mathbf{p}_1^\dagger \hat{\mathbf{M}}_1^{-1} \mathbf{y}_1\right|^2}{\left(\mathbf{p}_1^\dagger \hat{\mathbf{M}}_1^{-1} \mathbf{p}_1\right)\left(1 + \frac{1}{L_r} \mathbf{y}_1^\dagger \hat{\mathbf{M}}_1^{-1} \mathbf{y}_1\right)}.$$

When L_r is large, Kelly's Test is similar to the AMF. However, when L_r is small, the term $\mathbf{y}_1^\dagger \hat{\mathbf{M}}_1^{-1} \mathbf{y}_1$ is no longer negligible and Kelly's Test is more similar to the adaptive GLRT-LQ test. Thus, in Fig. 7, we see that the performance of the adaptive MIMO GLRT-LQ is actually slightly better than that of the MIMO AMF when $L_r = 2L$.

2) Under Non-Gaussian Clutter (Adaptive): Figs. 8 and 9 show the detection performance for both the adaptive MIMO GLRT-LQ with FPE and the MIMO AMF with SCM when the texture has Gamma and Weibull distribution, respectively.

As expected, the adaptive MIMO GLRT-LQ performs much better than the MIMO AMF when the clutter is impulsive. When the clutter is less impulsive, the detection performance of both detectors is similar but the adaptive MIMO GLRT-LQ still works better. Moreover, the MIMO AMF is more sensitive to the estimation of the covariance matrix than the adaptive MIMO GLRT-LQ. This is not surprising since the SCM is no longer the ML estimate under non-Gaussian clutter.

V. Conclusion

The CFAR GLRT-LQ detector for detection under non-Gaussian clutter has been extended to the MIMO case where all subarrays are considered jointly as a system such that only one detection threshold is used. Theoretical performance for the new detector is also derived and validated using Monte Carlo simulations. Detection performance is then analyzed through simulations. Compared to the classical OGD detector, the new detector shows significant improvements in detection performance under non-Gaussian clutter especially in very impulsive clutter. It has a slight loss in performance when the clutter is Gaussian. This is expected since the MIMO OGD is the optimal detector under Gaussian clutter. However, the robustness of the MIMO GLRT-LQ detector under non-Gaussian clutter more than compensates this slight loss under Gaussian clutter.

Next, the adaptive version of this new detector is considered. The FPE is used to estimate the covariance matrix as the classical SCM no longer works under non-Gaussian clutter. The theoretical performance of the adaptive version is shown to be texture-CFAR and matrix-CFAR for the case where there is only one subarray. On top of that, the detection performance tends to that of the case where the covariance matrix is known when the number of secondary data is large. Due to the complexity of equations, the case where there is more than one subarray is studied only empirically using simulations.

The detection performance of the adaptive non-Gaussian detector using the FPE is then compared to the MIMO AMF using the SCM through simulations. As expected, the new detector performs much better under non-Gaussian clutter, especially when the clutter is very impulsive and it has comparable performance under Gaussian clutter.

The main conclusion is that it is always preferable to use the adaptive GLRT-LQ with the FPE, whatever the clutter distribution, because of the robustness of these tools with respect to the covariance matrix and the texture. Even in the case where the clutter is Gaussian for all subarrays, the covariance matrix and clutter power for each subarray is expected to be different. Hence it is still better to use the new non-Gaussian detector.

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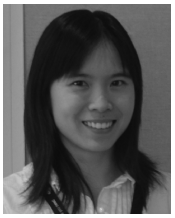

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Annexe M

Coherency Matrix Estimation of Heterogeneous Clutter in High-Resolution Polarimetric SAR Images

Coherency Matrix Estimation of Heterogeneous Clutter in High-Resolution Polarimetric SAR Images

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Frederic Pascal, *Member, IEEE*, and Céline Tison

***Abstract*—This paper presents an application of the recent advances in the field of spherically invariant random vector (SIRV) modeling for coherency matrix estimation in heterogeneous clutter. The complete description of the polarimetric synthetic aperture radar (POLSAR) data set is achieved by estimating the span and the normalized coherency independently. The normalized coherency describes the polarimetric diversity, while the span indicates the total received power. The main advantages of the proposed fixed-point (FP) estimator are that it does not require any *a priori* information about the probability density function of the texture (or span) and that it can directly be applied on adaptive neighborhoods. Interesting results are obtained when coupling this FP estimator with an adaptive spatial support based on the scalar span information. Based on the SIRV model, a new maximum-likelihood distance measure is introduced for unsupervised POLSAR classification. The proposed method is tested with both simulated POLSAR data and airborne POLSAR images provided by the Radar Aéroporté Multi-Spectral d'Etude des Signatures system. Results of entropy/alpha/anisotropy decomposition, followed by unsupervised classification, allow discussing the use of the normalized coherency and the span as two separate descriptors of POLSAR data sets.**

***Index Terms*—Estimation, heterogeneous clutter, polarimetry, segmentation, synthetic aperture radar (SAR).**

NOMENCLATURE

BN	Boxcar neighborhood.
$[C]$	Generic covariance matrix.
FP	Fixed point.
i.i.d.	Independent and identically distributed.
LLMMSE	Locally linear minimum mean-squared error.
$[M]$	Generic normalized polarimetric coherency matrix.
ML	Maximum likelihood.
MPWF	Multilook polarimetric whitening filter.
P	Generic span.
PDF	Probability density function.
PWF	Polarimetric whitening filter.
SCM	Sample covariance matrix.
SDAN	Span-driven adaptive neighborhood.
SIRP	Spherically invariant random process.
SIRVs	Spherically invariant random vectors.
$[T]$	Generic polarimetric coherency matrix.

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I. INTRODUCTION

A SYNTHETIC aperture radar (SAR) measures both the amplitude and phase of a backscattered signal, producing one complex image for each recording. With the sensors being able to emit or receive two orthogonal polarizations, fully polarimetric SAR (POLSAR) systems describe the interactions between the electromagnetic wave and the target area by means of the Sinclair matrix [1]. Among the difficulties encountered when using POLSAR imagery, one important feature is the presence of speckle. Occurring in all types of coherent imagery, the speckle is due to the random interference of the waves scattered by the elementary targets belonging to one resolution cell [2]. In general, POLSAR data are locally modeled by a multivariate zero-mean circular Gaussian PDF, which is completely determined by the covariance matrix [3].

The recently launched POLSAR systems are now capable of producing high-quality images of the Earth's surface with meter resolution. The decrease of the resolution cell offers the opportunity to observe much thinner spatial features than the decametric resolution of the up-to-now available SAR images. Recent studies [4] show that the higher scene heterogeneity leads to non-Gaussian clutter modeling, particularly for urban areas. One commonly used fully polarimetric non-Gaussian clutter model is the product model [5]: The spatial non-homogeneity is incorporated by modeling the clutter as a product between the square root of a scalar random variable (texture) and an independent zero-mean complex circular Gaussian random vector (speckle). If the texture random variable is supposed to be a Gamma spatial distributed intensity, the product model is equivalent to the well-known $\mathcal{K}$-distributed clutter model [6], [7].

For a Gaussian polarimetric clutter model, the estimation of the polarimetric coherency matrix is treated in the context of POLSAR speckle filtering. The POLSAR adaptive filtering

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techniques can roughly be divided into two main classes [8]: based on the optimization of the spatial support and based on the use of the local statistics to derive adaptive estimators. These two directions are not exclusive since both of them can be applied simultaneously [9], [10]. For example, the refined Lee filter couples eight edge-aligned directional neighborhoods with an adaptive estimator based on the LLMMSE criterion [9].

In the context of the non-Gaussian polarimetric clutter models, several studies tackled POLSAR parameter estimation using the product model. For deterministic texture, Novak *et al.* derived the PWF by optimally combining the elements of the polarimetric covariance matrix to produce a single scalar image [11], [12]. Using the complex Wishart distribution, the PWF for homogeneous surfaces has been generalized to an MPWF [13], [14]. In general, the texture random variable is specified by the PDF. For Gamma-distributed texture, Lopes and Sery [13] derived the ML estimator of the covariance matrix. Moreover, the vector spatial LLMMSE filter applied on the scalar ML texture estimator has also been introduced when the texture variance and spatial correlation functions are *a priori* known [13]. In [15], DeGrandi *et al.* performed an extensive study on the dependence of the normalized second-order moment of intensity on polarization state for a $\mathcal{K}$-distributed clutter model. This dependence was condensed in a graphical form by a formalism called the polarimetric texture signature. This study has been applied for target detection and texture segmentation using the discrete wavelet transform generated with the first derivative of a B-spline of order three as mother wavelet [16].

The POLSAR information allows the discrimination of different scattering mechanisms. In [17], Cloude and Pottier introduced the target entropy and the entropy–alpha–anisotropy $(H{-}\alpha{-}A)$ model by assigning to each eigenvector the corresponding coherent single scattering mechanism. Based on this decomposition, unsupervised classification for land applications was performed by an iterative algorithm based on the complex Wishart density function [18], [19].

The objective of this paper is to present a new coherency estimation technique [20] based on the SIRV model [21] and to analyze the consequences that this model has on the conventional POLSAR processing chain. This paper is organized as follows. Section II is dedicated to the presentation of the proposed estimation scheme. The heterogeneity of the polarimetric textured scenes is taken into account by coupling the ML normalized coherency estimator with adaptive neighborhoods (ANs) driven on the scalar ML span estimators. A new ML distance measure is also introduced for classifying normalized coherency matrices under the SIRV model. In Section III, the results obtained using the proposed approach are presented and compared to those given by the Gaussian ML estimator. Results of the $H{-}\alpha{-}A$ decomposition, followed by unsupervised POLSAR classification, allow discussing the use of the normalized coherency and the span as two separate descriptors of POLSAR data sets. Detailed discussion on the advantages and the limitations of the SIRV model is given in Section IV. Eventually, in Section V, some conclusions and perspectives are presented.

II. Heterogeneous Model for Polarimetric Textured Scenes

The goal of the estimation process is to derive the scene signature from the observed data set. In the case of spatially changing surfaces ("heterogeneous" or "textured" scenes), the first step is to define an appropriate model describing the dependence between the polarimetric signature and the observable as a function of the speckle. In general, the multiplicative model [5] has been employed for SAR data processing as a product between the square root of a scalar positive quantity (texture) and the description of an equivalent homogeneous surface (speckle) by means of the following:

1) the intensity descriptor for single-polarization SAR images [22], [23];
2) the complex SAR signal descriptor for single-polarization SAR data [24];
3) the polarimetric target vector descriptor in lexicographic basis for monostatic POLSAR images [11], [25], [26];
4) the normalized polarimetric target vector descriptor in lexicographic basis [6], [27], [28];
5) the polarimetric covariance matrix descriptor for POLSAR data [13], [29].

In this paper, the polarimetric descriptors used are the target vectors $\mathbf{k} = [k_1, k_2, k_3]^{\mathrm{T}}$ in the Pauli basis (monostatic acquisition). The following section presents an application of the recent advances, in the field of SIRV modeling [20], for estimating span and normalized coherency matrices of high-resolution POLSAR data.

A. Gaussian Model

The elements of a vector are generally modeled by a multivariate zero-mean complex Gaussian random process. The PDF is given by the following expression [2]:

$$p_m(\mathbf{k}) = \frac{1}{\pi^m \det\{[T]\}} \exp\left\{-\mathbf{k}^\dagger [T]^{-1} \mathbf{k}\right\} \tag{1}$$

where $[T] = E\{\mathbf{k}\mathbf{k}^\dagger\}$ is the polarimetric coherency matrix, $\det\{\ldots\}$ denotes the matrix determinant, $\dagger$ is the conjugate transpose operator, m is the dimension of the target vector ($m = 3$ for monostatic POLSAR acquisitions), and $E\{\ldots\}$ denotes the statistical mean over the polarimetric channels.

According to (1), a Gaussian stochastic process is completely characterized by the coherency matrix. In this case, the ML estimator of the polarimetric coherency matrix is the SCM obtained by replacing the statistical mean by spatial averaging

$$[\widehat{T}]_{\mathrm{SCM}} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \mathbf{k}_i^\dagger \tag{2}$$

where N is the number of samples. The SCM is statistically determined by the Wishart PDF [2].

Another POLSAR parameter is the span (or total power) P generally defined for each pixel as [1]

$$P_{\mathrm{SLC}} = \mathbf{k}^\dagger \mathbf{k}. \tag{3}$$

The corresponding multilook span can be estimated within a local neighborhood according to

$$P = E\left\{\mathbf{k}^{\dagger}\mathbf{k}\right\} = \mathrm{Tr}\left\{[T]\right\} \tag{4}$$

where $\mathrm{Tr}\{[T]\}$ denotes the trace of the matrix $[T]$. Hence, the common span estimator for the Gaussian case can directly be obtained from the SCM as

$$\widehat{P}_{\mathrm{SCM}} = \mathrm{Tr}\left\{[\widehat{T}]_{\mathrm{SCM}}\right\}. \tag{5}$$

B. SIRV Model

SIRVs and their applications to estimation and detection in communication theory were first introduced by Yao [21]. The SIRV is a class of nonhomogeneous Gaussian processes with random variance. The complex m-dimensional measurement $\mathbf{k}$ is defined as the product between the independent complex circular Gaussian vector $\mathbf{z}$ (speckle) with zero mean and covariance matrix $[M] = E\{\mathbf{z}\mathbf{z}^{\dagger}\}$ and the square root of the positive random variable τ (representing the texture)

$$\mathbf{k} = \sqrt{\tau}\mathbf{z}. \tag{6}$$

It is important to notice that, in the SIRV definition, the PDF of the texture random variable is not explicitly specified. As a consequence, SIRVs describe a whole class of stochastic processes defined by (6). This class includes the conventional clutter models having Gaussian, $\mathcal{K}$-distributed, Chi, Rayleigh, Weibull, or Rician PDFs [30].

For POLSAR data, the SIRV product model is the product of two separate random processes operating across two different statistical axes.

1) The polarimetric diversity is modeled by a multidimensional Gaussian kernel characterized by its covariance matrix $[M]$.
2) The randomness of spatial variations in the radar backscattering from cell to cell is characterized by τ. The corresponding random process operates along the spatial axis given by the image support. Relatively to the polarimetric axis, the texture random variable τ can be viewed as an unknown deterministic parameter from cell to cell.

One major advantage of the SIRV clutter model is the high degree of generality with respect to other texture-aware models employed in the literature [4], [30]. Nevertheless, this model is founded on the validity of three basic assumptions: The texture random variable affects the backscattered power only; it is multiplicative and spatially uncorrelated. When applied to high-resolution POLSAR clutter, the SIRV model postulates that the texture descriptor τ from (6) is identical for all polarization channels.

Now, let $p(\tau)$ be the texture PDF associated to the SIRV model. The SIRP corresponding to (6) has the following PDF [31]:

$$F\left\{p(\tau),[M]\right\} = p_m(\mathbf{k}) = \int_0^{+\infty} \frac{1}{(\pi\tau)^m \det\left\{[M]\right\}} \times \exp\left(-\frac{\mathbf{k}^{\dagger}[M]^{-1}\mathbf{k}}{\tau}\right) p(\tau)\, d\tau. \tag{7}$$

1) Model Identification: When using the product model, an identification problem can be pointed out: The SIRV model is uniquely defined with respect to the covariance matrix parameter up to a multiplicative constant. Let $[M_1]$ and $[M_2]$ be two covariance matrices such that $[M_1] = \kappa \cdot [M_2]\ \forall \kappa \in \mathbb{R}_+^*$. Notice that the two sets of parameters defined as $\{\tau_1, [M_1]\}$ and $\{\tau_2 = (\tau_1/\sqrt{\kappa}), [M_2]\}$ describe the same SIRV. For solving this identification problem, the covariance matrix has to be normalized. In the following, the covariance matrix $[M]$ is normalized such that $\mathrm{Tr}\{[M]\} = m$, with m being the dimension of the target vector.

One important consequence of the imposed normalization condition is that the resulting normalized polarimetric coherency matrix reveals information concerning the polarimetric diversity only: The total power information is transferred into the texture random variable. The POLSAR data can fully be characterized by coupling the normalized coherency matrix with the span descriptor

$$P_{\mathrm{SLC}} = \mathbf{k}^{\dagger}\mathbf{k} = \tau(\mathbf{z}^{\dagger}\mathbf{z}). \tag{8}$$

When operating on the polarimetric statistical axis, the span for the SIRV case is given by

$$P = E\left\{\tau(\mathbf{z}^{\dagger}\mathbf{z});\tau\right\} = \tau \cdot E\left\{\mathbf{z}^{\dagger}\mathbf{z}\right\} = \tau \cdot \mathrm{Tr}\left\{[M]\right\} = \tau \cdot m. \tag{9}$$

An estimate of P can be obtained when considering τ as an unknown deterministic parameter from cell to cell.

2) Stationarity Definition: In the following, several generic concepts are recalled. Given a SIRP, this process is wide-sense stationary if and only if both the texture random variable and the speckle random vector are wide-sense stationary. As the speckle is a zero-mean complex Gaussian vector, the latter means that the statistical samples $\mathbf{k}_i$ used in the estimation process must have the same theoretical covariance matrix $[M]$. This condition is called "matrix stationarity."

However, as the results presented in this section can be applied whatever the texture PDF $(\forall p(\tau))$, the previous properties can be reformulated using the SIRV class of stochastic processes. Given a "matrix stationary" stochastic process, this process is "SIRV homogeneous" if and only if the texture random variable is "texture homogeneous," where texture homogeneous means that it is possible to define a texture PDF $(\exists p(\tau))$ such that the stochastic process can be described by the product model from (6). We illustrate these properties using four local populations which often occur in practical POLSAR applications.

1) One zero-mean Gaussian process with covariance matrix $[M]$: $\mathcal{N}(0,[M])$. Being a "Gaussian stationary"[1] process, it is also "SIRP stationary" and SIRV homogeneous. This model is widely used for POLSAR data analysis [32].
2) Two adjacent Gaussian processes with different covariance matrix: $\mathcal{N} = \{\mathcal{N}^{(1)}(0,[M]_1), \mathcal{N}^{(2)}(0,[M]_2)\}$. The Gaussian mixture $\mathcal{N}$ is neither SIRP stationary nor SIRV homogeneous as the matrix stationarity condition is not

[1] A "Gaussian stationary" process is a stochastic process whose Gaussian PDF does not change when shifted in time or space.

respected. Generally, such cases are treated by employing adaptive estimation schemes [8], [9] in order to approximate the local "Gaussian stationarity" condition.

3) One $\mathcal{K}$-distributed process [33] with Gamma-distributed texture $p_G(\tau;\overline{\tau},\nu)$ and covariance matrix $[M]$: $F_{\mathcal{K}}\{p_G(\tau;\overline{\tau},\nu),[M]\}$. This process is SIRP stationary as it is "$\mathcal{K}$ stationary",[2] but obviously, it is not Gaussian stationary.
4) Two adjacent $\mathcal{K}$-distributed processes with two different Gamma texture PDFs $p_G^{(1)}(\tau;\overline{\tau}_1,\nu_1)$, $p_G^{(2)}(\tau;\overline{\tau}_2,\nu_2)$ and the same covariance matrix $[M]$: $F_{\mathcal{K}} = \{F_{\mathcal{K}}^{(1)}\{p_G^{(1)}(\tau;\overline{\tau}_1,\nu_1),[M]\}, F_{\mathcal{K}}^{(2)}\{p_G^{(2)}(\tau;\overline{\tau}_2,\nu_2),[M]\}\}$. The $\mathcal{K}$-distributed processes $F_{\mathcal{K}}^{(1)}$ and $F_{\mathcal{K}}^{(2)}$ are SIRP stationary and $\mathcal{K}$ stationary, but the mixture $F_{\mathcal{K}}$ is not $\mathcal{K}$ stationary. Despite this, the process $F_{\mathcal{K}}$ is SIRV homogeneous as it is possible to define a texture PDF which models the Gamma mixture. As a consequence, the results presented in this section can still be applied in this case.

In conclusion, the two properties to be verified in order to apply the SIRV model are the matrix stationarity and the "texture homogeneity." Moreover, the latter considerably relaxes the "texture stationarity" condition required when using explicit texture models such as the Gamma or the Fisher PDF.

3) SIRV Parameter Estimation: In the field of target detection for radar applications, the SIRV model led to many investigations [34]–[37]. In (6) and (7), the normalized covariance matrix is an unknown parameter which can be estimated from the ML theory. In [31], Gini and Greco derived the exact ML estimate $[\widehat{M}]$ of the normalized covariance matrix when τ_i are assumed to be unknown deterministic parameters. For N i.i.d. data, the likelihood function to maximize with respect to $[M]$ and τ_i is given by

$$L_{\mathbf{k}}\left(\mathbf{k}_1,\ldots,\mathbf{k}_N;[M],\tau_1,\ldots,\tau_N\right) = \frac{1}{\pi^{mN}\det\{[M]\}^N}\times\prod_{i=1}^{N}\frac{1}{\tau_i^m}\exp\left(-\frac{\mathbf{k}_i^\dagger[M]^{-1}\mathbf{k}_i}{\tau_i}\right). \quad (10)$$

For a given $[M]$, maximizing $L_{\mathbf{k}}(\mathbf{k}_1,\ldots,\mathbf{k}_N;[M],\tau_1,\ldots,\tau_N)$ with respect to τ_i yields the texture ML estimator

$$\widehat{\tau}_i = \frac{\mathbf{k}_i^\dagger[M]^{-1}\mathbf{k}_i}{m}. \quad (11)$$

Replacing τ_i in (10) by their ML estimates, the generalized likelihood is obtained as

$$L'_{\mathbf{k}}\left(\mathbf{k}_1,\ldots,\mathbf{k}_N;[M]\right) = \frac{1}{\pi^{mN}\det\{[M]\}^N}\times\prod_{i=1}^{N}\frac{m^m\exp(-m)}{\left(\mathbf{k}_i^\dagger[M]^{-1}\mathbf{k}_i\right)^m}. \quad (12)$$

[2]A "$\mathcal{K}$ stationary" process is a stochastic process whose $\mathcal{K}$ PDF does not change when shifted in time or space.

The ML estimator of the normalized covariance matrix in the deterministic texture case is obtained by canceling the gradient of $L'_{\mathbf{k}}$ with respect to $[M]$ as the solution of the following recursive equation:

$$\begin{aligned}[\widehat{M}]_{\mathrm{FP}} &= f\left([\widehat{M}]_{\mathrm{FP}}\right)\\ &= \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{k}_i\mathbf{k}_i^\dagger}{\mathbf{k}_i^\dagger[\widehat{M}]_{\mathrm{FP}}^{-1}\mathbf{k}_i}\\ &= \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{z}_i\mathbf{z}_i^\dagger}{\mathbf{z}_i^\dagger[\widehat{M}]_{\mathrm{FP}}^{-1}\mathbf{z}_i}.\end{aligned} \quad (13)$$

This approach has been used in [38] by Conte *et al.* to derive a recursive algorithm for estimating the matrix $[M]$. This algorithm consists in computing the FP of f using the sequence $([M]_i)_{i\geq 0}$ defined by

$$[M]_{i+1} = f\left([M]_i\right). \quad (14)$$

This study has been completed by the work of Pascal *et al.* [20], [39], which recently established the existence and the uniqueness, up to a scalar factor, of the FP estimator of the normalized covariance matrix, as well as the convergence of the recursive algorithm whatever the initialization. The algorithm can therefore be initialized with the identity matrix $[M]_0 = [I_m]$. One way to analyze the convergence of the FP estimator consists in evaluating the following criterion:

$$C(i) = \frac{\left\|[\widehat{M}](i+1) - [\widehat{M}](i)\right\|_F}{\left\|[\widehat{M}](i)\right\|_F} \quad (15)$$

where $\|\ldots\|_F$ represents the Frobenius norm. When computing the FP estimator, (14) is iterated until C becomes smaller than a predefined lower limit. Note that only a few iterations suffice to reach an error that is less than 10^{-15}[20].

It has also been shown in [31] and [38] that the recursive estimation scheme from (14) can be applied to derive an exact ML estimator of the normalized covariance matrix

$$[\widehat{M}]_{\mathrm{ML}} = \frac{m}{N}\sum_{i=1}^{N}\frac{h_{m+1}\left(\mathbf{k}_i^\dagger[\widehat{M}]_{\mathrm{ML}}^{-1}\mathbf{k}_i\right)}{h_m(\mathbf{k}_i^\dagger[\widehat{M}]_{\mathrm{ML}}^{-1}\mathbf{k}_i)}\mathbf{k}_i\mathbf{k}_i^\dagger$$

$$\text{with}\quad h_m(q) = \int_0^\infty \tau^p\exp\left(\frac{q}{\tau}\right)p(\tau)\,d\tau. \quad (16)$$

In the previous equation, the exact ML estimator depends on the texture PDF through the SIRV density-generating function $h_m(q)$. Chitour and Pascal [40] have recently demonstrated that (16) admits a unique solution and that its corresponding iterative algorithm converges to the FP solution for every admissible initial condition. Pascal *et al.* [20], [39] have also demonstrated that the normalized covariance ML estimator developed under the deterministic texture case (13) yields also an approximate ML estimator under stochastic texture hypothesis.

We propose to apply these results in estimating normalized coherency matrices for high-resolution POLSAR data. The main advantage of this approach is that the local "scene heterogeneity" can be taken into account without any *a priori* hypothesis regarding the texture random variable τ [(14) does not depend on τ]. The obtained FP is the approximate ML estimate under the stochastic τ assumption and the exact ML under the deterministic τ assumption. Moreover, the normalized polarimetric coherency matrix estimated using the FP method is unbiased and asymptotically Gaussian distributed [20], [39].

Note also that the texture estimator from (11) can directly be linked to the total scattered power (span) according to (9). By estimating the normalized coherency as the FP solution of (13), the derived estimate is independent of the total power, and it contains polarimetric information only. Using this matrix, it is possible to compute the SIRV span ML estimator for unknown deterministic τ as

$$\widehat{P}_{\mathrm{PWF}} = \mathbf{k}^{\dagger}[\widehat{M}]_{\mathrm{FP}}^{-1}\mathbf{k}. \tag{17}$$

One can observe that the span estimator from (17) has the same form as the PWF introduced by Novak and Burl in [11]. The only difference is the use of the normalized coherency estimate given by the FP estimator instead of the conventional SCM.

Finally, it is possible to derive an estimate of the conventional polarimetric coherency matrix according to (6)

$$[\widehat{T}]_{\mathrm{FP}} = \frac{\widehat{P}_{\mathrm{PWF}}}{m}[\widehat{M}]_{\mathrm{FP}}. \tag{18}$$

4) Gaussian Model in the SIRV Context: The multivariate Gaussian distribution presented in Section II-A is obviously a member of the SIRV class. Let us assume N i.i.d. realizations of the target vector $\mathbf{k}$. The SCM from (2) is the ML estimator of $[T]$ in the Gaussian clutter but not in the clutter described by the product model [41]. In the specific case of completely correlated texture ($\tau = \tau_i \ \forall i \in \{1, \ldots, N\}$), Richmond [42] proved that the SCM is, again, the exact ML estimator of $[M]$ provided that the M normalization is respected. In fact, the completely correlated τ case is equivalent to the Gaussian model for a given realization of data across all resolution cells [31]. Consequently, it is possible to define the normalized SCM as

$$[\widehat{M}]_{\mathrm{SCM}} = m\frac{[\widehat{T}]_{\mathrm{SCM}}}{\mathrm{Tr}\left\{[\widehat{T}]_{\mathrm{SCM}}\right\}}. \tag{19}$$

In other words, in Gaussian clutter, the local power P is no more random in (9), but $m \cdot \tau = P$ with probability one [31]. Based on this consideration and according to (17), the MPWF can be defined as

$$\widehat{P}_{\mathrm{MPWF}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{k_i}^{\dagger}[\widehat{M}_i]_{\mathrm{FP}}^{-1}\mathbf{k_i}. \tag{20}$$

The MPWF is the span ML estimator for Gaussian clutter with known power P, and it is unbiased [13], [14]. When compared to the span estimator from (5), the main advantage of the MPWF is that it takes into account the correlation between the different polarization channels (speckle) in the whitening process.

C. Spatial Support

In the estimation process, a certain number of samples must be gathered for deriving the observation vector. In this purpose, the boxcar sliding neighborhood is usually employed. The main drawback of nonadaptive BN is that the available number of samples is directly proportional with the loss of spatial resolution. In order to deal with this undesired effect, several strategies to obtain locally ANs were proposed for POLSAR data processing. In [8], three local neighborhoods are analyzed, and their performances are discussed with respect to different end-user applications (visual interpretation, classification, etc.). Experiments on real data sets have shown that the intensity-driven adaptive neighborhood (IDAN) represents, on the whole, a good tradeoff between preserving signal characteristics and gathering a significant number of samples for coherency and $H-\alpha-A$ parameter estimation [8], [43].

Recent studies have revealed that the original IDAN algorithm tends to introduce a bias with respect to the radiometry information [44]. The main reasons are the use of a symmetric confidence interval around the mean for the Gamma-distributed intensity and the estimation of the initial seed by the median computed within a 3×3 neighborhood. In order to deal with these problems, the SDAN algorithm has been introduced in [45]. It allows using heterogeneous scene models, such as SIRV, in the estimation step. Note that this approach is not optimal as the resulting AN is driven on the texture (span) information only. One may use other existing locally ANs (e.g., directional neighborhoods [9]), but, up to now, the existing AN algorithms are also tributary to the span information.

SDAN successively truncates the texture PDF using two symmetric confidence intervals around the mean. The truncation thresholds are expressed with Gamma-distributed texture. However, different PDFs can be truncated according to the same thresholds (initially set using a Gamma prior). In this paper, the SDAN is employed to eliminate eventual outliers from the local neighborhood. The main advantage of this approach consists in selecting spatially connected pixels within a certain confidence interval. Its main inconvenience is the estimation bias which can be induced by truncating the significant part of the unknown texture PDF.

Within the SIRV context, the SDAN algorithm operates under deterministic texture hypothesis: If τ is deterministic, the span statistics over matrix stationary areas is given by the Gamma PDF resulting from the complex Gaussian kernel. This is coherent with the general hypothesis adopted for POLSAR speckle filtering, stating that the local matrix stationarity property is revealed by changes in the span image when texture is absent [9].

D. Application to POLSAR Parameter Estimation

One way to derive the normalized coherency matrix is the normalized sample covariance estimator, obtained by locally

replacing the statistical mean by spatial average within the sliding neighborhood $\mathcal{W}$

$$[\widehat{M}]_{\mathrm{SCM}}(i,j) = \frac{m}{\mathrm{Tr}\left\{[\widehat{T}]_{\mathrm{SCM}}\right\}}[\widehat{T}]_{\mathrm{SCM}}$$

$$\text{with} \quad [\widehat{T}]_{\mathrm{SCM}} = \frac{1}{\mathrm{card}\left\{\mathcal{W}(i,j)\right\}} \sum_{(p,q)\in\mathcal{W}(i,j)} \mathbf{k}(p,q)\mathbf{k}^{\dagger}(p,q) \tag{21}$$

where (i,j) represents the current range/azimuth position and card$\{\mathcal{W}\}$ denotes the cardinal of $\mathcal{W}$. The main advantage of the $[\widehat{M}]_{\mathrm{SCM}}$ estimator consists in deriving the polarimetric covariance matrix independently of the span for the Gaussian case. The normalized SCM estimator presents also one major disadvantage: It is not SIRP stationary, and, as a consequence, this estimator is not consistent over textured areas. Although the derivation of the normalized SCM estimator from the standard SCM estimator is straightforward, we could not find any specific paper to report its use for POLSAR data.

In this paper, we propose to extend the estimation of the normalized polarimetric coherency matrix by using a heterogeneous scene model over the sliding neighborhood. The FP estimator of the normalized covariance matrix for the SIRV model is applied using the procedure described in Section II-B. More precisely, the FP normalized coherency matrix is computed iteratively as

$$[\widehat{M}_l]_{\mathrm{FP}}(i,j) = \frac{m}{\mathrm{card}\left\{\mathcal{W}(i,j)\right\}} \times \sum_{(p,q)\in\mathcal{W}(i,j)} \frac{\mathbf{k}(p,q)\mathbf{k}^{\dagger}(p,q)}{\mathbf{k}^{\dagger}(p,q)[\widehat{M}_{l-1}]^{-1}_{\mathrm{FP}}(i,j)\mathbf{k}(p,q)}$$

$$\text{with} \quad [\widehat{M}_0]_{\mathrm{FP}} = [I_m] \tag{22}$$

where l is the iteration index. Equation (22) gives the covariance matrix estimate of the SIRV complex Gaussian kernel without imposing any statistical constraint over the texture random variable τ. The resulting matrix $[\widehat{M}]_{\mathrm{FP}}$ is asymptotically Gaussian distributed. The proposed procedure (SDAN-FP) starts by computing the AN using the SDAN algorithm [45] at each range/azimuth position. The resulting AN is supposed to respect the matrix stationarity condition. Finally, the FP estimator is applied to derive the normalized polarimetric coherency matrix estimate under a compound Gaussian polarimetric clutter model (22).

Another physical parameter to be estimated is the total power. For the SIRV model, the PWF span estimator is the ML estimator; hence, it should be applied for textured areas. However, on Gaussian textureless areas, a stronger speckle reduction can be obtained using the MPWF estimator. In practical applications, the PWF and the MPWF estimators should be applied as follows: On Gaussian stationary regions, the best span estimator is the MPWF, while on SIRV homogeneous areas only, the PWF should be applied. We propose to deal with

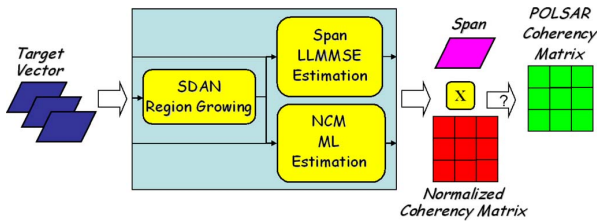


Fig. 1. Proposed estimation scheme.

this tradeoff by applying the LLMMSE criterion for the span estimation [9]

$$\widehat{P}_{\mathrm{LLMMSE}} = \widehat{P}_{\mathrm{MPWF}} + \alpha_{\mathrm{LLMMSE}}(\widehat{P}_{PWF} - \widehat{P}_{\mathrm{MPWF}})$$

$$\text{with} \quad \alpha_{\mathrm{LLMMSE}} = \frac{\sigma^2_{PWF}(1+\sigma^2_n)}{\sigma^2_{\mathrm{MPWF}} - \mu^2_{\mathrm{MPWF}}\sigma^2_n} \tag{23}$$

where μ_{MPWF}, σ_{PWF}, and σ_{MPWF} are the signal mean and standard deviations computed inside the local estimation neighborhood, respectively, and σ_n is the noise standard deviation (*a priori* known). In (23), the two span estimators can be computed according to (17) and (20).

In the last stage, it is also possible to unify these two descriptors by multiplying them according to the SIRV model from (6). An important remark is that, by multiplying the two descriptors, the separation between the total received power (span) and the polarimetric information (speckle normalized coherency) is lost. Finally, the resulting coherency matrix $[\widehat{T}]$ does not obey the Wishart PDF as it depends on the estimated span PDF.

In summary, this section introduces a novel estimation scheme (see Fig. 1) for deriving normalized polarimetric coherency matrices and resulting estimated span. The proposed algorithm couples span-driven multiresolution techniques [45] with heterogeneous SIRV scene models [20] to deal with the polarimetric texture inside the estimation neighborhood. It is important to notice that the proposed FP estimator uses normalized coherency matrix inversion, and thus, it works only with Hermitian positive definite normalized coherency matrices. This constraint is still acceptable since, in practice, image coherency matrices are generally of full rank (three for monostatic POLSAR data) [46]. However, in the specific case of a noninvertible matrix, which can correspond to a strongly polarized scattered signal, the SIRV model can be applied by using only the nonzero signal subspace.

E. Distance Measures for POLSAR Segmentation

Classification of ground cover with POLSAR data is an important application [17]–[19], [47]–[49]. Generally, one has to find a distance between the pixel covariance matrix $[C]$ and the class center $[C]_{\omega}$. Based on this distance, conventional clustering methods have already been introduced with POLSAR data: "naive" Bayesian ML classifier or K-means [18], fuzzy K-means, or expectation maximization [47].

When the POLSAR data are modeled by a stochastic process with a known PDF, it is possible to derive optimal ML distance measures (e.g., the Wishart distance for Gaussian processes). In [27], Yueh *et al.* derived an optimal ML distance measure

for terrain cover classification using the normalized target vector in the lexicographical basis. The adopted normalization condition was the Euclidian norm, and the distance measure was computed applying the Bayesian ML classifier with the PDF of the normalized polarimetric data. Note that, in Yueh's approach, the covariance matrix is estimated using the SCM (ML estimator only with Gaussian clutter). In consequence, the derived optimal distance is a generalized ML distance for Gaussian clutter only.

We propose the following general binary hypothesis test for a given class ω:

$$\begin{cases} H_0 : [C] = [C]_\omega \\ H_1 : [C] \neq [C]_\omega. \end{cases} \tag{24}$$

According to the Neyman–Pearson Lemma, the likelihood ratio test (LRT) provides the most powerful test [50]

$$\Lambda = \frac{p_m(\mathbf{k}_1, \ldots, \mathbf{k}_N / H_1)}{p_m(\mathbf{k}_1, \ldots, \mathbf{k}_N / H_0)}. \tag{25}$$

For Gaussian clutter, maximizing the LRT from (25) and replacing the pixel coherency matrix $[T]$ with the ML estimate $[\widehat{T}]_{\mathrm{SCM}}$ are equivalent to minimizing the conventional Wishart distance

$$D_{\mathrm{Wishart}}\left([\widehat{T}]_{\mathrm{SCM}}, [T]_\omega\right) = \ln \frac{\det\{[T]_\omega\}}{\det\left\{[\widehat{T}]_{\mathrm{SCM}}\right\}} + \mathrm{Tr}\left\{[T]_\omega^{-1}[\widehat{T}]_{\mathrm{SCM}}\right\}. \tag{26}$$

This distance has been widely used for supervised and unsupervised POLSAR data clustering [18], [19], [47].

In the case of the SIRV model, one can rewrite the hypothesis test as

$$\begin{cases} H_0 : [M] = [M]_\omega & \Leftrightarrow \quad \mathbf{k} = \sqrt{\tau}\mathbf{z}, \text{ with } \mathbf{z} \sim \mathcal{N}(0, [M]) \\ H_1 : [M] \neq [M]_\omega & \Leftrightarrow \quad \mathbf{k} = \sqrt{\tau}\mathbf{z}, \text{ with } \mathbf{z} \sim \mathcal{N}(0, [M]_\omega) \end{cases} \tag{27}$$

where τ is the unknown deterministic texture.

For a given class $[M]_\omega$, the LRT with respect to the texture τ and the normalized coherency matrix $[M]$ is given by

$$\Lambda_{\mathrm{SIRV}} = \frac{\prod_{n=1}^{N} \frac{1}{\pi^m \tau_n^m \det\{[M]_\omega\}} \exp\left\{-\frac{\mathbf{k}_n^\dagger [M]_\omega^{-1} \mathbf{k}_n}{\tau_n}\right\}}{\prod_{n=1}^{N} \frac{1}{\pi^m \tau_n^m \det\{[M]\}} \exp\left\{-\frac{\mathbf{k}_n^\dagger [M]^{-1} \mathbf{k}_n}{\tau_n}\right\}}. \tag{28}$$

Notice that the likelihood function in (28) does not use the stochastic texture description as the PDF $p(\tau)$ is supposed unknown in the SIRV model. As previously stated in Section II-B3, the texture parameter τ can be considered either as a random variable with unknown PDF $p(\tau)$ or as an unknown deterministic parameter with PDF $p(\tau) = \delta(\tau - \tau_n)$ which characterizes yet a particular SIRV process. It can be shown that the ML estimation of the coherency matrix yields a good approximate ML estimate in the first case and the true ML estimate in the second case [31], [38]. With the general PDF being unknown, it is therefore impossible to derive a texture-independent closed-form expression for the likelihood ratio of the test given by (27). This procedure is here simplified, considering a particular SIRV process with a texture characterized by an unknown deterministic parameter. Consequently, each resolution cell is now associated with its own $p(\tau) = \delta(\tau - \tau_n)$ in (7), where τ_n are the unknown deterministic texture variables. This way, the texture descriptor can be discarded for each pixel independently.

By taking the natural logarithm in (28), one obtains

$$\ln(\Lambda_{\mathrm{SIRV}}) = -N \ln \frac{\det\{[M]_\omega\}}{\det\{[M]\}} - \sum_{n=1}^{N} \frac{\mathbf{k}_n^\dagger \left([M]_\omega^{-1} - [M]^{-1}\right) \mathbf{k}_n}{\tau_n}. \tag{29}$$

Now, since τ_n's and $[M]$ are unknown, they are replaced by their ML estimates from (11) and (13). The resulting generalized LRT Λ'_{SIRV} is given by

$$\ln(\Lambda'_{\mathrm{SIRV}}) = -N \ln \frac{\det\{[M]_\omega\}}{\det\left\{[\widehat{M}]_{\mathrm{FP}}\right\}} - m \sum_{n=1}^{N} \frac{\mathbf{k}_n^\dagger [M]_\omega^{-1} \mathbf{k}_n}{\mathbf{k}_n^\dagger [\widehat{M}]_{\mathrm{FP}}^{-1} \mathbf{k}_n} + Nm. \tag{30}$$

Maximizing the generalized LRT over all classes is equivalent to minimizing the following SIRV distance:

$$D_{\mathrm{SIRV}}\left([\widehat{M}]_{\mathrm{FP}}, [M]_\omega\right) = \ln \frac{\det\{[M]_\omega\}}{\det\left\{[\widehat{M}]_{\mathrm{FP}}\right\}} + \frac{m}{N} \sum_{n=1}^{N} \frac{\mathbf{k}_n^\dagger [M]_\omega^{-1} \mathbf{k}_n}{\mathbf{k}_n^\dagger [\widehat{M}]_{\mathrm{FP}}^{-1} \mathbf{k}_n}. \tag{31}$$

Notice that computing the distance from (31) needs the original scattering vectors $\mathbf{k}_n$.

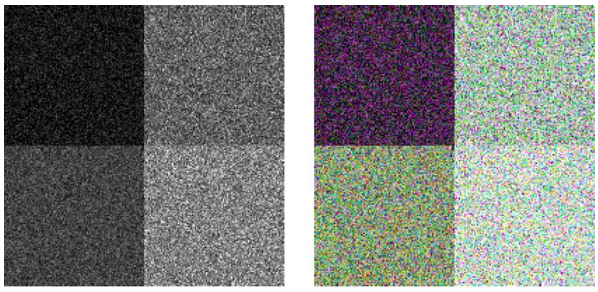
In this paper, the distance measure from (31) is used as a dissimilarity measure in the conventional K-means clustering for POLSAR data. The full description of the K-means algorithm can be found in [18].

In summary, this section introduces a new distance measure between normalized coherency matrices. The resulting approximate generalized ML distance is optimal for POLSAR data characterized by the SIRV model.

An interesting remark concerning the SIRV distance can be observed in (31). On the one hand, when the texture (span information) is high, the second term of the SIRV distance D_{SIRV} becomes small, and the distance measure is dominated by the determinant ratio. This usually corresponds to strongly polarized targets with a dominant scattering mechanism (e.g., dihedral, trihedral, etc.). On the other hand, with smaller span values, the distance is dominated by the second term which takes into account the N observed samples. This second case often corresponds to distributed targets.

III. Results and Discussion

This section has two main objectives. The first one consists in evaluating the performance of the normalized coherency estimation techniques presented in Section II. The second objective is to show the improvement in the conventional



(a) (b)

Fig. 2. Simulated POLSAR data, Gaussian case (200 × 200 pixels). (a) Initial one-look span estimated using (3). (b) Amplitude color composition of the target vector elements $k_1 - k_3 - k_2$.

POLSAR processing chain brought by introducing the normalized coherence matrix related to the SIRV model.

Three different estimation techniques are analyzed: the normalized SCM coupled with the 7 × 7 BN (BN-SCM) and the FP estimator coupled either with the 7 × 7 BN (BN-FP) or with the SDAN (SDAN-FP). In all three cases, the corresponding span image is estimated using the LLMMSE estimator from (23). The parameters used for the SDAN algorithm are $L_{\mathrm{eq}} = 3$ and $N_{\max} = 50$.

A. Simulated POLSAR Data

As, for real data, it is impossible to find reference regions with known coherency matrix, the effectiveness of the estimation schemes is demonstrated using simulated POLSAR data [30].

1) Gaussian Case: The first POLSAR data set consists of four adjacent Gaussian regions as presented in Fig. 2. Each of the four quadrants is associated with a known deterministic texture value and a known theoretical covariance matrix. Using these parameters, each component of the polarimetric target vector is simulated accordingly. Fig. 2(a) shows the initial span image computed using (3), and Fig. 2(b) shows the resulting amplitude color composition of the three target vector components.

The LLMMSE span P and the normalized coherency matrix $[M]$ are estimated using the three different estimation schemes. Note that, in the Gaussian case, the optimal ML estimation technique is the BN-SCM from Fig. 3(a) and (d). Inside each quadrant, the stochastic process characterizing the data is Gaussian stationary; hence, it is also SIRV homogeneous. The BN-FP estimation yields quite similar results, from the visual point of view, as shown in Fig. 3(b) and (e). However, the use of the BN is associated with the well-known edge-blurring effect as the matrix stationarity condition is not respected over the transitions between the quadrants. The SDAN-FP estimation reduces this undesired effect as presented in Fig. 3(c) and (f). As a general remark, the blurring is more present within the normalized coherency diagonal elements than within the span images due to the use of the adaptive LLMMSE span estimator.

In order to objectively assess the estimation performances, the mean and the variance for each element of the normalized coherency are computed over the SE quadrant. A global error measure ϵ for the normalized coherency matrix is also introduced as

$$\epsilon = \frac{1}{N} \sum_{i=1}^{N} \frac{\left\| [\widehat{M_i}] - [M_{\mathrm{ref}}] \right\|_F}{\| [M_{\mathrm{ref}}] \|_F} \tag{32}$$

where $[M_{\mathrm{ref}}]$ is the reference normalized coherency matrix used for data simulation. As observed in Table I, the best results are obtained using the BN-SCM estimator. Being the ML estimator for Gaussian stationary regions, the SCM is used as a benchmark for the SDAN and the FP estimator. The mean value is well preserved for both BN-FP and SDAN-FP estimates, while the variance of the BN-FP is smaller than the measured variance of the SDAN-FP. The latter observation is explained by the fact that the BN is optimal on such SIRV homogeneous regions. One can also note that, despite the mean of each element of the normalized coherency being quite similar, a better error measure is provided by the ϵ parameter. Using the Frobenius norm, which is a norm associated to the inner product on the ring of all complex matrices, the corresponding error ϵ shows that the smallest error is obtained for the optimal BN-SCM estimator. When introducing the FP estimator, ϵ increases, and it increases even more by using the SDAN adaptive spatial support. This behavior corresponds to the expected theoretical observations. However, it is important to notice that, for both BN-FP and SDAN-FP, the error is not increased by more than 7% with respect to the ML estimator. This is acceptable for the POLSAR applications where the clutter is characterized by a Gaussian stationary stochastic process.

A similar objective performance assessment is carried out for the estimation of the span image. Table II shows the span mean ratio and the speckle coefficient of variation computed for the same Gaussian stationary region. As for the normalized coherency, the bias in the estimated radiometry is less than 7% for all three estimation techniques. An interesting remark consists in the fact that, when using the FP estimator, the bias from Table II is also linked with the average computed over the corresponding homogeneous region. Even if the mean ratio is a standard parameter for evaluating the speckle filter radiometric bias, this parameter is not so well adapted for the FP estimation. Although being asymptotically Gaussian distributed, the FP estimator is outperformed by the SCM (ML estimator for Gaussian clutter) with a fixed number of samples. Consequently, the average over a homogeneous area should be coupled with the estimation of the FP normalized coherency over the same homogeneous population for optimal performances. This effect can be noticed with the SDAN-FP span, where the local SDAN can gather more than 49 samples over Gaussian stationary areas. The resulting SDAN-FP span estimator exhibits a radiometric bias that is less than 1% (same as for the BN-SCM).

In summary, the subjective and objective performance assessment carried out for Gaussian POLSAR clutter shows that, despite being suboptimal, the proposed FP estimator and the SDANs give good overall performances. The corresponding error measure is less than 7% for all estimation schemes.

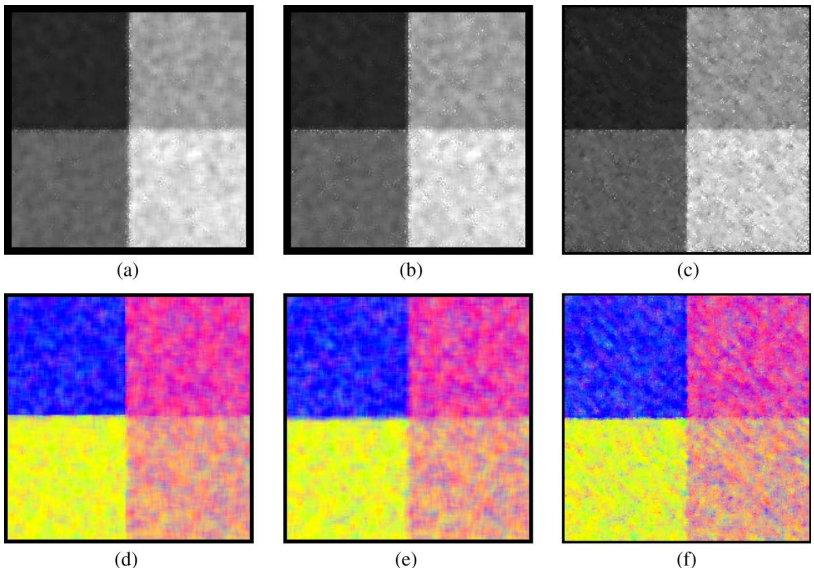


Fig. 3. Simulated POLSAR data, Gaussian case (200 × 200 pixels). Square root of the LLMMSE span image using the normalized coherency estimated by (a) BN-SCM, (b) BN-FP, and (c) SDAN-FP. Color composition of the normalized coherency diagonal elements $[M]_{11} - [M]_{33} - [M]_{22}$ estimated by (d) BN-SCM, (e) BN-FP, and (f) SDAN-FP.

TABLE I
SIMULATED POLSAR DATA, GAUSSIAN CASE: MEAN AND STANDARD DEVIATION OF THE NORMALIZED COHERENCY ELEMENTS OVER GAUSSIAN STATIONARY AREAS

Parameter (3378 pixels)	Value	Mean			Standard deviation		
		BN-SCM	**BN-FP**	**SDAN-FP**	**BN-SCM**	**BN-FP**	**SDAN-FP**
M_{11}	1.79	1.78	1.77	1.77	0.13	0.15	0.19
M_{22}	0.77	0.78	0.78	0.78	0.11	0.12	0.15
M_{33}	0.43	0.44	0.44	0.44	0.06	0.07	0.09
$\Re\{M_{12}\}$	0.01	0.01	0.01	0.01	0.10	0.13	0.16
$\Im\{M_{12}\}$	-0.19	-0.17	-0.17	-0.17	0.12	0.13	0.16
$\Re\{M_{13}\}$	0.07	0.08	0.08	0.08	0.09	0.10	0.11
$\Im\{M_{13}\}$	0.03	0.03	0.03	0.03	0.09	0.10	0.12
$\Re\{M_{23}\}$	0.16	0.15	0.15	0.15	0.06	0.07	0.08
$\Im\{M_{23}\}$	0.02	0.02	0.02	0.02	0.06	0.07	0.08
Mean normalized error (ϵ)		0.17	0.19	0.23			

TABLE II
SIMULATED POLSAR DATA, GAUSSIAN CASE: SPAN MEAN RATIO $\widehat{\mu}/\mu_{\text{ref}}$ AND COEFFICIENT OF VARIATION OVER GAUSSIAN STATIONARY AREAS

Span (3378 pixels)	Mean ratio			Coefficient of variation		
	BN-SCM	**BN-FP**	**SDAN-FP**	**BN-SCM**	**BN-FP**	**SDAN-FP**
LLMMSE	0.99	1.07	0.99	0.10	0.13	0.16

2) SIRV Case: The second simulated POLSAR data set proposes the same four quadrants but with Gamma-distributed texture [Fig. 4(a)]: Each quadrant is $\mathcal{K}$ distributed. The texture coefficient of variation used for simulation is equal to three, which corresponds to a highly non-Gaussian clutter (urban areas). Fig. 4(b) shows the initial span image. Fig. 4(c) shows the corresponding amplitude color composition of the three target vector components.

The overall data set is not SIRV homogeneous as the matrix stationarity condition is not respected on the boundaries; however, each quadrant is SIRP stationary. In the following, we shall use only the "SIRV homogeneity" assumption over each quadrant, namely, the texture PDF is supposed unknown. Fig. 5 shows the LLMMSE span P and the normalized coherency matrix $[M]$ estimated using the three different estimation schemes.

As the data set is not Gaussian, the PWF span estimator is dominant in the LLMMSE criterion, and the corresponding speckle reduction is performed using only three samples. Hence, concerning the LLMMSE span estimation, BN-SCM, BN-FP, and SDAN-FP [Fig. 5(a)–(c)] look similar from the visual point of view.

The effectiveness of the FP estimator in compound Gaussian clutter can be observed in Fig. 5(e) and (f). While the BN-SCM normalized coherency [Fig. 5(d)] presents a "patchy" appearance, the BN-FP estimation [Fig. 5(e)] provides better visual homogeneity within each quadrant. The adaptive SDAN

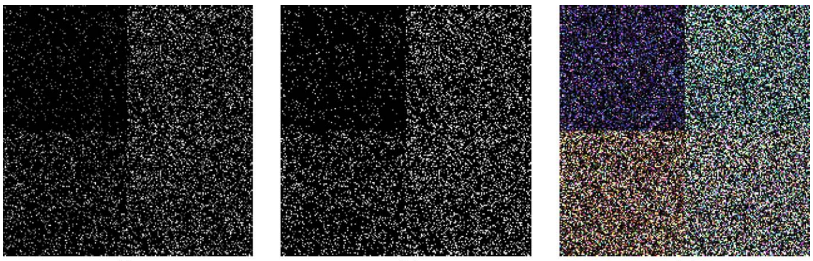


Fig. 4. Simulated POLSAR data, SIRV case (200 × 200 pixels). (a) Texture image. (b) Initial one-look span estimated using (3). (c) Amplitude color composition of the target vector elements $k_1 - k_3 - k_2$.

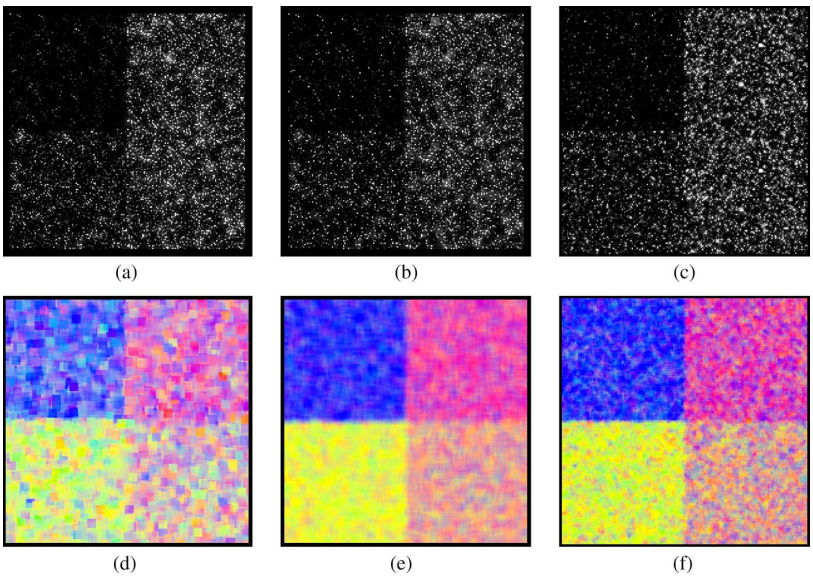


Fig. 5. Simulated POLSAR data, SIRV case (200 × 200 pixels). Square root of the LLMMSE span image using the normalized coherency estimated by (a) BN-SCM, (b) BN-FP, and (c) SDAN-FP. Color composition of the normalized coherency diagonal elements $[M]_{11} - [M]_{33} - [M]_{22}$ estimated by (d) BN-SCM, (e) BN-FP, and (f) SDAN-FP.

spatial support [Fig. 5(f)] assures better edge preservation for the transitions between quadrants. One important issue is that the diagonal elements of the BN-FP normalized coherency for the SIRV case [Fig. 5(e)] have the same visual aspect as for the previous Gaussian POLSAR data set [Fig. 3(e)]. This shows that the FP estimate of the covariance matrix does not depend on the texture PDF.

Using the same reference region as for the Gaussian case, Table III presents the mean and the variance for each element of the normalized coherency and also the overall error measure ϵ computed for the three estimation schemes. BN-FP and SDAN-FP outperform BN-SCM in retrieving the reference value and also in terms of variance reduction. Since the matrix stationarity is always assured within the reference region, BN-FP outperforms the SDAN-FP also. Finally, another interesting result consists in the fact that Table III indicates the same BN-FP value for the ϵ error parameter as in the Gaussian case (Table I). This objective issue confirms the visual comparison mentioned in the previous paragraph.

Objective performance assessment has been carried out for the LLMMSE span estimation also. Table IV presents the Kolmogorov–Smirnov (KS) test with respect to the reference span used for simulation. The resulting KS values, computed over the entire span image, indicate that BN-FP outperforms BN-SCM. The best results are reported when using the SDAN-FP estimator. Note that the KS distance is rather small $\epsilon \in (0.07, 0.12)$ in all three cases.

B. Airborne POLSAR Data

To illustrate the improvements in the standard POLSAR processing chain, the results obtained with high- and very high resolution airborne data are reported. Both data sets were acquired by the airborne French Aerospace Laboratory (ONERA) RAMSES system [51].

1) High-Resolution POLSAR Data: The first POLSAR data set was acquired in Brétigny, France. The mean incidence angle is 30°. It represents a fully polarimetric (monostatic mode) X-band acquisition with a spatial resolution of approximately 1.5 m in range and azimuth.

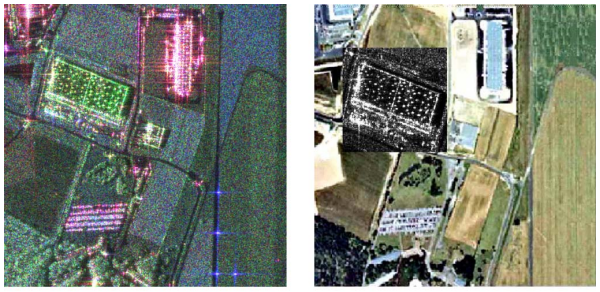
Fig. 6(a) shows the color composition of the target vector amplitudes. The target area is composed of three buildings, a

TABLE III
SIMULATED POLSAR DATA, SIRV CASE: MEAN AND STANDARD DEVIATION OF THE NORMALIZED COHERENCY ELEMENTS OVER SIRP STATIONARY AREAS

Parameter (3378 pixels)	Value	Mean			Standard deviation		
		BN-SCM	**BN-FP**	**SDAN-FP**	**BN-SCM**	**BN-FP**	**SDAN-FP**
M_{11}	1.79	1.75	1.80	1.76	0.35	0.14	0.24
M_{22}	0.77	0.75	0.75	0.79	0.27	0.10	0.20
M_{33}	0.43	0.49	0.44	0.44	0.23	0.08	0.11
$\Re\{M_{12}\}$	0.01	0.04	0.01	0.02	0.36	0.14	0.22
$\Im\{M_{12}\}$	-0.19	-0.17	-0.19	-0.17	0.35	0.11	0.24
$\Re\{M_{13}\}$	0.07	0.09	0.07	0.06	0.30	0.11	0.17
$\Im\{M_{13}\}$	0.03	0.04	0.04	0.04	0.26	0.09	0.17
$\Re\{M_{23}\}$	0.16	0.18	0.16	0.15	0.18	0.06	0.10
$\Im\{M_{23}\}$	0.02	0.03	0.02	0.02	0.19	0.06	0.10
Mean normalized error (ϵ)		0.51	0.19	0.28			

TABLE IV
SIMULATED POLSAR DATA, SIRV CASE: THE KS TEST $KS_n = \max_x |F_n(x) - F_{\text{ref}}(x)|$ FOR THE SPAN DISTRIBUTION

Span (3378 pixels)	K-S test		
	BN-SCM	**BN-FP**	**SDAN-FP**
LLMMSE	0.111	0.100	0.068



(a) (b)

Fig. 6. Brétigny, RAMSES POLSAR data, X-band (501 × 501 pixels). (a) Amplitude color composition of the target vector elements $k_1 - k_3 - k_2$. (b) Optical image (137 × 137 pixel zoom of the initial span superposed for illustrating the region of interest).

parking lot, and the surrounding agricultural areas. For further illustration, a non-Gaussian urban (building) region has been selected, namely, the span image superposed over the airborne photograph from Fig. 6(b).

The LLMMSE span and the normalized coherency matrix are estimated using the three different estimation schemes (Fig. 7). The BN-FP span illustrated in Fig. 7(b) exhibits better whitening in the estimation process than the BN-SCM span from Fig. 7(a). This can be observed on the isolated brilliant pointwise structures surrounding the building. However, both BN-SCM and BN-FP are tributary to the "ring effect" (two large dips on a spatial profile near the boundaries of a pointwise target) induced by coupling the BN spatial support with the LLMMSE estimator [52]. This effect is reduced in the SDAN-FP span image as it can be observed over the metallic structures that are present on the roof of the building from Fig. 7(c).

Visual assessment is carried out also with the normalized coherency $[M]$ estimates. Color compositions, constructed from either the diagonal elements of $[M]$ or the corresponding $H-\alpha-A$ parameters [17], are computed for the three estimation techniques [Fig. 7(d)–(i)]. Both parameters exhibit the same behavior:

1) BN-SCM: patchy appearance mainly due to the texture;
2) BN-FP: blurring as the matrix stationarity condition is not respected;
3) SDAN-FP: higher spatial feature preservation but more variance.

As the target area is highly heterogeneous, the SDAN-FP estimation is a good tradeoff between robust estimation and spatial resolution preservation.

Finally, it is possible to derive the SDAN-FP polarimetric coherency matrix as a product between the span image [Fig. 7(c)] and the corresponding normalized coherency [Fig. 7(f)]. In Fig. 8(a)–(c), the SDAN-FP coherency is compared with the conventional coherency matrices obtained by the SCM estimator coupled with two spatial supports: the BN and the IDAN [8]. Subjective visual assessment can be expressed in terms of the hue–saturation–lightness color space [53] by associating the lightness to the span and the saturation to the polarimetric diversity. The SDAN-FP coherency from Fig. 8(c) exhibits better performances in terms of lightness and saturation, which means that both the span and the normalized coherency are better estimated. The corresponding $H-\alpha$ classification maps [17] are shown in Fig. 8(d)–(f). For $H-\alpha$ classification also, the SDAN-FP coherency provides better performances as it achieves stronger noise reduction than the IDAN filter.

One key issue to be discussed is whether the normalized coherency matrix and the span should be aggregated in the final estimation step or not (the question mark from Fig. 1). Most of the existing processing chains use the conventional coherency matrix for representing POLSAR data for unsupervised land cover classification [18], [19], [43], [47] and for target detection applications [12], [33]. Due to the SIRV model identification problem discussed in Section II-B, the complete description of the POLSAR data set is achieved by estimating the span and the normalized coherency independently. The latter describes the polarimetric diversity, while the span indicates the total received power. Moreover, the FP estimation of the normalized coherence does not depend on the span information. Given these facts, we propose to investigate this problem in the framework of unsupervised POLSAR classification. The classification scheme discussed in the following is the standard Wishart $H-\alpha$ segmentation [18]. For segmenting the

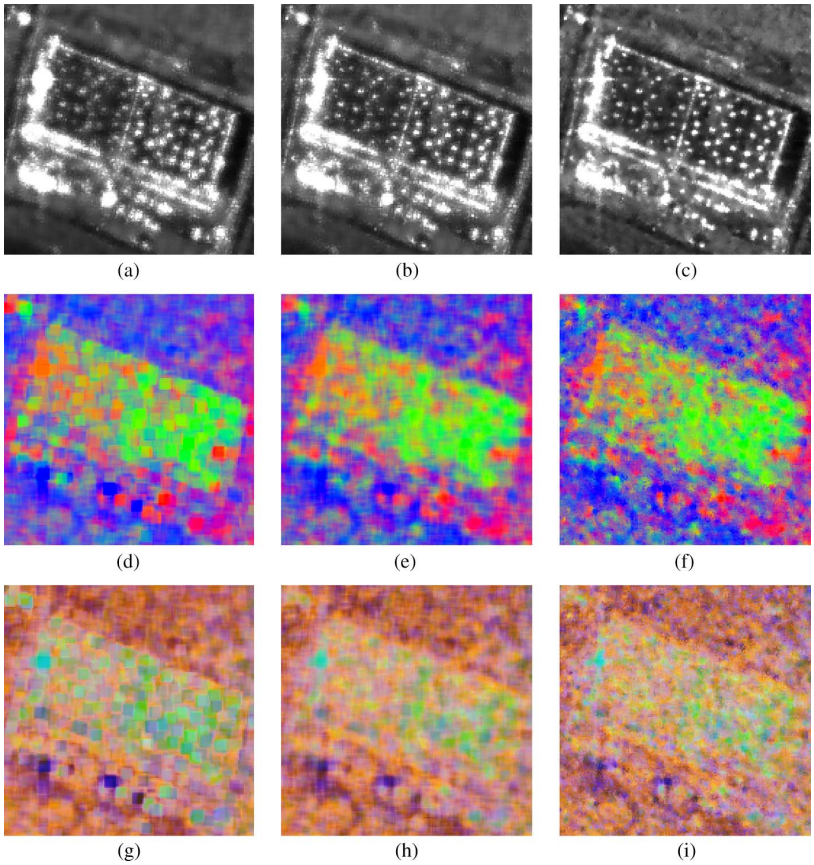


Fig. 7. Brétigny, RAMSES POLSAR data, X-band (137 × 137 pixels). Square root of the LLMMSE span image using the normalized coherency estimated by (a) BN-SCM, (b) BN-FP, and (c) SDAN-FP. Color composition of the normalized coherency diagonal elements $[M]_{11} - [M]_{33} - [M]_{22}$ estimated by (d) BN-SCM, (e) BN-FP, and (f) SDAN-FP. Color composition of the polarimetric $H-\alpha-A$ parameters estimated by (g) BN-SCM, (h) BN-FP, and (i) SDAN-FP.

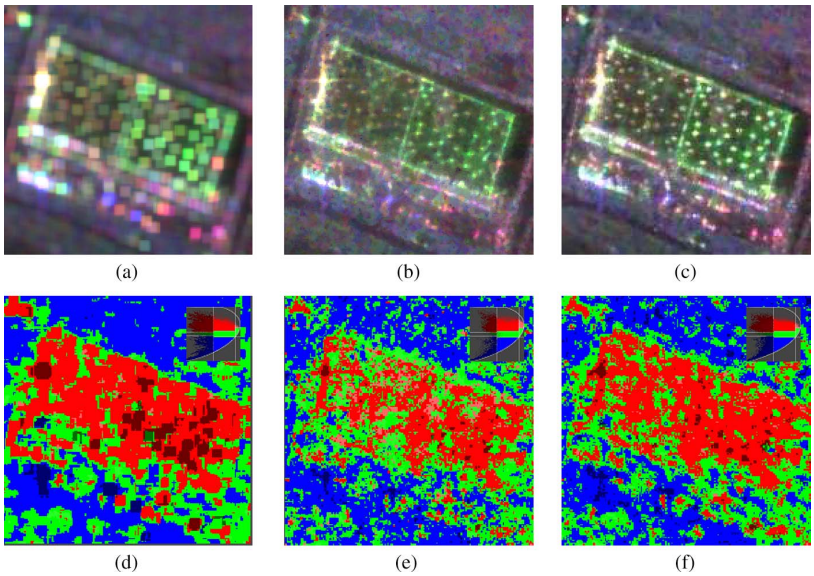


Fig. 8. Brétigny, RAMSES POLSAR data, X-band (137 × 137 pixels). Color composition of the coherency diagonal elements $[T]_{11} - [T]_{33} - [T]_{22}$ estimated by (a) BN, (b) IDAN, and (c) SDAN-FP after multiplication with the LLMMSE span from Fig. 7(c). $H-\alpha$ classification results using (d) BN, (e) IDAN, and (f) SDAN-FP.

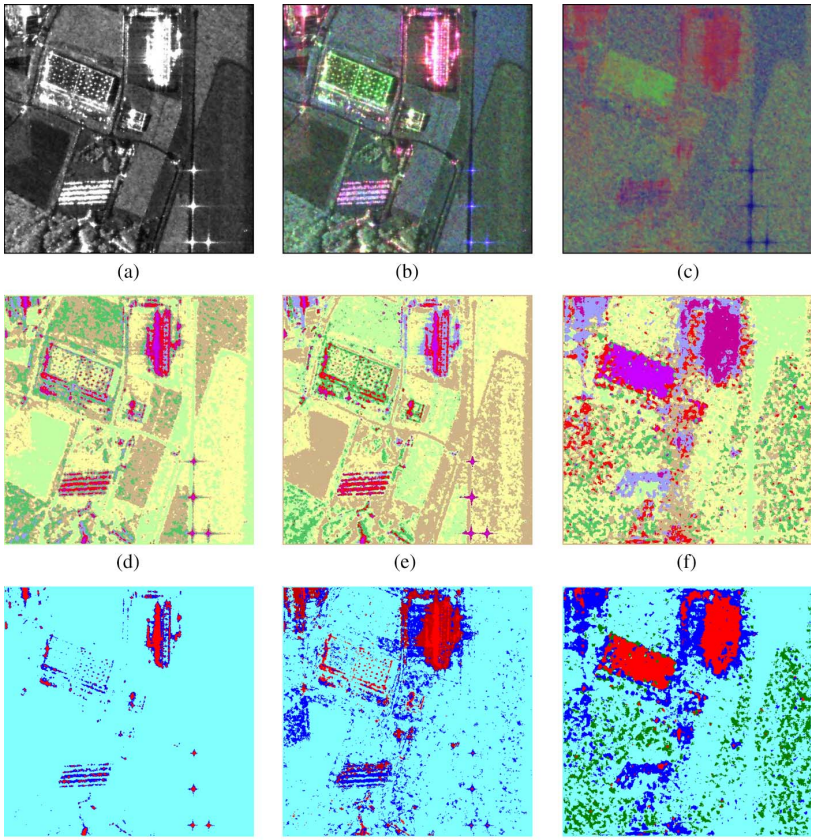


Fig. 9. Brétigny, RAMSES POLSAR data, X-band (501 × 501 pixels). LLMMSE span using the normalized coherency estimated by SDAN-FP: (a) Span image, (d) Gamma unsupervised classification, and (g) physical mechanism identification (odd-bounce classes, even-bounce classes, and volume class) using the SDAN-FP normalized coherency. SDAN-FP coherency matrix after span multiplication: (b) Color composition of the diagonal elements $[T]_{11} - [T]_{33} - [T]_{22}$, (e) Wishart unsupervised classification, and (h) physical mechanism identification. SDAN-FP normalized coherency matrix: (c) Color composition of the diagonal elements $[M]_{11} - [M]_{33} - [M]_{22}$, (f) SIRV unsupervised classification, and (i) physical mechanism identification.

normalized coherency, we have modified the Wishart $H-\alpha$ algorithm by replacing the Wishart distance with the SIRV ML distance discussed in Section II-E. For comparison, we have also used the scalar Gamma K-means classification with $H-\alpha$ initialization.

Fig. 9 shows the POLSAR unsupervised classification results using three descriptors estimated by SDAN-FP: span [Fig. 9(a)], coherency [Fig. 9(b)], and normalized coherency [Fig. 9(c)]. The selected scene is composed of both Gaussian (agricultural fields) and non-Gaussian (urban) areas. This case is encountered in many practical POLSAR classification applications. Fig. 9(e) shows the eight-class segmentation map obtained using the SDAN-FP coherency matrix. When compared to the scalar unsupervised classification map [Fig. 9(d)] obtained using the span only, one can observe the high degree of similarity between them. This leads to the following conclusion concerning the Brétigny data set: The Wishart $H-\alpha$ classification is mainly influenced by the information contained in the span image. The same behavior has been reported over alpine glaciers, with L-band E-SAR data [45]. Regarding the polarimetric information, Fig. 9(f) shows the classification map computed using the normalized coherency matrix and the associated SIRV distance. The visual assessment in Fig. 9(e) and (f) reveals that a significant part of the polarimetric information is lost when using the standard coherency matrix: The building class separation is lost, as well as the natural canonical targets (trihedral, dihedral, etc.) that are present over different "field" classes. One important remark concerning the Wishart $H-\alpha$ classification is that a large number of samples are usually

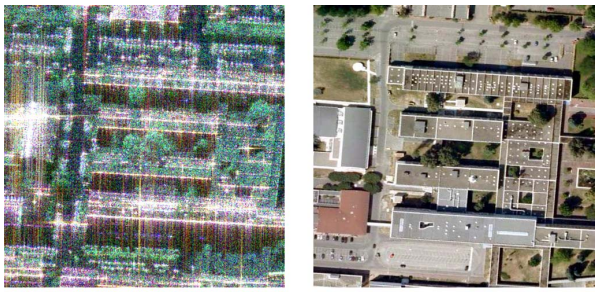


Fig. 10. Toulouse, RAMSES POLSAR data, X-band (500 × 500 pixels), resolution azimuth and range of 50 cm. (a) Amplitude color composition of the target vector elements $k_1 - k_3 - k_2$. (b) Optical image.

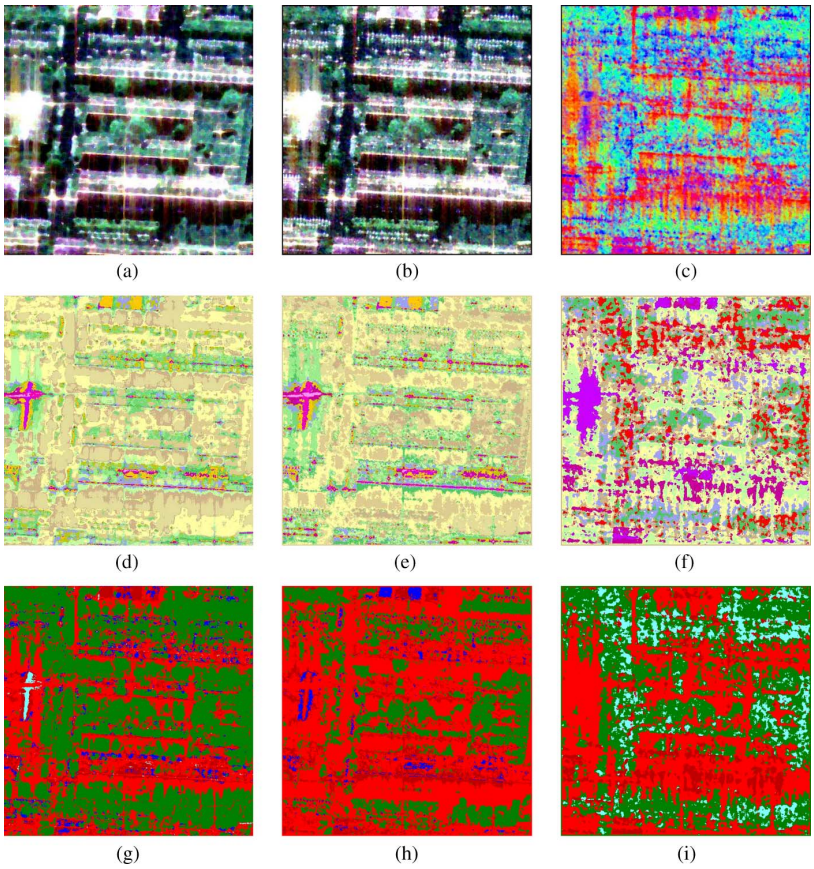


Fig. 11. Toulouse, RAMSES POLSAR data, X-band (500 × 500 pixels), resolution azimuth and range of 50 cm. Color composition of the diagonal elements $[T]_{11} - [T]_{33} - [T]_{22}$: (a) Lee refined filter and (b) SDAN-FP coherency matrix after span multiplication. (c) Color composition of the diagonal elements $[M]_{11} - [M]_{33} - [M]_{22}$ estimated by SDAN-FP. Wishart unsupervised classification: (d) Coherency estimated by the Lee refined filter and (e) SDAN-FP coherency matrix after span multiplication. (f) Unsupervised classification of the SDAN-FP normalized coherency based on the SIRV distance measure. Physical mechanism identification (odd-bounce classes, even-bounce classes, and volume class) using (g) coherency estimated by the Lee refined filter, (h) SDAN-FP coherency matrix after span multiplication, and (i) SDAN-FP normalized coherency.

assigned to the class feature vector when iterating the K-means clustering algorithm. Due to this, locally Gaussian areas (agricultural fields) may become heterogeneous regions as neither the matrix stationarity nor the texture homogeneity conditions are respected.

The same behavior can also be observed in Fig. 9(g)–(i) with the classification maps obtained after basic scattering mechanism identification [54]. The use of polarimetric indicators, derived from the eigenvector–eigenvalue decomposition of the normalized coherency matrix, allows the interpretation of each cluster scattering mechanism from Fig. 9(d)–(f). In all three cases, the POLSAR parameters were computed using the SDAN-FP normalized coherency from Fig. 9(f). The observed scene is then classified into three canonical scattering types: even bounce (blue or cyan), odd bounce (red or dark red), and volume scattering (green) [55].

2) Very High Resolution POLSAR Data: The second POLSAR data (Fig. 10) set was acquired in Toulouse, France, with a mean incidence angle of $50°$. It represents a fully polarimetric (monostatic mode) X-band acquisition with a spatial resolution of approximately 50 cm in range and azimuth.

Fig. 11 shows the visual assessment of the proposed estimation scheme applied to very high resolution POLSAR data acquired in an urban environment. The obtained results are visually compared to those obtained by the refined Lee filter operating under Gaussian clutter hypothesis [9]. With a 50-cm spatial resolution, the SDAN-FP normalized coherency from Fig. 11(c) reveals higher variability in polarimetric signatures than with a 1.5-m spatial resolution [Fig. 7(f)]. Fig. 11(b) shows the color composition of the diagonal elements of the SDAN-FP coherency matrix after multiplication by the corresponding LLMMSE span. When compared to the polarimetric coherency derived by the refined Lee filter [Fig. 11(a)], the SDAN-FP coherency better preserves the polarimetric and radiometric signatures over thin spatial features (brilliant points and edges), while over larger structures (buildings, fields, and roads), the two images look similar. This can also be observed in the eight-class segmentation maps obtained by Wishart $H-\alpha$ clustering [Fig. 11(d) and (e)]. It is important to stress that, for very high resolution urban POLSAR data, the polarimetric coherency matrix is not Wishart distributed. Hence, the unsupervised $H-\alpha$ classification based on the SIRV distance measure can

properly be applied using the SDAN-FP normalized coherency. The result is shown in Fig. 11(f). Finally, the three classification maps are interpreted according to the basic scattering mechanism identification procedure [54]. The subjective visual assessment indicates that quite realistic results are obtained using the SDAN-FP normalized coherency descriptor [Fig. 11(i)]: Buildings and cars are mainly retrieved in the red "odd-bounce" class, while "even-bounce" scattering mechanism (cyan class) appears on the flat regions (roads).

In conclusion, the joint analysis of the span and the normalized coherency presents several advantages with respect to the coherency matrix descriptor: separation between the total received power and the polarimetric information, estimation of the normalized coherency matrix independently of the span, and the existence of the SIRV distance measure for unsupervised ML classification of normalized coherencies. However, the span–normalized-coherency description of POLSAR images raises new problems which still remain under investigation. The first issue concerns the use of span for testing the matrix stationarity condition for the normalized coherency estimation. This test is currently used for POLSAR data speckle filtering, and it is founded on the basic principle that changes within the polarimetric signature are revealed by changes in the total received power. Consequently, one may envisage other estimation schemes dedicated to the SIRV model with stochastic texture by considering external estimators of matrix stationarity. The second important remark concerns the Wishart unsupervised classification scheme. Although all statistical requirements employed for unsupervised classification are met, the polarimetric information is quite difficult to extract using the K-means clustering. As it can be noticed in Fig. 9(c), the polarimetric signatures are strongly mixed, and the class boundaries are smoothed within high-resolution POLSAR images (even for highly heterogeneous target areas). Therefore, other clustering strategies should be better suited to capture the spatial distribution of different polarimetric signatures. One starting point could be the POLSAR segmentation by likelihood approximation [56], spectral clustering ensemble [57], or the support vector machines kernel-based nonlinear classification [58].

Finally, one can observe that span information does also, in some cases, contribute to classification quality (e.g., discrimination of roads in Fig. 9 and buildings in Fig. 11), although the polarimetric signature clustering suffers. Based on the SIRV model, the separation span/polarimetric signature is achieved. Future work is needed to objectively assess the classification potential of these two descriptors separately.

IV. General Remarks

One critical point of the SIRV model is linked to the scalar texture (span) descriptor τ. The validity of the product model for POLSAR data has been investigated in many papers over the last decades [7], [11]–[13], [28].

Yueh *et al.* [27] derived the generalized likelihood of the normalized polarimetric target vector in Gaussian clutter. This approach has been extended to the $\mathcal{K}$-distributed clutter in [6] and [28]. Note that this extension is not optimal since the covariance matrix parameter is replaced by the SCM, or the SCM depends on the texture PDF $p(\tau)$, and it is not the ML estimator of the covariance matrix in the $\mathcal{K}$-distributed clutter. The exact ML normalized covariance estimator can be derived using Yao's representation theorem for SIRVs, and its exact expression is given in (16).

The product model has also been used by Novak *et al.* [11], [12] for deriving the PWF. Based on this result, Lopes and Sery [13] derived the MPWF as well as the adaptive LLMMSE filter for Gaussian and $\mathcal{K}$-distributed clutter. The SIRV representation theorem allows the derivation of the PWF as an ML estimator of the deterministic texture. Once the texture parameter is obtained for every resolution cell, further statistical processing can be applied over a population of texture parameters (e.g., the proposed LLMMSE span filter).

For Gaussian clutter, Lee *et al.* [18], [19], [47] introduced optimal polarimetric covariance matrix classification schemes based on the Wishart distance. The proposed methods can be extended to the SIRV model by using the SIRV distance presented in Section II-E. Moreover, the asymptotic distribution of the FP estimator from (13) has been derived in [39]. The FP estimator computed with N samples (secondary data) converges in distribution to the normalized SCM computed with $N[m/(m+1)]$ secondary data. Since the normalized SCM is the SCM up to a scale factor, we may conclude that, in problems invariant with respect to a scale factor on the covariance matrix, the FP estimate is asymptotically equivalent to the SCM computed with $N[m/(m+1)]$ secondary data.

We can conclude that Yao's representation theorem allows optimal multivariate signal processing of POLSAR data in a general framework. The SIRV model provides the methodology for retrieving the conventional cases (multivariate Gaussian and multivariate $\mathcal{K}$ distribution). This methodology can also be generalized to other heterogeneous clutter models defined by explicit texture PDFs (inverse Gamma, Fisher, etc.) [59].

More recent studies have revealed the presence of different scattering characteristics between the cross- and copolar terms of the Sinclair matrix [16], [60]. In consequence, the POLSAR clutter could be modeled by different texture random variables for each polarization channel. Such a stochastic model already exists in the literature, and it is known as the generalized SIRV model [61]. Unfortunately, the covariance matrix generalized SIRV estimator of the Gaussian kernel could not be found, without taking into account any *a priori* information about the texture multivariate PDF. Future work should investigate the coupling between SIRVs and multiple single-channel spatial texture descriptors, such as the nonstationary anisotropic Gaussian-kernel model [62].

Despite being quite general, the SIRV clutter model supposes the matrix stationarity condition to be verified over the observation vector. We proposed the use of an adaptive spatial support based on the scalar span information. The resulting SDAN operates under deterministic texture hypothesis, and it states that the local matrix stationarity property is revealed by changes in the span image.

One limitation of the proposed estimation scheme concerns the determination of the SIRV homogeneous neighborhood surrounding a pixel. The strategy adopted for this paper consists

in testing the matrix stationarity condition using the span, under deterministic texture assumption. Despite not being optimal in the context of the SIRV model, the proposed approach does not require additional *a priori* information regarding the local clutter statistics.

Finally, the SDAN-FP algorithm is more computation intensive than other existing POLSAR speckle filtering algorithms developed for Gaussian clutter [8]–[10], and it handles single-look complex data only. Further work should address the extension of the proposed approach to adaptive nonlinear filtering of multilook POLSAR data.

V. Conclusion and Perspectives

This paper has presented a new estimation scheme for deriving normalized coherency matrices and the resulting estimated span with high-resolution POLSAR images. The proposed approach couples nonlinear ML estimators with SDANs for taking the local scene heterogeneity into account.

The heterogeneous clutter in POLSAR data was described by the SIRV model. Two estimators were introduced for describing the POLSAR data set: the FP estimator of the normalized coherency matrix and the corresponding LLMMSE span. The FP estimation is independent on the span PDF and represents an approximate ML estimator for a large class of stochastic processes obeying the SIRV model. Moreover, the derived normalized coherency is asymptotically Gaussian distributed.

For the SIRV clutter, a new ML distance measure was introduced for unsupervised POLSAR classification. This distance was used in conventional K-means clustering initialized by the $H-\alpha$ polarimetric decomposition. Other extensions of the existing unsupervised or supervised POLSAR clustering methods (e.g., Bayes ML or fuzzy K-means) can be derived by replacing the conventional Wishart distance with the proposed SIRV distance.

The effectiveness of the proposed estimation scheme was illustrated by high- and very high resolution ONERA RAMSES X-band POLSAR data. The reliability of the obtained results was demonstrated by quantitative performance assessments using simulated POLSAR data.

This work has many interesting perspectives. We believe that this paper contributes toward the description and the analysis of heterogeneous clutter over scenes exhibiting complex polarimetric signatures. The proposed approach presents a high degree of generality as no explicit stochastic texture model is needed. Finally, the proposed estimation scheme can be extended to other multidimensional SAR techniques using the covariance matrix descriptor, such as the following: multibaseline interferometry, polarimetric interferometry, or multifrequency polarimetry. Future work should address the quantitative performance analysis of classification and target detection algorithms based on these estimators.

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Résumé

Ce mémoire d'HDR présente une synthèse des travaux de recherche ou d'études menés depuis une vingtaine d'années au sein de l'unité Traitement du Signal du Département Électromagnétisme et Radar de l'ONERA, The French Aerospace Lab à Palaiseau. Ces travaux ont, pour la plupart, été essentiellement consacrés à l'apport des techniques de Traitement du Signal pour le radar et l'imagerie radar. Le premier axe sur lequel je me suis penché provient des perspectives logiques issues de mes travaux de thèse sur la transformation de Mellin et consiste à étendre à l'imagerie SAR ou l'imagerie ISAR (Inverse Synthetic Aperture Radar) les techniques temps-fréquence utilisées pour l'analyse des signaux non-stationnaires. L'Analyse Temps-Fréquence permet de pouvoir étudier la dispersivité et l'anisotropie des réflecteurs dans une image SAR ou ISAR. Principalement basé sur des analyses en sous bandes et sous ouvertures, ce type d'analyse a également permis d'analyser la non-stationnarité angulaire et spectrale des mécanismes polarimétriques de rétrodiffusion des réflecteurs d'une image SAR polarimétrique mais également d'améliorer l'estimateur de la cohérence interférométrique (INSAR) voire la cohérence polarimétrique et interférométrique (POLINSAR) qui permet l'estimation de la hauteur des points du sol.
Mon deuxième axe de recherche concerne la détection et l'estimation en environnement hétérogène et/ou non gaussien. Les modèles gaussiens composés SIRV (Spherically Invariant Random Vectors), vecteurs multivariés gaussiens dont la variance est aléatoire, permettent d'étendre de manière très élégante et efficace toutes les méthodes de détection classiques basées sur l'hypothèse restrictive gaussienne. Couplées à de nouveaux estimateurs puissants et robustes des paramètres de l'environnement (matrice de covariance par exemple), ces techniques peuvent s'appliquer à la détection radar multi-voies (antenne réseau pour la localisation de sources, techniques de type STAP (Space Time Adaptive Processing), MIMO, interférométrie, polarimétrie, imagerie hyperspectrale, ...).
Ces deux axes de recherches qui ont été construits et exploités de manière indépendante, peuvent en fait se combiner pour résoudre de nouveaux problèmes développés dans mes perspectives de recherche. Les applications dérivées de ces deux thèmes de recherche sont assez nombreuses : l'analyse des images SAR mono et multi voies (segmentation, classification), détection de cibles mobiles dans les images SAR, l'imagerie hyperspectrale, la détection de changement.

Mots-clés: Détection, Estimation, Radar, Imagerie Radar, SIRV, STAP, SAR

Abstract

This HDR thesis presents a synthesis of works I have been conducting since more than twenty years in the Signal Processing Unit (Electromagnetism and Radar Department) of ONERA, the French Aerospace Lab. All these works have mainly been devoted to the Signal Processing techniques for radar and imaging radar. The fist part comes logically from my Ph.D. works conducted on the Mellin Transform and consists in extending Time Frequency Distributions for radar imaging. Usually used for analyzing non stationary signals, these techniques allow to study the coloration and the anisotropy of the scatterers in a SAR or ISAR image. Mainly based on sub-band and sub-look decompositions, these Time-Frequency distributions allow also to study the angular and spectral non-stationarity of the polarimetric mechanisms of scatterers in polarimetric SAR images but also to improve the quality of the interferometric coherency estimate (INSAR) and of the polarimetric coherency estimate (POLINSAR).
The second part concerns radar detection and estimation in non homogeneous and/or non-Gaussian environment. The compound Gaussian SIRV (Spherically Invariant Random Vectors)modelling, allows nicely to extend all the classical detection schemes based only on the Gaussian assumption. Jointly used to robust and powerful estimators of the unknown environment parameters (e.g. covariance matrix), these modelling techniques can be applied to multi channels radar detection (array processing for source

localization, STAP (Space Time Adaptive Processing), MIMO, interferometry, polarimetry, hyperspectral imaging, ...).
In fact, these two research axes that have been built and exploited independently can be jointly be used to solve new problems that are developed in my research perspectives. All the applications derived from these two axes are quite numerous : mono and multi channels SAR images analysis (detection, segmentation, classification), moving target detection in SAR images, hyperspectral imaging, change detection, ...

Keywords: Detection, Estimation, Radar, Radar Imaging, SIRV, STAP, SAR