A Joint Optimization for Coherent MIMO Radar

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Abstract

This paper deals with the optimization of a set of transmitted sequences and their associated mismatched filters, for the coherent MIMO radar. A minimization problem may be considered, with an objective function that measures for instance the correlations within the set. In the literature, it is usually solved alternatively, meaning that the optimization is performed on the sequences while the mismatched filters are set, and *vice-versa*. In this paper, an iterative method that proceeds jointly is introduced. Sequences are derived with a gradient descent, and mismatched filters are chosen in such a way that they are optimal in the ISL (*Integrated Sidelobe Level*) sense. Simulations show interesting results, as a joint optimization performs better than a separate one, even with relatively short sequences.

1 Introduction

Wireless communications face among other challenges an increasing number of users and data demands [1]. An intuitive solution consists in using multiple antennas: it is the MIMO (*Multiple-Input Multiple-Output*) concept. This concept has been adopted in radar in the late 70's [2]; a MIMO radar employs several transmitters and several receivers. Two configurations of MIMO radars are usually considered, known as statistical and coherent. In a statistical MIMO radar, antenna elements are widely separated, improving detection performance [3], while they are sufficiently close in a coherent (or co-located) MIMO radar, providing a better spatial resolution [4]. This article focuses on the latter, and especially on waveform design.

Figure 1 illustrates the concept of the coherent MIMO radar. Contrary to a phased array, each transmitter produces its own signal, represented by different colors. Each target receives a coherent sum of these elementary signals, that is distinct according to its position. In other words, it appears that each direction is explored simultaneously with a different signal, implying that this radar may broadcast wide beams, unlike (again) phased arrays.

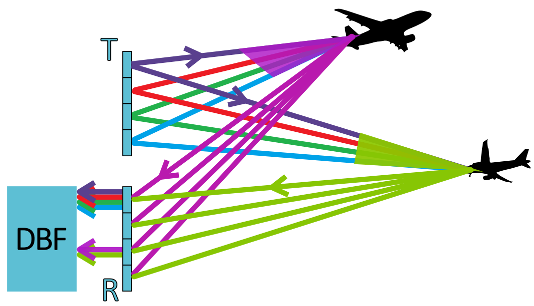

Fig. 1: Concept of the coherent MIMO radar (DBF: *Digital Beamforming*)

In this way, the coherent processing on receive is obtained via a matched filter in range (*i.e.,* in delay) and in angle. It means that the received signal is not compared with each elementary signal, but with signals transmitted in different directions. The MIMO ambiguity function [5] measures the correlations of signals that are radiated by the radar. So, in wireless communications or in radar processing, a lot of studies have been made on waveform design (see [6] and [7] for instance). Besides, MIMO waveforms can be classified into several categories [8]. Among them, phase codes seem present the best range/angle coupling (in the ambiguity function), but at a cost of high sidelobes [9]. These sidelobes should thus be reduced in order to improve detection performances: a weak target response can indeed be buried into sidelobes.

This is one of the reason why radar processing is not done in practice with a matched filter, but with another signal, known as a mismatched filter. This flexibility may thus provide lower sidelobes, but at a cost of a containable *Loss-in-Processing Gain* (LPG). Through optimization techniques, it is possible to compute the optimal mismatched filter that minimizes the PSLR (*Peak-to-Sidelobe Level Ratio*) or the ISL (*Integrated Sidelobe Level*) [10].

A joint optimization of a set of transmitted sequences and of some mismatched filters is proposed in this article, in the coherent MIMO framework. An iterative procedure is suggested: each mismatched filter is chosen ISL-optimal and is expressed as a function of the transmitted sequences. Sequences and filters are then both computed with a gradient descent (the gradient calculation depends on the filter choice).

This article is organized as follows. It begins with a short reminder on the matched filter and the mismatched filter. Section 3 formulates the joint optimization problem, and submits a method in order to solve it. Some results are given in the last section.

Notations: In the following, bold letters designate matrices and vectors. $(.)^*$, $(.)^T$ and $(.)^H$ denote the conjugate, the transpose and the transpose conjugate operator, respectively.

2 Matched Filter, Mismatched Filter

This section reminds some definitions on the matched filter and the mismatched filter. It also explains how an optimal mismatched filter can be obtained through an optimization problem, for a given sequence. More details are given in [10].

2.1 Definitions

Let $\boldsymbol{s}$ be a discrete signal containing N samples:

$$\boldsymbol{s} = [s_1, s_2, \ldots, s_N]^T. \tag{1}$$

In the following developments, the sequence $\boldsymbol{s}$ is of constant modulus (which is a non-convex constraint). Let $\alpha_k \in [-\pi, \pi]$ be the phase angle of the element s_k:

$$s_k = \begin{cases} \exp(j2\pi\alpha_k)/\sqrt{N} & \text{if } k \in [\![1, N]\!] \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Matched filtering consists in a comparison of the signal $\boldsymbol{s}$ and a time-shifted version of itself, *i.e.,* generating the sequence $\boldsymbol{y}$ of length $2N-1$ such that:

$$\boldsymbol{y} = \boldsymbol{\Lambda}_N(\boldsymbol{s})\boldsymbol{s}^*, \tag{3}$$

where $\boldsymbol{\Lambda}_K(\boldsymbol{s})$ is a matrix of size $K+N-1 \times K$ containing delayed versions of the sequence $\boldsymbol{s}$, such that:

$$\boldsymbol{\Lambda}_K(\boldsymbol{s}) := \begin{bmatrix} s_N & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & s_N & \ddots & & & & \vdots \\ s_2 & & \ddots & 0 & & & \vdots \\ s_1 & s_2 & \cdots & s_N & 0 & \cdots & 0 \\ 0 & s_1 & \ddots & \vdots & s_N & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 & & \ddots & 0 \\ 0 & \cdots & 0 & s_1 & s_2 & & s_N \\ \vdots & & & 0 & s_1 & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & s_2 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & s_1 \end{bmatrix} \tag{4}$$

On the other hand, processing the signal $\boldsymbol{s}$ with a different filter $\boldsymbol{q}$, of length K, is called mismatched filtering:

$$\boldsymbol{y} = \boldsymbol{\Lambda}_K(\boldsymbol{s})\boldsymbol{q}. \tag{5}$$

Note that, in both cases, the central element is designated as the mainlobe, while the others are sidelobes.

The mismatched filter is less constrained than the matched filter, since $\boldsymbol{q}$ can take any value in $\mathbb{C}^K$. Moreover, its length may also differ from N, and can be in particular chosen to contain more elements, thus providing additional degrees of freedom compared to the matched filter. It will be assumed without loss of generality that $K = N + 2p$, $p \in \mathbb{N}$, so that the length of $\boldsymbol{y}$ is odd.

The matched filter is known for maximizing the SNR (*Signal-to-Noise Ratio*) at the peak response, under a white noise hypothesis. Using a mismatched filter implies inevitably a loss-in-processing gain (LPG), expressed by:

$$\begin{aligned} \text{LPG} &= 10\log_{10}\left(\frac{\text{SNR}_{\text{mismatched}}}{\text{SNR}_{\text{matched}}}\right) \\ &= 10\log_{10}\left(\frac{|\boldsymbol{q}^H\boldsymbol{s}|^2}{(\boldsymbol{q}^H\boldsymbol{q})(\boldsymbol{s}^H\boldsymbol{s})}\right) \leq 0. \end{aligned} \tag{6}$$

This loss-in-processing gain can be inserted as a convex constraint in optimization problems, depending on $\boldsymbol{s}$ and $\boldsymbol{q}$, as in [10]:

$$\text{LPG} \geq -10\log_{10}\alpha \Leftrightarrow \boldsymbol{q}^H\boldsymbol{q} \leq \alpha\boldsymbol{s}^H\boldsymbol{s}. \tag{7}$$

2.2 Optimal Mismatched Filters

Several criteria have been introduced in order to measure the performance of these filters: the Merit Factor [11], the Peak-to-Sidelobe Level Ratio (PSLR) [10] or the Integrated Sidelobe Level (ISL). In this article is studied the latter, defined by:

$$\text{ISL}(\boldsymbol{s}, \boldsymbol{q}) := \boldsymbol{y}^H\boldsymbol{F}\boldsymbol{y}, \tag{8}$$

where $\boldsymbol{y}$ is defined as in (5) and $\boldsymbol{F}$ is a diagonal matrix of order $K+N-1$, defined by the vector $[1, \ldots, 1, 0, 1, \ldots, 1]$, with ones except for a 0 at the entry $N+p$.

The ISL can be considered as an objective function of an optimization problem, called in this article (P_{ISL}):

$$(P_{\text{ISL}}) \begin{cases} \min_{\boldsymbol{q}} & \text{ISL}(\boldsymbol{s}, \boldsymbol{q}) \\ \text{s.t.} & \boldsymbol{s}^H\boldsymbol{q} = \boldsymbol{s}^H\boldsymbol{s}, \end{cases} \tag{9}$$

that can be solved analytically, using Lagrangian multipliers [12]:

$$\boldsymbol{q}_{\text{ISL}}(\boldsymbol{s}) = \frac{\left(\boldsymbol{s}^H\boldsymbol{s}\right)\left(\boldsymbol{\Lambda}_K(\boldsymbol{s})^H\boldsymbol{F}\boldsymbol{\Lambda}_K(\boldsymbol{s})\right)^{-1}\boldsymbol{s}}{\boldsymbol{s}^H\left(\boldsymbol{\Lambda}_K(\boldsymbol{s})^H\boldsymbol{F}\boldsymbol{\Lambda}_K(\boldsymbol{s})\right)^{-1}\boldsymbol{s}}. \tag{10}$$

This last definition means that, for a given sequence $\boldsymbol{s}$, there exists an optimal filter $\boldsymbol{q}_{\text{ISL}}(\boldsymbol{s})$ that minimizes the ISL. Note here that this analytic solution does not guarantee an acceptable loss-in-processing gain. If the LPG is added as a second constraint in the constrained optimization problem (P_{ISL}), then the new problem does not seem to present any analytic solution (but its solution can be found numerically).

3 A Joint Optimization for Coherent MIMO Radar

This section deals with a joint optimization of the transmitted sequences of a coherent MIMO radar, and their associated mismatched filters. It begins with some reminders on the coherent MIMO radar. The problem formulation is given in a second part, while an algorithm is proposed in the last one.

3.1 The Coherent MIMO Radar

Consider a transmitting array of N_E antennas and a receiving array of N_R antennas. Each of these antennas transmits its particular waveform, possibly different from the others. The signal radiated by the radar in the direction θ_c, denoted by $\boldsymbol{s}(\theta_c)$, is [5]:

$$s_l(\theta_c) = \sum_{m=1}^{N_E} \mathrm{e}^{j\boldsymbol{x}_{E,m}^T \boldsymbol{k}(\theta_c)} s_l^m \,, \tag{11}$$

where:

- $\boldsymbol{x}_{E,m}^T$ is the position of the m-th transmission antenna,
- $\boldsymbol{k}(\theta_c)$ is the wave vector in the direction θ_c,
- $\boldsymbol{s}^m := [s_1^m, ..., s_N^m]^T$ is the waveform of length N assigned to the m-th antenna.

It can be worth noticed that a colocated MIMO radar produces different signals at different angles, unlike classical phased arrays. Hence, even with isotropic antennas, transmit beampattern might not be uniform. The power density is incidentally defined by [13]:

$$P(\theta) = \frac{1}{4\pi} \boldsymbol{k}(\theta)^H \boldsymbol{R} \boldsymbol{k}(\theta), \tag{12}$$

where $\boldsymbol{R}$ is the matrix of signal cross-correlations, that is to say $\boldsymbol{R}_{k,l} := (\boldsymbol{s}^k)^H \boldsymbol{s}^l$.

The usual ambiguity function measures the correlation between a signal and a delayed version of it (in this paper, the Doppler shift is ignored). However, in the coherent MIMO background, the angular aspect should also be considered. Hence, the MIMO ambiguity function is defined by [5]:

$$A(k,\theta_l,\theta_{l'}) := \sum_{k'=1}^{N} \sum_{n=1}^{N_R} \left(\mathrm{e}^{j\boldsymbol{x}_{R,n}^T \boldsymbol{k}(\theta_l)} \sum_{m=1}^{N_E} \mathrm{e}^{j\boldsymbol{x}_{E,m}^T \boldsymbol{k}(\theta_l)} s_{k'}^m \right) \left(\mathrm{e}^{j\boldsymbol{x}_{R,n}^T \boldsymbol{k}(\theta_{l'})} \sum_{m'=1}^{N_E} \mathrm{e}^{j\boldsymbol{x}_{E,m'}^T \boldsymbol{k}(\theta_l')} s_{k+k'-N}^m \right)^* \tag{13}$$

where $\boldsymbol{x}_{R,n}^T$ denotes the position of the n-th reception antenna. As said in the previous section, radar processing can also be performed with mismatched filters. These mismatched filters should be associated with a direction, like the signal transmitted by the radar. Remark that they are thus not directly related to the probing signals.

3.2 An Optimization Problem

In this section, an optimization problem is developed. Obtained solutions should be interesting waveform/filter pairs for the coherent MIMO radar.

As previously said, a mismatched filter is associated to a given direction. It could thus be convenient to discretise the angular domain: let $\Theta = \{\theta_1, \ldots, \theta_\nu\}$, $\nu \in \mathbb{N}^*$, be a set of directions of interest. The signal transmitted by the radar in the direction θ_l is denoted by $\boldsymbol{s}(\theta_l) = [s_1(\theta_l), \ldots, s_N(\theta_l)]^T$ while its counterpart mismatched filter is denoted by $\boldsymbol{q}(\theta_l)$, $l \in [\![1, \nu]\!]$.

In addition, their autocorrelation function should present low sidelobes. In the ISL sense, it is equivalent to minimize the following, with $l \in [\![1, \nu]\!]$:

$$\mathrm{E_a}(\boldsymbol{s}(\theta_l), \boldsymbol{q}(\theta_l)) = \mathrm{ISL}(\boldsymbol{s}(\theta_l), \boldsymbol{q}(\theta_l)). \tag{14}$$

In practice, the position of the target (*i.e.,* the origin of the received signal) is unknown. Reception processing is performed by filtering the received signal with each mismatched filter. So it is important that sidelobes generated by the correlation between a signal backscattered by a target in a given direction and a mismatched filter optimized for a different direction, remain low. In other words, sidelobes should be minimized along both range and angle dimensions. This is equivalent to a criterion on cross-correlations:

$$\mathrm{E_b}(\boldsymbol{s}(\theta_l), \boldsymbol{q}(\theta_{l'})) = \sum_{k'=1}^{K+N-1} |y_{k'}|^2 \,, \quad l \neq l', \tag{15}$$

with $\boldsymbol{y} = \boldsymbol{\Lambda}_K\left(\boldsymbol{s}(\theta_l)\right) \boldsymbol{q}(\theta_{l'})$ defined as in (5).

Finally, the power density has to be similar for each $\theta_k \in \Theta$, so that no direction is favored:

$$\mathrm{E_c}(\{\boldsymbol{s}^k\}) = \sum_{l=1}^{\nu} \left| P(\theta_l) - \bar{P} \right|^2 , \tag{16}$$

with $\bar{P}$ the mean power density in Θ, $\bar{P} := \sum_{l=1}^{\nu} P(\theta_l)/\nu$.

Gathering all these expressions gives an objective function E on the set of transmitted sequences $\{\boldsymbol{s}^k\}_{k \in [\![1, N_E]\!]}$, and on the set of mismatched filters $\{\boldsymbol{q}^l\}_{l \in [\![1,\nu]\!]}$:

$$\mathrm{E}\left(\{\boldsymbol{s}^k\}, \{\boldsymbol{q}^l\}\right) := \sum_{l=1}^{\nu} \mathrm{E_a}(\boldsymbol{s}(\theta_l), \boldsymbol{q}(\theta_l)) + \sum_{\substack{l,l'=1 \\ l \neq l'}}^{\nu} \mathrm{E_b}(\boldsymbol{s}(\theta_l), \boldsymbol{q}(\theta_{l'})) + \mathrm{E_c}(\{\boldsymbol{s}^k\}), \tag{17}$$

and its associated optimization problem (P_1):

$$(P_1) \begin{cases} \min_{\{\boldsymbol{s}^k\},\{\boldsymbol{q}^l\}} \mathrm{E}\left(\{\boldsymbol{s}^k\}, \{\boldsymbol{q}^l\}\right) \\ \text{s.t. } \boldsymbol{s}(\theta_l)^H \boldsymbol{q}(\theta_l) = \boldsymbol{s}(\theta_l)^H \boldsymbol{s}(\theta_l), \forall l \in [\![1, \nu]\!]. \end{cases} \tag{18}$$

3.3 The Proposed Algorithm

How can the previous optimization problem be solved? An iterative algorithm is proposed herein, that includes a gradient descent and computations of optimal filters in the ISL sense.

The optimization problem (18) is quite complicated as it may have a lot of variables ($N\,N_E + K\nu$ elements, to be precise). But especially, it is a non-convex problem, because of the constant modulus constraint on the transmitted sequences.

A lot of methods have been reviewed in order to solve this sort of problem [14]. Among them, the steepest descent has shown interesting capabilities, even if its convergence is local. An example of optimization of phase codes for the coherent MIMO radar is explained in [15].

A joint optimization of a sequence and its mismatched filter with a gradient-based method (L-BFGS) has already been proposed [16]. This procedure implies computing a gradient for the mismatched filter as well as for the signal. In this article is proposed on the contrary to exploit the existence of an analytical expression of the optimal filter that minimizes the ISL.

The suggested procedure is based on a modified version of the problem (P_1). Separating both optimization variables gives the following:

$$(P_2)\left\{\begin{array}{l}\min_{\{\boldsymbol{s}^k\}}\min_{\{\boldsymbol{q}^l\}}\mathrm{E}\left(\{\boldsymbol{s}^k\},\{\boldsymbol{q}^l\}\right)\\ \text{s.t. } \boldsymbol{s}(\theta_l)^H\boldsymbol{q}(\theta_l)=\boldsymbol{s}(\theta_l)^H\boldsymbol{s}(\theta_l),\forall l\in[\![1,\nu]\!].\end{array}\right. \quad (19)$$

A part of this expression may be identified as (P_{ISL}), as:

$$(P_{2a})\left\{\begin{array}{l}\min_{\{\boldsymbol{q}^l\}}\mathrm{E_a}\left(\{\boldsymbol{s}^k\},\{\boldsymbol{q}^l\}\right)=\left\{\boldsymbol{q}_{\mathrm{ISL}}\left(\boldsymbol{s}^k\right)\right\}\\ \text{s.t. } \boldsymbol{s}(\theta_l)^H\boldsymbol{q}(\theta_l)=\boldsymbol{s}(\theta_l)^H\boldsymbol{s}(\theta_l),\forall l\in[\![1,\nu]\!].\end{array}\right. \quad (20)$$

As mentioned earlier, there exists indeed a global solution of (P_{2a}) that can be computed, denoted here $\left\{\boldsymbol{q}_{\mathrm{ISL}}\left(\boldsymbol{s}^k\right)\right\}$. Its explicit definition is defined in Section 2. Based on that, the proposed method introduces this solution into the objective function, becoming a function of a sequence-only variable:

$$(P_3)\left\{\begin{array}{l}\min_{\{\boldsymbol{s}^k\}}\tilde{\mathrm{E}}\left(\{\boldsymbol{s}^k\}\right):=\mathrm{E}\left(\{\boldsymbol{s}^k\},\left\{\boldsymbol{q}_{\mathrm{ISL}}\left(\boldsymbol{s}^k\right)\right\}\right)\\ \text{s.t. } \boldsymbol{s}(\theta_l)^H\boldsymbol{q}_{\mathrm{ISL}}\left(\boldsymbol{s}(\theta_l)\right)=\boldsymbol{s}(\theta_l)^H\boldsymbol{s}(\theta_l),\forall l\in[\![1,\nu]\!].\end{array}\right. \quad (21)$$

A feasible solution of the problem (P_3) can classically be found with a gradient descent. The gradient vector, $\nabla\tilde{\mathrm{E}}$, should be composed of partial derivatives of $\tilde{\mathrm{E}}$ with respect to the phase of each element s_k^m, denoted α_k^m:

$$\left(\nabla\tilde{\mathrm{E}}\right)_k^m=\frac{\partial\tilde{\mathrm{E}}}{\partial\alpha_k^m}\quad m\in[\![1,N_E]\!],k\in[\![1,N]\!]. \quad (22)$$

It can be viewed from Eq.(10) that the optimal solution $\boldsymbol{q}_{\mathrm{ISL}}$ to the problem P_{ISL} requires a matrix inversion. Thus this gradient cannot be easily computed analytically and will be computed with finite differences here. Note that this implies such a computation for each partial derivative to compute.

The resolution of (P_3) suggests that the initial sequences $\left\{\boldsymbol{s}^k\right\}$ have been modified. Therefore, all this process should be repeated, until convergence. The algorithm is summarized in Table 1.

Algorithm A joint optimization algorithm	
Given	Transmitted sequences $\{\boldsymbol{s}^k\}$, $k\in[\![1,N_E]\!]$ Directions of interest Θ
Repeat	1. Computation of $\left\{\boldsymbol{q}_{\mathrm{ISL}}\left(\boldsymbol{s}^k\right)\right\}$, optimal mismatched filters of $\{\boldsymbol{s}^k\}$ 2. Gradient descent search — Computation of the gradient vector $\nabla\tilde{\mathrm{E}}$ — Search of the best step μ — Update of $\{\boldsymbol{s}^k\}$: $s_k^m=s_k^m\exp(-j\mu(\nabla\tilde{\mathrm{E}})_k^m)$, $m\in[\![1,N_E]\!]$, $k\in[\![1,N]\!]$
Until	A stopping criterion is satisfied.

Table 1: Proposed algorithm

4 Results

In this section, some simulations are made in order to illustrate the efficiency of the proposed algorithm, under the following parameters:

- Consider a simulated radar antenna with four transmitters ($N_E = 4$) and eight receivers ($N_R = 8$).
- Transmitted phase codes are of length $N = 32$, while the mismatched filters are three times longer.
- The loss-in-processing gain (LPG) is set to 1 dB.
- In order to emphasize the effect of the optimization, only one direction of interest will be considered: $\theta = 0$.

Figures 2 to 4 present the ambiguity function range/elevation of signals obtained with several methods. Sidelobes differences can easily be observed.

- Figure 2 represents the ambiguity function of a random initialization. Sidelobes are well-distributed into each elevation angle, and are correct in the direction of interest (around -22 dB).
- The autocorrelation property of the transmitted sequences has been optimized in Figure 3, as in [15]. An ISL-optimal filter is then applied. It can be observed that, apart from the cut in the direction of interest, most of the sidelobes are greater than -13 dB. This was expected since in that case, the optimization only considered one direction angle, thus potentially leading to undesired uncontrolled effects in the other directions.
- In Figure 4, the proposed method is applied in order to jointly optimize the transmitting sequences and the set of angular mismatched filters used for the range/angle processing. Results are quite encouraging: in the direction

of interest, the highest sidelobe is at -37 dB. Besides, the Peak Sidelobe – in all angles – is at -18 dB...

- After all, a joint optimization (Figure 3) seems to perform better than a separate one (Figure 2).

However, these observations are mainly qualitative. Hence, Figure 5 compares the cut $\theta = 0$ of the ambiguity function, which was a direction of interest in the optimization procedure. In these figures, "Random" indicates that a mismatched filter has been applied on a sequence drawn randomly. Results may not be that significant – a more substantial number of simulations should be more thorough – but they highlight well the behaviour of these algorithms.

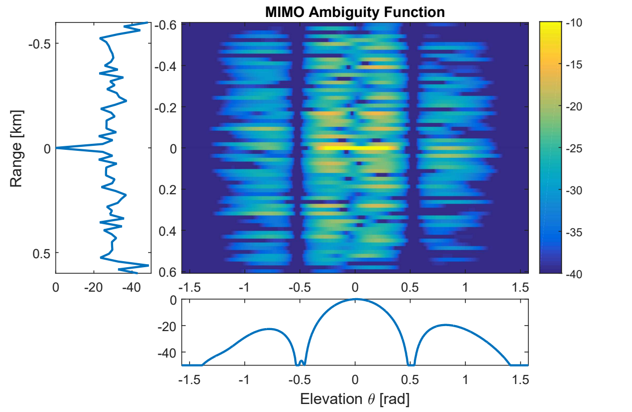

Fig. 2: Range/elevation ambiguity function of a random signal

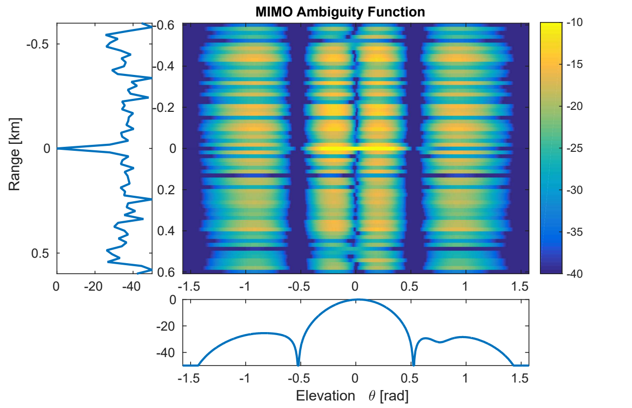

Fig. 3: Range/elevation ambiguity function after an optimization of the matched filter, in the direction $\theta = 0$

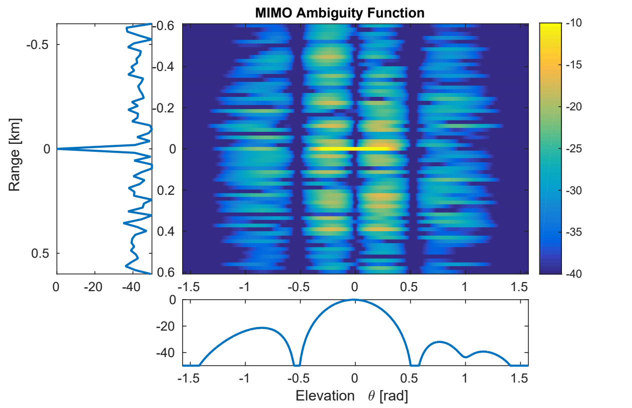

Fig. 4: Range/elevation ambiguity function after a joint optimization

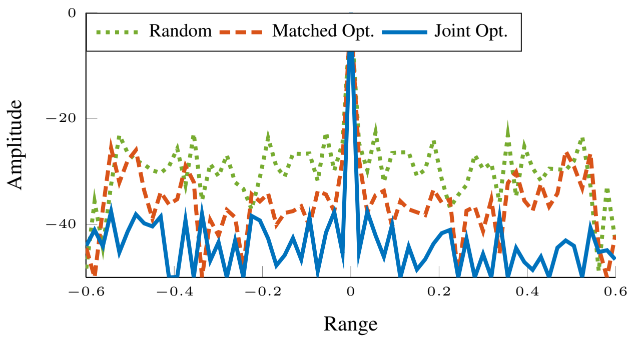

Fig. 5: Comparison of the sidelobes at the cut $\theta = 0$.

5 Conclusion

In this article, an iterative algorithm that jointly optimizes the transmitted sequences of a coherent MIMO radar and several mismatched filters has been proposed. This algorithm is based on the existence of an optimal mismatched filter in the ISL sense. Simulations have highlighted really promising results. As expected, mismatched filtering provides a noticeable gain compared to the usual matched filtering. Moreover, a joint optimization performs better than a separate one.

Ongoing works will be focused on:

- Speeding up the algorithm, by computing analytically the gradient of the modified optimization problem
- Applying the same procedure, but with the mismatched filter that minimizes the PSLR criterion
- Considering the angular domain as a continuous domain, and not as a discrete one

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