A New Method for High Resolution Polarimetric SAR Data Classification Based on the M-Box Test

Pierre Formont, Jean-Philippe Ovarlez
ONERA, DEMR/TSI, BP 72, 92322 Chatillon Cedex, France

Frédéric Pascal
SONDRA / Supélec, 3 rue Joliot Curie, 91192 Gif-sur-Yvette Cedex, France

Gabriel Vasile
GIPSA-Lab, 961 rue de la Houille Blanche, BP 46, 38402 Grenoble Cedex, France

Laurent Ferro-Famil
IETR, Campus de Beaulieu, 263 Av du Général Leclerc, 35042 Rennes Cedex, France

Abstract

Radar backscattering is classically modeled with a Gaussian process. However, in recent PolSAR imaging, the increase of resolution leads to some clutter heterogeneity, which calls for a non-Gaussian representation. Based on such a modeling, the SIRV model, this paper first compares the Wishart and the SIRV classifications on experimental polarimetric SAR data. This illustrates the strong dependency of the Wishart classification to the texture information. To fill this gap, this paper proposes a new algorithm, based on the M-Box test, to classify pixels solely on their polarimetric properties.

1 Introduction

Recent PoLSAR systems offer very high resolution images of the Earth's surface and consequently, much thinner spatial features. This increase in resolution leads to the heterogeneity of the backscattering clutter from cell to cell, especially in urban areas. One of the solutions is then to consider a model taking this heterogeneity into account such as the Spherically Invariant Random Vector product model, introduced first by Yao in [1] for detection and estimation in information theory and validated on real data measurements [2].

The polarimetric information provided by the PoLSAR system can be used to obtain several physical properties of the illuminated scene. In [3], Cloude and Pottier introduced the target entropy and the entropy-alpha-anisotropy ($H/\alpha/A$) model, based on the eigenvalue decomposition of the covariance matrix of the samples. It does not rely on a particular statistical distribution of the clutter and the underlying assumption is that there is a dominant average scattering mechanism in each cell, whose H and α parameters are to be found. Once these parameters are computed, the pixels of the image can be separated into 9 zones, depending on the values of H and α. An unsupervised classification algorithm was proposed by Lee et al. in [4], which used both the $H - \alpha$ decomposition and a distance measure based on the statistical distribution of the clutter.

Previous approach provided a distance by assuming that the data are complex multivariate Gaussian distributed. The main contribution of this paper is to propose an improved method to test if two pixels (i_1, j_1) and (i_2, j_2) from a SAR polarimetric image have the same covariance matrix. For that purpose, we assume that the covariance matrix $\mathbf{T}$ of each pixel (i, j) can be estimated from an i.i.d. N-sample $(\mathbf{k}_1, ..., \mathbf{k}_N)$. The paper is organized as follows. Section 2 presents the statistical framework; the SIRV model and the covariance matrices estimators are introduced. Section 3 is devoted to the classical Wishart and SIRV classifications and shows some results obtained on experimental polarimetric SAR data. As these methods exhibit some drawbacks, Section 4 proposes an improved procedure based on a M-Box test and provides a new classification. Finally, Section 5 concludes this work.

2 Statistical framework

2.1 SIRV model

The polarimetric clutter obtained with recent SAR systems tends to have non-Gaussian characteristics. One of the most general and elegant non-Gaussian noise model is provided by the so-called Spherically Invariant Random Vectors (SIRV). A SIRV [1] is a compound Gaussian process defined as the product of a multivariate circular Gaussian process and a scalar random variable:

$$\mathbf{k} = \sqrt{\tau}\,\mathbf{x} \tag{1}$$

where τ, a positive random variable, called *texture*, whose Probability Density Function (PDF) is unknown and $\mathbf{x}$ is a complex circular zero-mean Gaussian m-vector with covariance matrix $\mathbf{T} = E\left[\mathbf{x}\mathbf{x}^H\right]$, called *speckle* where $E[.]$ denotes the statistical expectation. For POLSAR data, the polarimetric diversity is modeled by the speckle $\mathbf{x}$ containing the 3 polarization channels HH, HV and VV, i.e. $m = 3$ and the random variation of the power from cell to cell corresponds to the texture τ. By letting $g(.)$ the texture PDF, the resulting SIRV PDF is given by:

$$p(\mathbf{k}) = \int_0^{+\infty} \frac{1}{(\pi\tau)^m\,|\mathbf{T}|}\exp\left(\frac{\mathbf{k}^H\,\mathbf{M}^{-1}\,\mathbf{k}}{\tau}\right) g(\tau)\,d\tau\,. \tag{2}$$

2.2 Covariance matrices estimators

To ideally estimate the covariance matrix of a pixel $\mathbf{k}$, one needs several realizations of the pixel at different times. As it is impossible for SAR images, a spatial neighborhood of the pixel $(\mathbf{k_1}, ..., \mathbf{k_N})$ is required for the estimation process.

To test the equality of a population covariance matrix and a known matrix, the classical hypothesis test is defined as:

$$\begin{cases} H_0 : \mathbf{T} = \mathbf{T}_\omega \\ H_1 : \mathbf{T} \neq \mathbf{T}_\omega \end{cases} \tag{3}$$

The Likelihood Ratio Test (LRT) is given by:

$$\Lambda = \frac{p\left(\mathbf{k}_1, ..., \mathbf{k}_N | H_1\right)}{p\left(\mathbf{k}_1, ..., \mathbf{k}_N | H_0\right)} \tag{4}$$

where $\mathbf{k}_i$ are the vectors relative to spatial neighbors of the pixel under test and N the size of this neighborhood.
In the classical Gaussian model, the Maximum Likelihood Estimator (MLE) of the covariance matrix is called the Sample Covariance Matrix (SCM):

$$\widehat{\mathbf{T}}_{SCM} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{k}_i\,\mathbf{k}_i^H \tag{5}$$

Under SIRV assumption, the covariance matrix can be estimated from Eq. (2) thanks to ML theory. Considering a deterministic texture, Gini *et al.* derived in [5] the exact ML estimate, solution of the implicit equation:

$$\widehat{\mathbf{T}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{k}_i\,\mathbf{k}_i^H}{\mathbf{k}_i^H\,\widehat{\mathbf{T}}_{FP}^{-1}\,\mathbf{k}_i} \tag{6}$$

Existence and uniqueness of the above equation solution $\widehat{\mathbf{T}}_{FP}$, the Fixed Point (FP) estimate, have been investigated in [6], and its statistical properties (consistency, unbiasedness and asymptotic Gaussianity) have been studied in [7]. In practice, it is obtained by the associated recursive algorithm which converges whatever the initialization (see for details [6]).

3 Wishart and SIRV classifications

3.1 Wishart and SIRV distances

Under Gaussian assumption, from the LRT (Eq. (4)), Lee *et al.* [8] derived a distance measure, called the Wishart distance and defined by

$$d_W = \ln\frac{|\mathbf{T}_\omega|}{|\widehat{\mathbf{T}}_{SCM}|} + \mathrm{Tr}\left(\mathbf{T}_\omega^{-1}\widehat{\mathbf{T}}_{SCM}\right) \tag{7}$$

On the other hand, for the SIRV model, Vasile *et al.* [9] derived the SIRV distance based on the FP estimate (Eq. (6)):

$$d_S = \ln\frac{|\mathbf{T}_\omega|}{|\widehat{\mathbf{T}}_{FP}|} + \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{k}_i^H\,\mathbf{T}_\omega^{-1}\,\mathbf{k}_i}{\mathbf{k}_i^H\,\widehat{\mathbf{T}}_{FP}^{-1}\,\mathbf{k}_i} \tag{8}$$

In [4], Lee *et al.* developed an algorithm, called the Wishart Classifier, to classify pixels using the Wishart distance of Eq. (7). This is described below:
1) Classify the image into 8 zones according to the H/α decomposition.
2) For each class, compute the class center as the mean of all matrices which belong to this class.
3) For each pixel, compute the distance between its covariance matrix and every class center and classify the pixel in the class whose center minimizes the distance measure.
4) Repeat steps 2-3 until a given stopping criterion is achieved.

3.2 Applications on experimental data

Experimental data were acquired in X-band by the ONERA RAMSES system in the area of Brétigny, France, with a spatial resolution of approximatively 1.5 meter in range and azimuth, and a mean incidence angle of 30°.

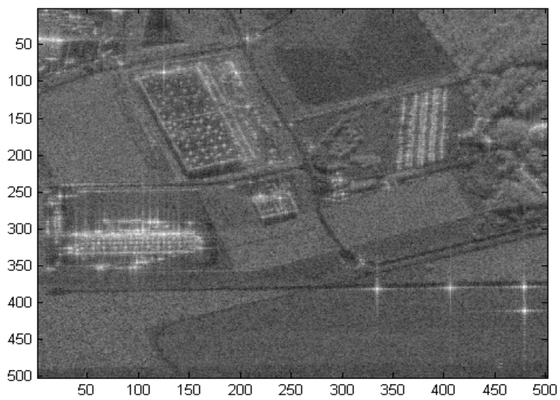

Figure 1: Polarimetric Span Image

Fig.1 represents the span of the backscattering signal. The polarimetric diversity on this image is high. This means that one can observe several buildings, different kinds of fields, forest areas and a parking lot.

Fig.2 shows the result of the Wishart Classifier, built with the SCM estimate, the Wishart distance d_W and a H/α initialization. Notice that Gaussian-distributed areas are clearly identified (yellow and blue areas at the bottom of the picture).

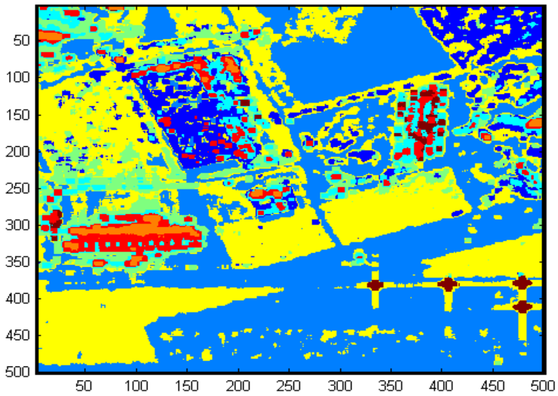

Figure 2: Wishart Classifier with SCM, Wishart distance d_W and H/α initialization

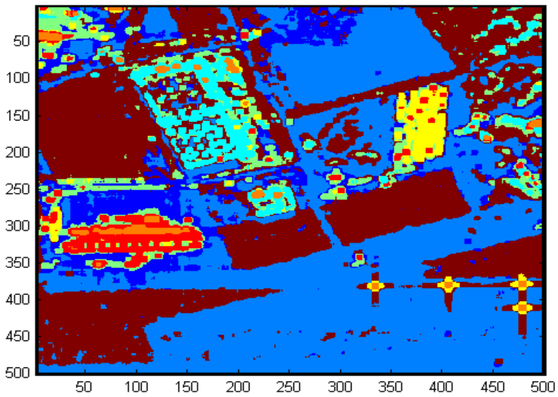

Figure 3: Wishart Classifier with SCM, Wishart distance d_W and random initialization

Moreover, **Fig.3** presents the same classification as on **Fig.2** but for a random initialization of the algorithm. This original result shows the robustness of the Wishart classification to the initialization scheme. However, in non-Gaussian areas, like buildings, the parking lot and even the forest areas, the classification scheme is degraded and it does not separate each mechanism, see [9] for more details. The Wishart classification results are totally corrupted by the presence of the texture in the SAR image. This span dependency can be eliminated by using the FP estimate (Eq. (6)) with the SIRV distance (Eq. (8)). This is the purpose of **Fig.4**.

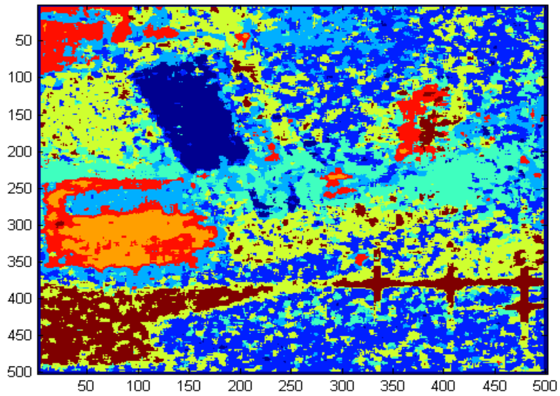

Figure 4: Wishart Classifier with FP, SIRV distance d_S and H/α initialization

Fig.4 provides a better classification of the non-Gaussian areas (the building in dark blue, building, parking and urban area in orange/red) while the classification of Gaussian parts still remains satisfactory.

4 M-Box test

In previous section, the number of classes has been fixed to 8 according to the H/α decomposition. We now propose an original approach, by testing the equality of two covariance matrices associated to two different pixels, which does not require a predefined number of classes. This is one of the main contribution of this paper.

The procedure is to test if the covariance matrices of two different pixels populations are equal. For that purpose, we consider two pixels, $\mathbf{k}^{(1)}$ and $\mathbf{k}^{(2)}$ of covariance matrices $\mathbf{T}_1$ and $\mathbf{T}_2$, randomly chosen. The resulting hypothesis test is thus:

$$\begin{cases} H_0 : \mathbf{T}_1 = \mathbf{T}_2 \\ H_1 : \mathbf{T}_1 \neq \mathbf{T}_2 \end{cases}$$

In opposite to the test defined by Eq. (3), both matrices $\mathbf{T}_1$ and $\mathbf{T}_2$ are unknown and an estimate is required. The test statistic t is a modification of Bartlett's criterion [10] and is defined as follows:

$$t = \frac{|\widehat{\mathbf{T}}_1|^{\frac{\nu_1}{2}} |\widehat{\mathbf{T}}_2|^{\frac{\nu_2}{2}}}{|\widehat{\mathbf{T}}_t|^{\nu_t}}$$

where ν_i is the degree of freedom of $\widehat{\mathbf{T}}_i$. For the SCM, $\nu_1 = \nu_2 = N$, $\nu_t = \nu_1 + \nu_2$ and $\widehat{\mathbf{T}}_t$ is the pooled sample covariance matrix defined by:

$$\widehat{\mathbf{T}}_t = \frac{\nu_1 \widehat{\mathbf{T}}_1 + \nu_2 \widehat{\mathbf{T}}_2}{\nu_1 + \nu_2}$$

Box [11] proposes the following approximation for the distribution of t:

$$u = -2(1 - c_1)\ln(t) \sim \chi^2 \left(\frac{1}{2} m(m+1) \right) \quad (9)$$

where $c_1 = N\left(\frac{2m^2+3m-1}{12\,(m+1)}\right)$ and $\chi^2(a)$ denotes the χ^2 distribution with a degrees of freedom. This approximation is valid for Wishart-distributed matrices. Pascal *et al.* proved that the FP estimate is assymptotically Wishart-distributed with $\nu = (m/m+1)N$ degrees of freedom in [7]. Therefore the χ^2 approximation holds for the FP estimate.

Now, the classification algorithm associated to the M-Box test is introduced.
1) Select a random pixel $\mathbf{k}^{(\mathbf{i})}$, which will be the class C_i center.
2) For each other pixel $\mathbf{k}$, compute u defined in Eq. (9).
3) If $\mathbf{k}$ has not been classified yet, compare u to a threshold (which is the χ^2 quantile for a given false-alarm probability). If u is below the threshold, $\mathbf{T}_1$ and $\mathbf{T}_2$ are considered equal and $\mathbf{k}$ is classified in C_i else the pixel is not classified and the algorithm loops back.
4) If $\mathbf{k}$ has already been classified, u is compared to the value used for the previous classification. If the new value is smaller, switch $\mathbf{k}$ from its old class to C_i else the pixel stay in its class.

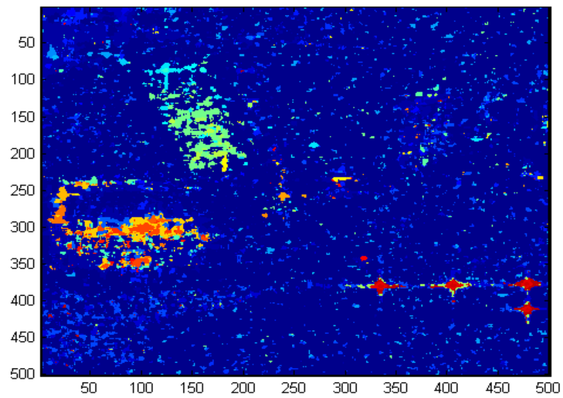

Figure 5: Results of the Box M-Test algorithm with FP

Since the initialization process has no influence on the Wishart classifier, we apply the M-Box test algorithm with a random initialization and with the FP estimate. Furthermore, since the number of classes is unknown, it has no sense to define some class centers. **Fig.5** presents the results. These preliminary results show that polarimetric phenomena tend to be classified in the same class for random areas like the fields or the forest (blue parts) while man-made objects seem to be classified in different classes (green, orange and red).

5 Conclusion

The paper presents a new classification algorithm for High Resolution POLSAR images. Polarimetric data are assumed to be non-homogeneous, which motivates the use of a SIRV model. Since the Wishart classification suffers from several drawbacks, we first investigate the SIRV classification. Then, the main contribution of this paper is to develop a new classification approach based on the M-Box test. Preliminary results are encouraging especially in the physical interpretation of the polarimetric phenomena. However, the algorithm needs to be improved, particularly, its robustness against reference pixels.

References

[1] Yao, Kung: *A representation theorem and its applications to spherically-invariant random processes*, IEEE Trans.-IT, Vol. 19, No. 5, 1973, pp 600-608.

[2] Billingsley, J.B.: *Ground Clutter Measurements for Surface-Sited Radar*, Technical Report 780, MIT, Feb. 1993.

[3] Cloude, S.R.; Pottier, E.: *An entropy-based classification scheme for land applications of polarimetric SAR*, IEEE Trans.-GRS, Vol. 35, No. 1, January 1997.

[4] Lee, J.S.; Grunes, M.R.; Ainsworth, T.L.; Du, L.J.; Schuler, D.L.; Cloude, S.R.: *Unsupervised classification using polarimetric decomposition and the complex Wishart classifier*, IEEE Trans.-GRS, Vol. 37, No. 5, Sept. 1999, pp. 2249-2258.

[5] Gini, F.; Greco, M.V.: *Covariance matrix estimation for CFAR detection in heavily correlated heavy tailed clutter*, Signal Processing, Vol.82, No.12, 2002, pp. 1847-1859.

[6] Pascal, F.; Chitour, Y.; Ovarlez, J.-P.; Forster, P.; Larzabal, P.: *Covariance Structure Maximum Likelihood Estimates in Compound Gaussian Noise: Existence and Algorithm Analysis*, IEEE Trans.-SP, Vol. 56, No. 1, pp. 34-38, January 2008.

[7] Pascal, F.; Forster, P.; Ovarlez, J.-P.; Larzabal, P.: *Performance Analysis of Covariance Matrix Estimate in Impulsive Noise*, IEEE Trans.-SP, Vol. 56, No. 6, pp. 2206-2217, June 2008.

[8] Lee, J.S.; Grunes, M.R.; Kwok, R.: *Classification of Multi-Look Polarimetric SAR imagery based on comlpex Wishart distribution*, International Journal of Remote Sensing, Vol. 15, No. 11, 1994, pp. 2299-2311.

[9] Vasile, G.; Ovarlez, J.-P.; Pascal, F.; Tison, C.: *Coherency matrix estimation of heterogeneous clutter in high resolution polarimetric SAR images*, To appear in IEEE Trans.-GRS

[10] Rencher, A. C.: *Methods of Multivariate Analysis*, John Wiley and Sons, 2002.

[11] Box, G.E.P.: *A General Distribution Theory for a Class of Likelihood Criteria*, Biometrika, Vol. 36, 1949, pp. 317-146.