H/α UNSUPERVISED CLASSIFICATION FOR HIGHLY TEXTURED POLINSAR IMAGES USING INFORMATION GEOMETRY OF COVARIANCE MATRICES

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ABSTRACT

We discuss in the paper the use of the Riemannian mean given by the differential geometric tools. This geometric mean is used in this paper for computing the class centers in the polarimetric H/α unsupervised classification process. We show that the class centers remain more stable during the iteration process, leading to a different interpretation of the H/α/A classification. This technique can be applied both on classical Sample Covariance Matrix and on Fixed Point covariance matrices. Used jointly with the Fixed Point covariance matrix estimate, this technique can give more robust results when dealing with high resolution and highly textured polarimetric SAR images classification.

***Index Terms*—** SAR, Polarimetry, Classification, Estimation, Differential Geometry.

1. INTRODUCTION

The recently launched POLSAR systems are now capable of producing high quality polarimetric SAR images of the Earth surface under meter resolution. The additional polarimetric information allows the discrimination of different scattering mechanisms. Cloude and Pottier introduced in [1] the entropy-alpha-anisotropy (H/α/A) classification based on the eigenvalues of the polarimetric (or coherency) Covariance Matrix (CM). This CM is usually estimated, under homogeneous and Gaussian assumptions, with the well known Sample Covariance Matrix (SCM) which is Wishart distributed. Based on this decomposition, the unsupervised classification of the SAR images can be performed by an iterative algorithm based on complex Wishart density function. It uses the H/α decomposition results to get an initial segmentation into eight clusters (or more), then the K-mean clustering is implemented by considering the polarimetric CM as the polarimetric descriptor. This technique needs however to derive the average CM of each class by a classical Euclidian mean operation and to compute by Wishart distance the mininal distance between each pixel CM and with all the class centers.

The decrease of the resolution cell offers the opportunity to observe much thinner spatial features than the decametric resolution of the up-to-now available SAR images but also leads to more complicated effects like spatial heterogeneity, non-Gaussianity. Hence, some areas (grass, trees, ...) usually considered as random backscattering mechanisms can become pointwise deterministic backscattering mechanisms. The usual techniques of classification, detection, speckle filtering, used for decametric resolution have to be adapted to these new challenging problems. For high resolution SAR images, recent studies have shown that the spatial heterogeneity of the observed scene leads to non-Gaussian clutter modeling. Some techniques have been recently proposed to handle such problems. One commonly used fully polarimetric non-Gaussian clutter model is the compound Gaussian model: the spatial heterogeneity of the SAR image intensity is taken into account by modeling the polarimetric clutter information m-vector $\mathbf{k}$ (in the Pauli Basis, $m = 3$) as a SIRV (Spherically Invariant Random Vector), i.e. the product between the square root of a scalar random variable τ (called the texture) and an independent, m-dimensional, zero-mean and complex circular Gaussian random vector $\mathbf{z}$ (called the polarimetric speckle) with Covariance Matrix $\mathbf{M}$:

$$\mathbf{k} = \sqrt{\tau}\,\mathbf{z}\,. \tag{1}$$

In this model, the variable τ can represent the spatial variation of the intensity of the wave vector $\mathbf{k}$ from pixel to pixel. All the polarimetric information (phase relationships within the wave vector) is so contained only in the normalized covariance matrix $\mathbf{M}$ ($\mathrm{Tr}(\mathbf{M}) = m$ where $\mathrm{Tr}(.)$ is the trace operator). Conditionally to a given pixel (equivalently to a given τ), the wave vector is then fully Gaussian distributed.

The aim of this paper is twofold. Firstly, we propose in this paper to briefly recall original results, recently obtained in [2] for the joint Maximum Likelihood estimation of the texture and the polarimetric CM. These results, based on the Fixed Point estimate, allowed to derive a new distance for SIRV CM and allowed to propose a new technique of speckle filtering in heterogeneous environment. Secondly, we introduce a metric-based mean for the space of positive-definite Hermitian covariance matrices. An emerging theory [3, 4, 5] allows to take into account the fact that Euclidian space does not describe the space of positive-definite Hermitian CM.

2. ESTIMATION OF THE COHERENCY COVARIANCE MATRIX

2.1. Gaussian homogeneous case

In homogeneous and Gaussian clutter assumption, the texture τ is assumed to be constant and to be the same for all the pixels. In that case, the statistic of the secondary data is Gaussian and the SCM can be estimated by the Maximum Likelihood (ML) theory with a set of N secondary data $\mathbf{k}_n$, $n \in [1, N]$ as:

$$\widehat{M}_{SCM} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{k}_n \, \mathbf{k}_n^H \, . \tag{2}$$

This unbiased CM estimate is Hermitian, Wishart-distributed with N degrees of freedom and is therefore widely used in the SAR community. Another well known POLSAR parameter maximizing the mean-to-variance ratio is the Polarimetric Whitening Filter (PWF) as:

$$\text{PWF}^n_{\text{SCM}} = \mathbf{k}_n^\dagger \, \widehat{M}_{SCM}^{-1} \, \mathbf{k}_n \, . \tag{3}$$

For supervised and unsupervised POLSAR data clustering in Gaussian case, a LR distance D_W between a given pixel i characterized by its SCM $\widehat{M}^i$ and a center of class characterized by the SCM $\widehat{M}_\omega$ has been derived [6], [7] :

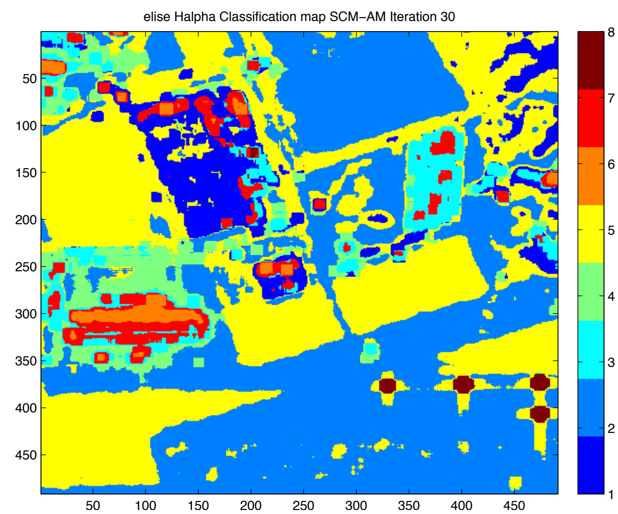
$$D_W\left(\widehat{M}_i, \widehat{M}_\omega\right) = \ln \frac{|\widehat{M}_\omega|}{|\widehat{M}_i|} + \text{Tr}\left(\widehat{M}_\omega^{-1}\, \widehat{M}_i\right) \, . \tag{4}$$

When dealing with heterogeneous and/or non-Gaussian clutter, this SCM is no longer robust. Under SIRV assumption, the SCM takes the form:

$$\widehat{M}_{SCM} = \frac{1}{N} \sum_{n=1}^{N} \tau_n \, \mathbf{z}_n \, \mathbf{z}_n^H \, . \tag{5}$$

The presence of all the textures τ_n of the N secondary wave vectors k_n makes this estimate really polluted. The figure 1 presents the set of data used for the discussion (polarimetric SAR images of ONERA RAMSES system in Bretigny in France) and the conventional H/α classification using SCM. We can see clearly that the conventional classification is very connected to the the intensity variation of the RGB Pauli representation. It can be explained by the fact that the SCM (and so, the Wishart distance, the class centers) is really dependent of the intensity (or texture) present in the reference cells. The figure 2 shows, after 30 iterations, the locations of all the pixels in the H-α plan (for height classes). The location of the pixel belonging to a given class seems very unordered.

(a) ONERA RAMSES polarimetric SAR Image in RGB Pauli basis



(b) Conventional H/α classification after 30 iterations

Fig. 1. Comparison between Pauli representation and conventional unsupervised H/α classification after 10 iterations.

2.2. Non-Gaussian and Heterogeneous Case

In the SIRV model, the covariance matrix is generally an unknown parameter which can be estimated from ML theory. In [8], Gini et al. derived the ML estimate $\widehat{\mathbf{M}}_{FP}$ of the covariance matrix $\mathbf{M}$ for deterministic texture, which is the solution of the following equation:

$$\widehat{\mathbf{M}}_{FP} = f(\mathbf{M}) = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \, \mathbf{k}_i^H}{\mathbf{k}_i^H \, \widehat{\mathbf{M}}_{FP}^{-1} \, \mathbf{k}_i} \, . \tag{6}$$

This approach has been used in [9] by Conte et al. to derive a recursive algorithm for estimating the solution matrix $\mathbf{M}_{FP}$ called the Fixed Point Covariance matrix. This algorithm consists in computing the Fixed Point of f using the sequence defined by $\mathbf{M}_{i+1} = f(\mathbf{M}_i)$ and $\mathbf{M}_0 = \mathbf{I}$. It has

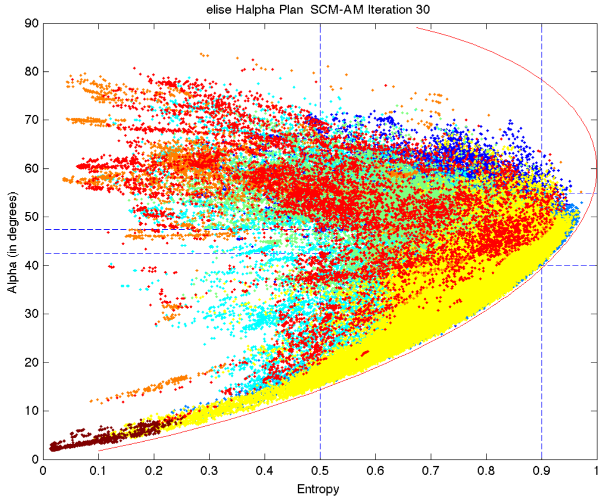


Fig. 2. Pixel locations in the H/α plan after 30 iterations in the Gaussian case.

been shown in [8] and [9] that the estimation scheme from (6), developed under the deterministic texture case, yields also an Approximate ML (AML) estimator under stochastic texture hypothesis. This study has been completed by the work of Pascal et al. [10], which recently established the existence and the uniqueness of the Fixed Point estimator of the normalized covariance matrix, as well as the convergence of the recursive algorithm whatever the initialization. This matrix estimate is shown to be robust, unbiased, consistent and asymptotically distributed as Gaussian probability density function [11]. One could suppose legitimately (not yet proven) that it has the same behavior as a Wishart distributed matrix with $m\,N/(m+1)$ degrees of freedom.

The generalized ML estimator for the τ_n texture for the primary data $\mathbf{k}_n$ is given by:

$$\widehat{\tau}_n = \frac{\mathbf{k}_n^\dagger\, \widehat{M}_{FP}^{-1}\, \mathbf{k}_n}{m}\,. \tag{7}$$

One can note that this later equation is proportional to the SIRV Polarimetric Whitening Filter.

$$\mathrm{PWF}_{\mathrm{SIRV}} = \mathbf{k}_n^\dagger\, \widehat{M}_{FP}^{-1}\, \mathbf{k}_n\,. \tag{8}$$

The role of the matrix $\widehat{M}_{FP}$ is here to whiten the possible correlation existing within the polarimetric channels without being polluted by the power of the secondary data used to estimate this matrix. A new ML SIRV distance D_S between two FP $\widehat{M}_i$ and $\widehat{M}_\omega$ has also been derived in [2]:

$$D_S\left(\widehat{M}_i, \widehat{M}_\omega\right) = \ln \frac{|\widehat{M}_\omega|}{|\widehat{M}_i|} + \frac{m}{N} \sum_{n=1}^{N} \frac{\mathbf{k}_n^\dagger\, \widehat{M}_\omega^{-1}\, \mathbf{k}_n}{\mathbf{k}_n^\dagger\, \widehat{M}_i^{-1}\, \mathbf{k}_n}\,. \tag{9}$$

This later equation depends on the secondary data $(\mathbf{k}_n)_{n=1,N}$ but can nicely be simplified as:

$$D_S\left(\widehat{M}_i, \widehat{M}_\omega\right) = D_W\left(\widehat{M}_i, \widehat{M}_\omega\right)\,. \tag{10}$$

This result shows that, again, the FP CM could be considered as a Wishart CM.

The figure 3 shows, after 30 iterations, the locations of all the pixels in the H-α plan (for height classes). The location of the pixel belonging to a given class seems much better ordered.

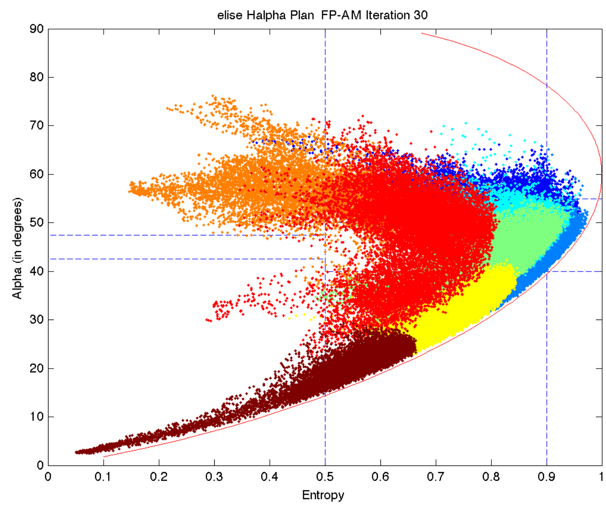


Fig. 3. Pixel locations in the H/α plan after 30 iterations in the SIRV case for the classical mean.

3. RIEMANNIAN DISTANCE

Rigorously, the averaged covariance matrix $\mathbf{M}_{\omega_l}$ (SCM or Fixed Point) of a H/α/A cluster l can not be computed with the Euclidean metric, i.e. usual arithmetic mean as:

$$\mathbf{M}_{\omega_l} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{M}_k^l\,, \tag{11}$$

where $\mathbf{M}_k^l$, $k \in [1, K]$ are the K covariances matrices of all pixels belonging to the class ω_l in the H/α plane. It is well known that after few iterations of the unsupervised classification, all the class centers move significantly within the H/α plane leading a more difficult physical interpretation to the final classification. The mean associated with the Riemannian metric corresponds to the geometric mean:

$$\mathbf{M}_{\omega_l} = \underset{\mathbf{M}_\omega \in \mathcal{P}(m)}{\arg\min} \sum_{k=1}^{K} \left|\left| \log\left({\mathbf{M}_k^l}^{-1} \mathbf{M}_\omega \right) \right|\right|_F^2\,, \tag{12}$$

where $||.||_F$ stands for the Frobenius norm and $\mathcal{P}(m)$ is the set of the Hermitian definite-positive covariance matrices of size m. The solution $\mathbf{M}_{\omega_l}$ can easily be found using a simple gradient algorithm given by the iterative procedure:

$$\mathbf{M}_{k+1} = \mathbf{M}_k - \epsilon\,\mathbf{M}_k \sum_{i=1}^{K} \log\left(\mathbf{M}_i^{l^{-1}}\mathbf{M}_k\right), \qquad (13)$$

where $k \in \mathbb{N}$, where ϵ controls the speed of the gradient descent and where:

$$\mathbf{M}_0 = \mathbf{M}_1^l\left(\mathbf{M}_1^{l^{-1}}\mathbf{M}_2^l\right)^{1/2} = \mathbf{M}_2^l\left(\mathbf{M}_2^{l^{-1}}\mathbf{M}_1^l\right)^{1/2}$$

can be initialized, for example, as the geometrical mean of the two first matrices.

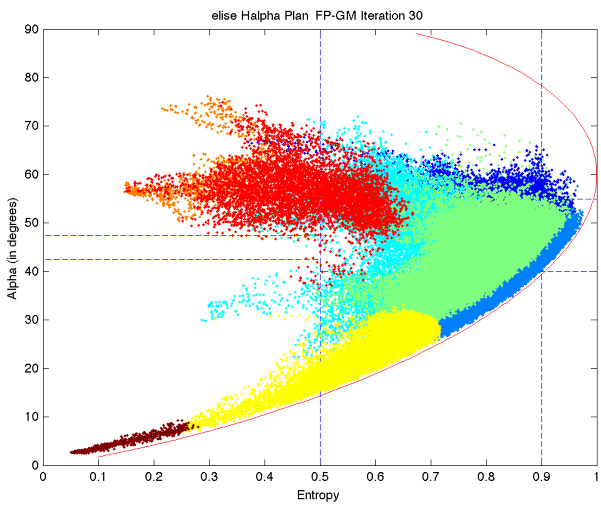
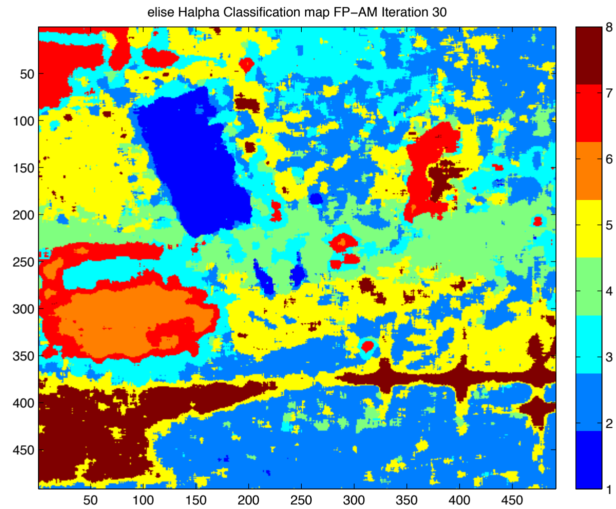


Fig. 4. Pixel locations in the H/α plan after 30 iterations in the SIRV case for the geometrical mean.

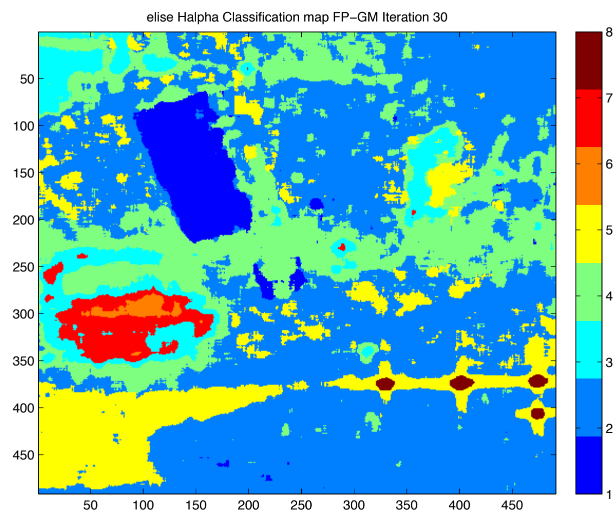
The figure 4 shows, after 30 iterations, the locations of all the pixels in the H-α plan (for height classes). Figure 5 represents a comparison between the unsupervised H/α classification results obtained with classical arithmetical mean and the Riemannian mean (after 10 iterations) with the Fixed Point matrices.

4. CONCLUSION

After recalling some recent techniques (e.g. the Fixed Point Covariance Matrix estimation) useful for modeling the heterogeneity and/or the non-Gaussian behavior of the polarimetric SAR images, this paper has presented the use of a new technique, based on the differential geometry, for computing the barycenter of K Hermitian covariances matrices. This mean is used in the H/α unsupervised process and allows, jointly with the Fixed Point estimate, to propose a new classification map. This latter which seems promising (much more stable location in the H/α plan) but needs however to be validated with ground truth.



(a) Fixed Point and arithmetical Mean



(b) Fixed Point and Geometrical Mean

Fig. 5. Comparison of unsupervised H/α classifications after 30 iterations for arithmetical and geometrical mean

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