HYPERIMAGE CONCEPT: MULTIDIMENSIONAL TIME-FREQUENCY ANALYSIS APPLIED TO SAR IMAGING

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ABSTRACT

This paper deals with the analysis of non-stationary scatterers in SAR images. Indeed, SAR imaging makes the assumptions that the scatterers are isotropic and white in the emitted frequency band. However, new SAR applications use a large bandwidth and a strong angular excursion. These assumptions become obsolete and the behavior of scatterers becomes non-stationary. The basic tool to study non-stationary signals is the time-frequency analysis. Recent studies based on multidimensional Time-Frequency Analysis describing the angular and frequency behavior of scatterers has highlighted anisotropic and dispersive behavior of bright points. This paper generalizes the hyperimage concept to study scatterers. Multidimensional Time-Frequency distributions are tested on simulations, then they are applied to very high resolution SAR images and show some scatterers are anisotropic and dispersive.

***Index Terms*—** Wavelet Transform, Radar Imaging, Target Classification

1. INTRODUCTION

The classical model used in radar imaging is the model of bright points. In radar imaging, the reflectors are assumed to have a constant behavior for the angle from which they are viewed and within the emitted frequency bandwidth. A target is considered as a set of isotropic independent sources with a constant response in the frequency band.

Nevertheless, new capacities in Radar imaging (large bandwidth and large angular excursions) makes this assumption non valid. For example, a SAR image is built using three sub-bands centered on the frequencies $f_c = 8.8$ GHz, $f_c = 9.4$ GHz and $f_c = 10$ GHz which are coded in the red/green/blue channels, (see Fig. 1). If a scatterer is white in the frequency band, so it is not colored on the image. Otherwise if the scatterer is dispersive, so it is colored. On the Very High Resolution (VHR) SAR image of Fig.1, the roofs are colored in red and blue, so the scatterers of the roofs are dispersive. The model of bright points is non valid.

To study the non-stationary behavior of scatterers the basic tool is the time frequency analysis. Bertrand and Ovarlez introduce the hyperImage concept based on the multidimensional continuous wavelet [1], [2]. So, the hyperImage concept allows to represent for each frequency and each angle of illumination, a spatial repartition of reflectors which respond at this frequency and this angle.

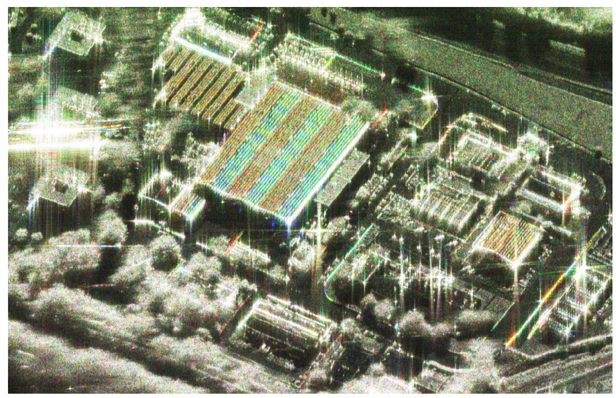

Fig. 1. A SAR image which highlights dispersive scatterers.

Inversely, for each reflector location, it is possible to analyse its behavior in the frequency and in the angular domain. Other studies use different time-frequency distributions, the short time fourier transform [3], the smoothed pseudo Wigner-Ville transform [4]. The aim of this letter is to generalize the hyperImage concept to other time-frequency distributions.

In this paper, the hyperImage formation is explained from the classical radar imaging. Then, the bidimensional Cohen class is presented. The affine class study is limited, here, to the continuous wavelets transform. The hyperImage concept is validated on simulations. All in all, the hyperImage principle is applied on VHR SAR data to show some scatterers are anisotropic and dispersive.

2. CLASSICAL RADAR IMAGING

The backscattering coefficient $H(\mathbf{k})$ for a given object illuminated by a radar is characterized, for a distance R going to infinity, as the ratio between the incoming field E_r and the emitted field E_i (spherical waves):

$$|H(\mathbf{k})| = \lim_{R\to\infty} \sqrt{4\pi R^2}\,\frac{E_r}{E_i}. \tag{1}$$

The squared modulus of $H(\mathbf{k})$ is called the Radar Cross Section (RCS) of the object for the wave vector $\mathbf{k}$ and is expressed in squared meter. Wave vector $\mathbf{k}$ is related to the frequency f and to the direction θ of illumination by $|\mathbf{k}| = k = 2f/c$ and $\theta = \arg(\mathbf{k})$

in two-dimensional approximation.

The model usually used in radar imaging is the model of bright points [5]. The object under analysis can be seen as a set of bright points, i.e. a set of independent sources which reflect in the same way for all frequencies (white points) and all directions of presentation (isotropic points). Let $S(\mathbf{r})$ be the complex amplitude of the bright point response located at $\mathbf{r} = (x, y)^T$ in a set of cartesian axes related to the object. Under far field conditions (decomposition into planes waves), the complex backscattering coefficient for the whole object is then given by the in-phase summation of each reflector contribution:

$$H(\mathbf{k}) = \int S(\mathbf{r})\, e^{-2i\pi\mathbf{k}.\mathbf{r}}\, d\mathbf{r}. \tag{2}$$

After a Fourier Transform of (2), one can obtain the spatial complex amplitude repartition $S(\mathbf{r})$ of the reflectors for a mean frequency (the center frequency) and for a mean angle of presentation:

$$S(\mathbf{r}) = \int H(\mathbf{k})\, e^{2i\pi\mathbf{k}.\mathbf{r}}\, d\mathbf{k}. \tag{3}$$

When a target is illuminated by a broad-band signal and/or for a large angular extent, it is realistic to consider that the amplitude spatial repartition $S(\mathbf{r})$ of the reflectors depends on frequency f and on aspect angle θ. This repartition depending on the wave vector $\mathbf{k}$, it will be noted in the following by $S(\mathbf{r}, \mathbf{k})$.

3. BIDIMENSIONAL AFFINE CLASS

3.1. Construction of the hyperImage based on the continuous wavelet

Let $\phi(\mathbf{k})$ be a mother wavelet supposed to represent the signal reflected by a reference target. This target is supposed located around $\mathbf{r} = \vec{0}$ and backscatters the energy in the direction $\theta = 0$ and at the frequency f given by $k = \frac{2f}{c} = 1$. A family of function is built $\Psi_{\mathbf{r_0},\mathbf{k_0}}$ from $\phi(\mathbf{k})$ by the similarity group S [1], [2]:

$$\Psi_{\mathbf{r}_o,\mathbf{k}_o}(\mathbf{k}) = \frac{1}{k_o}\, e^{-j2\pi\mathbf{k}.\mathbf{r}_o}\, \phi\left(\frac{k}{k_o}, \theta - \theta_o\right). \tag{4}$$

The wavelet coefficient $C_H(\mathbf{r}_o, \mathbf{k}_o)$ is defined as the scalar product $C_H(\mathbf{r}_o, \mathbf{k}_o) =< H, \Psi_{\mathbf{r}_o,\mathbf{k}_o} >$ between the complex backscattering coefficient H and the wavelet $\Psi_{\mathrm{r_o,k_o}}$. It is defined as following [8]:

$$C_H(\mathbf{r}_o, \mathbf{k}_o) = \int_0^{2\pi} d\theta \int_0^{+\infty} k\, H(k, \theta)\, \frac{1}{k_o} \\ e^{+j2\pi\mathbf{k}.\mathbf{r}_o}\phi^*\left(\frac{k}{k_o}, \theta - \theta_o\right)\, dk \tag{5}$$

So we define in the following the hyperImage $S_H(\mathbf{r}, \mathbf{k})$ as the wavelet coefficients $C_H(\mathbf{r}, \mathbf{k})$.

3.2. Properties

The continuous wavelet transform has two interesting properties. The first is the reconstruction. It is possible to build the complex backscattering coefficient $H(\mathbf{k})$ from the wavelet coefficient $C_H(\mathbf{r}_o, \mathbf{k}_o)$:

$$H(\mathbf{k}) = \frac{1}{K_\phi} \int_S d\mathbf{r}_o \int C_H(\mathbf{r}_o, \mathbf{k}_o)\, \Psi_{\mathbf{r}_o,\mathbf{k}_o}(\mathbf{k})\, d\mathbf{k}_o \tag{6}$$

where K_ϕ is defined as the *admissibility coefficient* of the mother wavelet and has, to build $H(\mathbf{k})$ from the wavelet coefficients, to check:

$$K_\phi = \int |\phi(\mathbf{k})|^2\ \frac{d\mathbf{k}}{k^2} < +\infty \tag{7}$$

The second property is the isometry:

$$\frac{1}{K_\phi} \int_S d\mathbf{r}_o \int |C_H(\mathbf{r}_o, \mathbf{k}_o)|^2\ d\mathbf{k}_o = \|H\|^2 \tag{8}$$

3.3. Limitations

The continuous wavelet is limited by the Heisenberg principle. Indeed, this concept tells that we cannot obtain a spatial good resolution with a good resolution in the frequency domain and reciprocally. However, the continuous wavelet offers a resolution which changes with the frequency and the spatial domain. It allows multiresolution analysis [6].

4. BIDIMENSIONAL COHEN CLASS

Cohen has shown that a number of time-frequency bilinear distributions which respect the covariance diagram in translation, could be written in a generalized form. It is the Cohen class.

4.1. Short time Fourier transform

4.1.1. Definition

By applying the classical bidimensional short time Fourier transform on the SAR image, $S(\mathbf{r})$, the following hyperImage can be defined:

$$F_S(\mathbf{r_0}, \mathbf{k_0}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S(\mathbf{r})\, w^*(\mathbf{r} - \mathbf{r_0})\, e^{-2i\pi\mathbf{k_0}\cdot\mathbf{r}}\, \mathbf{dr} \tag{9}$$

where w is a window function. This transform allows to define the associated spectrogram (in intensity):

$$I(\mathbf{r_0}, \mathbf{k_0}) = |F_S(\mathbf{r_0}, \mathbf{k_0})|^2 \tag{10}$$

Equivalently, by applying the classical bidimensional short time Fourier transform on the backscattering complex coefficient $H(\mathbf{k})$, the same hyperImage can be retrieved:

$$F_H(\mathbf{r_0}, \mathbf{k_0}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H(\mathbf{k})\, W^*(\mathbf{k} - \mathbf{k_0})\, e^{2i\pi\mathbf{k}\cdot\mathbf{r_0}}\, \mathbf{dk} \tag{11}$$

where W is the Fourier transform of w. This transform allows to define the associated spectrogram (in intensity):

$$I(\mathbf{r_0}, \mathbf{k_0}) = |F_H(\mathbf{r_0}, \mathbf{k_0})|^2 \tag{12}$$

4.1.2. Properties

The spectrogram satisfies the energy conservation but not the marginal properties.

4.1.3. Limitations

The resolution in spatial domain is limited by the window $w(\mathbf{r})$. Similarly, the resolution in the frequency and angle domains is limited by the width of the frequency window $W(\mathbf{k})$. The window width in spatial and the window width in frequency are inversely proportional to each other by the Heisenberg's inequality. Therefore, good resolution in spatial domain (small window w) necessarily implies poor resolution in frequency and angle (large frequency and angle window). Conversely, good resolution in frequency and angle implies poor resolution in spatial [6]. It is the Heisenberg principle. The gaussian window achieves the best resolution compromise among all the possible window function.

4.2. Wigner-Ville Distribution

4.2.1. Definition

By applying the classical bidimensional Wigner-Ville distribution on the SAR image $S(\mathbf{r})$, the following hyperImage is defined:

$$W_S(\mathbf{r_0}, \mathbf{k_0}) = \int_{-\infty}^{+\infty} S\left(\mathbf{r_0} + \frac{\mathbf{r}}{2}\right) S^*\left(\mathbf{r_0} - \frac{\mathbf{r}}{2}\right) e^{-2i\pi\mathbf{k_0}\cdot\mathbf{r}}\, \mathbf{dr} \tag{13}$$

Equivalently, by applying the classical bidimensional Wigner-Ville distribution on the backscattering complex coefficient $H(\mathbf{k})$, the same hyperImage can be retrieved:

$$W_H(\mathbf{r_0}, \mathbf{k_0}) = \int_{-\infty}^{+\infty} H\left(\mathbf{k_0} + \frac{\mathbf{k}}{2}\right) H^*\left(\mathbf{k_0} - \frac{\mathbf{k}}{2}\right) e^{2i\pi\mathbf{k}\cdot\mathbf{r_0}}\, \mathbf{dk} \tag{14}$$

4.2.2. Properties

The Wigner-Ville distribution is real but non-positive in all the time-frequency plane. It is a pseudo distribution of energy. The Wigner-Ville distribution has a number of desirable properties. It satisfies the marginal conditions (in spatial and frequency). It satisfies the energy conservation. It checks the instantaneous frequency property.

4.2.3. Limitations

Although, the Wigner-Ville distribution has many nice properties, and gives nearly the best resolution among all the time-frequency techniques, its main drawback comes from cross-term interferences [6]. Indeed, the Wigner-Ville transform of the sum of two signals is not the sum of the Wigner-Ville distributions. Let the backscattering complex coefficient H received by the radar and it is the sum of coefficients H_1 et H_2 backscattered by two reflectors: $H(\mathbf{k}) = H_1(\mathbf{k}) + H_2(\mathbf{k})$. So, the Wigner-Ville distribution of this signal is explained by:

$$\begin{aligned} W_H(\mathbf{r}, \mathbf{k}) = W_{H_1}(\mathbf{r}, \mathbf{k}) + W_{H_2}(\mathbf{r}, \mathbf{k}) \\ +2\mathcal{R}_e\left[\int_{\mathbb{R}^2} H_1\left(\mathbf{k} + \frac{\xi}{2}\right) H_2^*\left(\mathbf{k} - \frac{\xi}{2}\right) e^{j2\pi\xi\mathbf{r}}\, d\xi\right] \end{aligned} \tag{15}$$

So, two scatterers create cross-term interferences. The solution is to filter the Wigner-Ville transform to suppress cross-term interferences.

5. INTERPRETATION OF THE HYPERIMAGES

Let us rewritten $I(\mathbf{r}, \mathbf{k}) = I(x, y, f, \theta)$: for each frequency f_0 and each angle of radar illumination θ_0, $I(x, y, f_0, \theta_0)$ represents a spatial repartition of reflectors which respond at this frequency and this angle. Inversely, for each reflector located at $\mathbf{r}_0 = (x_0, y_0)$, we can extract its feature $I(x_0, y_0, f, \theta)$ in frequency f and in angular θ. This is this aspect that we decided to point out in order to see if this quantity can be interpretable in terms of target characteristics. To analyse this 4D structure, a visual display interface called i4D has been developed and allows to carry out an interactive and dynamic analysis [9].

6. APPLICATION ON VHR SAR IMAGES

The VHR image (see Fig. 2) chosen for the experiment is an helicopter. The sampling step in range is 10 cm and in azimuth 10 cm. The frequency band used to build this images is 1.4 GHz and the angular excursion is around 8 deg.

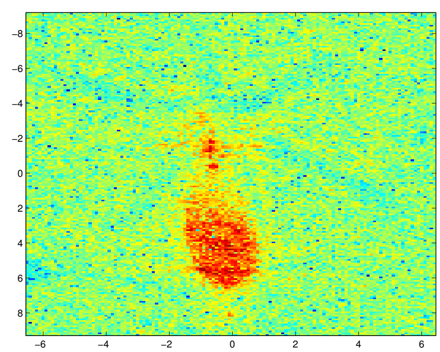

Fig. 2. VHR SAR image of an helicopter.

The hyperImage concept has been applied for the 2D spectrogram (Fig. 3), the 2D Wigner-Ville (Fig. 4) and the 2D continuous wavelet (Fig. 5). The results show some scatterers are anisotropic and dispersive.

7. CONCLUSION

Now, new SAR applications make obsolete the basic assumption of the isotropy and non-dispersive scatterer. The basic tool to study non-stationary signals is time frequency analysis. In this paper, the hyperImage concept is generalized from the affine class to the Cohen class. The hyperimages have been tested on simulation data. The results are that anisotropy and dispersive behaviors are found again according to the time-frequency distributions properties. The application on VHR image shows some scatterers are anisotropic and dispersive. Future work relates to the extraction of physical target attributes and to the use of hyperImage concept for imaging and detection of targets in SAR.

8. REFERENCES

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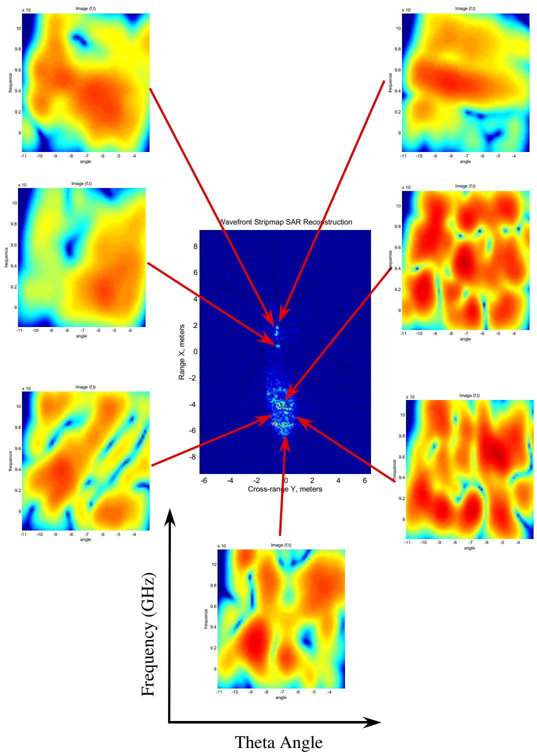

Fig. 3. Results of the 2D Spectrogram applied to SAR image.

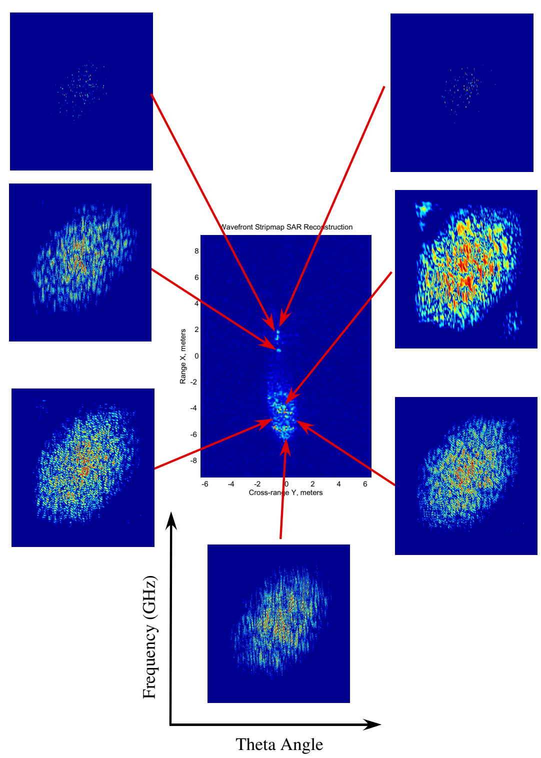

Fig. 4. Results of the 2D Wigner-Ville applied to SAR image.

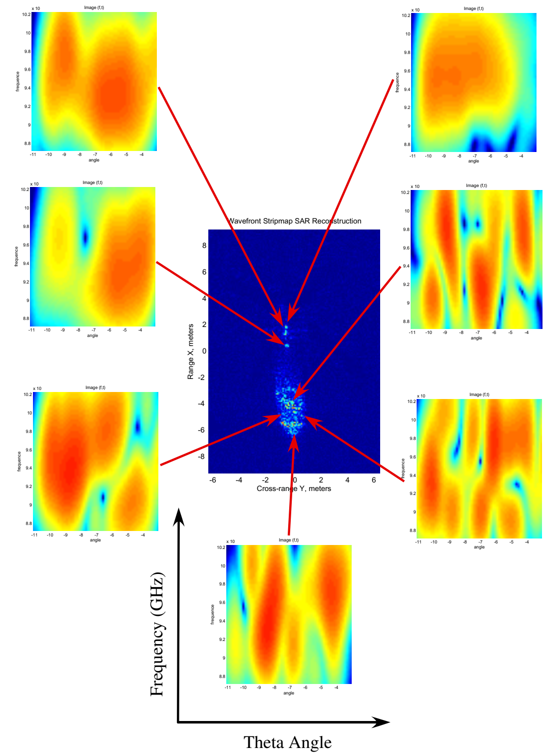

Fig. 5. Results of the continuous wavelet applied to SAR image.

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