NEW METHODS OF RADAR DETECTION PERFORMANCES ANALYSIS

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ABSTRACT

Original methods of radar detection performances analysis are derived for a fluctuating or non-fluctuating target embedded in additive and *a priori* unknown noise. This kind of noise can be, for example, the sea or ground clutter encountered in surface-sited radar for the detection of target illuminated at low grazing angles or in high resolution radar. For these cases, the spiky clutter tends to have a statistic which strongly differs from the gaussian assumption. Therefore, the detection theory becomes difficult to perform since the nature of statistics has to be known. The new methods proposed here are based on the parametric modelisation of the moment generating function of the noise envelope by Padé approximation and lead to a powerful estimation of its probability density function. They allow to evaluate the radar detection performances of target embedded in any noise without knowledge of the closed form of its statistic and allow in the same way to take into account any possible fluctuation of the target. These methods have been tested successfully on synthetic signals and have been performed on experimental signals such as ground clutter.

1. DESCRIPTION OF THE PROBLEM

The radar detection of a target against a background of unwanted clutter due to echoes from the sea or land is a problem of interest in the radar field. For many years, the statistic of quadrature components of the radar clutter was supposed to be jointly gaussian because of the low radar resolution capabilities : in this case, the clutter was viewed as a sum of responses from a very large number of elementary scatterers (Central Limit Theorem). The current systems have now improved their resolution capabilities and hence their performances of detection. However, as resolution has increased, the statistic of the additive noise have no longer been observed to be gaussian. Recent experimentations conducted in ONERA indicate that large deviation from Rayleigh statistics are observed in situations such as low grazing angle illumination or with high resolution radars. In such cases, due to the spiky nature of the clutter, the empirical distribution exhibits both higher tails and larger standard deviation to mean than predicted by the Rayleigh distribution. Therefore, many works have been devoted to fit empirical models of distribution to experimental data. This is the case of the compound gaussian processes [1, 2], also called Spherically Invariant Random Processes (SIRP) which allow to modelise the multivariate probability density function (PDF) of the envelope of the clutter returns, taking into account the possible spatial or temporal correlation of the processes. The well known log-normal, Weibull and K-distribution densities [3] belong to this class of distributions but the main problems for this kind of parametrization are the quality of the estimation of the SIRP parameters and the complexity of the optimal detector implementation. We propose in this paper to analyze the performances of radar detection of a target embedded in any combination of clutter and thermal noise without the knowledge of the closed form of the densities of the noises. The estimation of the noise envelope density is only performed according to the modelisation by Padé approximation of the moment generating function (MGF) of the noise envelope. This method is based on the estimation of all the n-order moments of the noise envelope. The goal of this paper being not to derive a method of evaluating the best estimation of the moments, they will be supposed exactly estimated. This kind of modelisation allows to derive, for a constant false alarm rate, the simple form of the probability of detection of a target with constant or fluctuating envelope embedded in a complex noise fully characterized by the moments of its envelope.

2. GENERAL RELATIONS OF THE DETECTION THEORY

We consider here the basic problem of detecting the presence or absence of a complex signal $s(t)$ with envelope A in a set of measurements $y(t) = y_I(t) + i\, y_J(t)$ corrupted by a sum of independent additive complex noises corresponding to the clutter echoes $c(t)$ and white gaussian thermal noise $n(t)$. This problem can be described mathematically in terms of a hypothesis test between the following pair of statistical hypothesis :

$$H_0 \quad : \quad y(t) = n(t) + c(t) \tag{1}$$

$$H_1 \quad : \quad y(t) = s(t) + n(t) + c(t) \tag{2}$$

If we note $p_{H_0}(r)$ the probability density of the noise envelope $|n(t) + c(t)|$, the detection threshold θ is fixed by the value of the given probability P_{fa} of false alarm.

$$P_{fa} = \int_{\theta}^{+\infty} p_{H_0}(r)\, dr \tag{3}$$

while, denoting $p_{H_1}(r)$ the PDF of the envelope of the complex signal embedded in noise $|s(t) + n(t) + c(t)|$, the detection probability P_d is classically given by :

$$P_d = \int_{\theta}^{+\infty} p_{H_1}(r)\, dr \tag{4}$$

Since the phases between quadrature components of the clutter, thermal noise and the target are unknown, they are commonly supposed to be uniformly distributed on $[-\pi, \pi]$. In this case, each two-dimensional density function of the quadrature component is hence a circular symmetric distribution. This implies that, with a change of variable in polar notation, each two-dimensional characteristic function becomes a function of a single radial variable ρ. This function of one variable is called the coherent radial characteristic function. Denoting $J_n(.)$, the ordinary Bessel function of order n, the characteristic function $C(\rho)$ and the density $p(r)$ are related by the following relation pair :

$$p(r) = \int_0^{+\infty} r\,\rho\, C(\rho)\, J_0(\rho\, r)\, d\rho \tag{5}$$

$$C(\rho) = \int_0^{+\infty} p(r)\, J_0(\rho\, r)\, dr \tag{6}$$

For a deterministic signal with constant envelope A characterized by its PDF $p(r) = \delta(r - A)$, the coherent radial characteristic function is given by $C(\rho) = J_0(\rho A)$. The characteristic function $C_{H_1}(\rho)$ of the sum of the signal $s(t)$ and unwanted noise $c(t) + n(t)$, is equal to the product of the characteristic function of the signal $C_s(\rho)$ and the characteristic function of the noise $C_{c+n}(\rho)$. We are now able to define a relation between the density $p_{H_0}(r)$ of the envelope under hypothesis H_0 (noise only) and the density $p_{H_1}(r/A)$ of the envelope under hypothesis H_1 (noise and signal with constant envelope A) :

$$p_{H_1}(r/A) = \int_0^{+\infty} r\,\rho\, C_s(\rho)\, C_{c+n}(\rho)\, J_0(\rho\, r)\, d\rho \tag{7}$$

$$= \int_0^{+\infty} r\,\rho\, J_0(\rho\, A)\, C_{c+n}(\rho)\, J_0(\rho\, r)\, d\rho \tag{8}$$

with

$$C_{c+n}(\rho) = \int_0^{+\infty} p_{H_0}(r)\, J_0(\rho\, r)\, dr \tag{9}$$

Replacing (9) in (8) gives the following important relation :

$$p_{H_1}(r/A) = \int_0^{+\infty}\int_0^{+\infty} r\rho\, J_0(\rho A)\, J_0(\rho r)\, J_0(\rho r')\, p_{H_0}(r')\, d\rho\, dr' \tag{10}$$

For example, this relation connects the pdf of the envelope of a complex gaussian noise (Rayleigh distribution) $p_{H_0}(r) = \dfrac{r}{\sigma^2} \exp\left(-r^2/2\sigma^2\right)$ to the pdf of the envelope of a constant signal embedded in this complex noise (Rice-Nakagami distribution) $p_{H_1}(r/A) = \dfrac{r}{\sigma^2} \exp\left(-\dfrac{A^2 + r^2}{2\sigma^2}\right) I_0\left(\dfrac{r\,A}{2\sigma^2}\right)$.

In the case of fluctuating target with density envelope fluctuation law $p(A/A_0)$ (with parameter A_0 representing the mean value of the fluctuation), the relation (10) becomes more general :

$$p_{H_1}(r/A_0) = \int_0^{+\infty} p_{H_1}(r/A)\, p(A/A_0)\, dA \tag{11}$$

The two relations (10) and (11) are quite difficult to compute numerically for evaluating the performances of detection for several signal-to-noise ratio. We propose in the next section to use an interesting method proposed and developed in [4, 5] which allows to estimate the pdf of any noise from its n-order moments and to give very useful relations to compute the pair P_d, P_{fa} for a given signal-to-noise ratio.

3. DESCRIPTION OF THE PADÉ APPROXIMATION

This method is based on the parametric construction of the Moment Generating Function (MGF) of the envelope of the noise by Padé Approximation. The MGF $\Phi(u)$ of a random process is defined by the mono-lateral Laplace transform of its envelope PDF $p(r)$:

$$\Phi(u) = \int_0^{+\infty} p(r)\, e^{-u\, r}\, dr \tag{12}$$

If we note $\mu_n = \int_0^{+\infty} r^n\, p(r)\, dr$, the moments of order n of the noise envelope are obtained by developing $\Phi(u)$ in Taylor series :

$$\Phi(u) = \sum_{n=0}^{\infty} \mu_n \frac{(-u)^n}{n!} = \sum_{n=0}^{\infty} c_n\, u^n \tag{13}$$

If we suppose all the moments μ_n perfectly known up to order $L+M+1$. The main idea of [4, 5] is to truncate the infinite series at the order $L+M+1$ and to approximate it by a rational function $P^{[L/M]}(u)$ $(L < M)$ defined by :

$$P^{[L/M]}(u) = \frac{\sum_{n=0}^{L} a_n\, u^n}{\sum_{n=0}^{M} b_n\, u^n} \tag{14}$$

where the coefficients $\{a_n\}$ et $\{b_n\}$ are determined so that the following equality be verified :

$$\frac{\sum_{n=0}^{L} a_n\, u^n}{\sum_{n=0}^{M} b_n\, u^n} = \sum_{n=0}^{L+M} c_n\, u^n \tag{15}$$

The moments matching conditions fix in a first step the set of coefficients $\{b_n\}$ by solving a simple set of M linear equations for the M unknown denominator coefficients and in a second step the set $\{a_n\}$ by a simple convolution of the $\{b_n\}$ and the $\{c_n\}$ coefficients :

$$\sum_{n=0}^{M} b_n\, c_{L-n+j} = 0, \qquad 1 \le j \le M \tag{16}$$

$$a_j = c_j + \sum_{i=1}^{\min(M,j)} b_i\, c_{j-i}, \qquad 0 \le j \le L \tag{17}$$

The set of coefficients $\{a_n\}$ and $\{b_n\}$ so determined, forms, thanks to the Padé Approximation, the parametric modeling of the MGF. If we suppose the rational fraction approximation has M

distinct poles with negative real part to assume its convergence for $u = \infty$, the relation (14) can be rewritten as :

$$P^{[L/M]}(u) = \sum_{k=1}^{M} \frac{\lambda_k}{u - \alpha_k} \qquad \mathrm{Re}(\alpha_k) < 0 \tag{18}$$

To determine the PDF $p(r)$ and the Cumulative Density Function (CDF) $F(r)$ from the Padé approximation of the MGF, the Inverse Laplace Transform is performed by residue inversion formula and the result leads to a sum of decaying exponentials :

$$p(r) = \sum_{k=1}^{M} \lambda_k \, e^{\alpha_k \, r} \tag{19}$$

$$F(r) = 1 + \sum_{k=1}^{M} \frac{\lambda_k}{\alpha_k} \, e^{\alpha_k \, r} \tag{20}$$

In the case of some poles with positive real part, it is necessary to stabilize the rational fraction without changing the moments [4]. The Padé approximation gives good results in the estimation of a probability density function and an example is shown at figure 1 for the K-distribution density defined by :

$$p(r) = \frac{b^{\nu+1}}{2^{\nu-1}\,\Gamma(\nu)} \, x^{\nu} \, K_{\nu-1}(b\,x) \tag{21}$$

where $K_{\nu-1}(.)$ is the modified Bessel function of the third kind, ν is the shape of the density and the parameter b is related to the second order moment σ^2 by :

$$b = 2\sqrt{\frac{\nu}{\sigma^2}} \tag{22}$$

This method has to be compared with a non-parametric method, based on the estimation of the pdf by kernel method and described in [6]. However, this one is not here well adapted to our problem because of its poor quality of estimation in the tail of distribution.

4. EVALUATION OF DETECTION PERFORMANCES

The general relations given by (10) and (11) can be simplified when using Padé approximation for the pdf of the noise envelope and for the pdf of the envelope fluctuation. With the knowledge of experimental data of the envelope of the noise, $p_{H_0}(r)$ can be approximated by a sum of complex decaying exponentials :

$$p_{H_0}(r) = \sum_{k=1}^{M} \lambda_k \, e^{\alpha_k \, r} \tag{23}$$

where the set of coefficients $\{\lambda_k\}$ and $\{\alpha_k\}$ is determined by the Padé approximation. Replacing (23) in (10) leads to :

$$p_{H_1}(r/A) = \int_0^{+\infty} \rho \, r \sum_{k=1}^{M} \lambda_k \frac{J_0(\rho\,A)\,J_0(\rho\,r)}{\sqrt{\rho^2 + \alpha_k^2}} \, d\rho \tag{24}$$

Recalling that $\int_0^{\theta} r\,J_0(\rho\,r)\,dr = \theta\,J_1(\rho\,\theta)/\rho$, the probability of detection P_d defined by (4) takes the simple form :

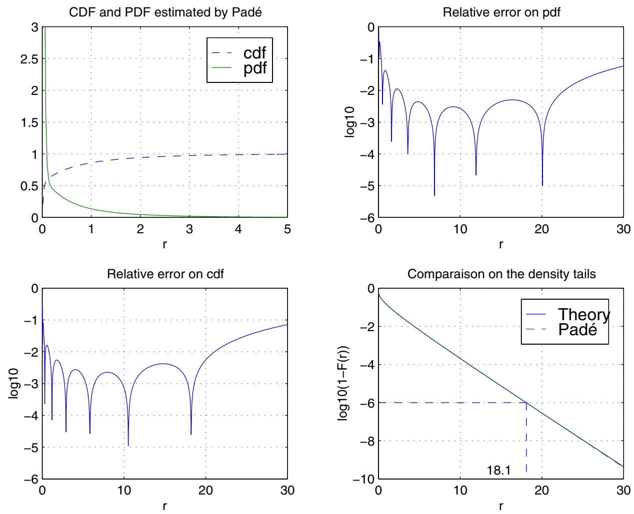

Figure 1: Analysis of Padé K-distribution approximation ($\nu = 0.1$) and ($\sigma^2 = 1$). The detection threshold value $\theta = 18.1$ is shown for $P_{fa} = 10^{-6}$

$$P_d = 1 - \int_0^{+\infty} \sum_{k=1}^{M} \lambda_k \, \theta \frac{J_0(\rho\,A)\,J_1(\rho\,\theta)}{\sqrt{\rho^2 + \alpha_k^2}} \, d\rho \tag{25}$$

where the detection threshold θ is perfectly defined by the determination (Newton find root algorithm) of the non-linear equation (see (3)) :

$$P_{fa} = -\sum_{k=1}^{M} \frac{\lambda_k}{\alpha_k} \, e^{\alpha_k \, \theta} \tag{26}$$

In the case of fluctuating target (Swerling fluctuations for example), it is possible to estimate the envelope fluctuation density $p(A/A_0)$ by Padé approximation :

$$p(A/A_0) = \sum_{i=1}^{N} \gamma_i \, e^{\delta_i \, A} \tag{27}$$

where the parameters $\{\gamma_i\}$ and $\{\delta_i\}$ depend implicitly on the mean value of fluctuation A_0. The equation (11) can be transformed as :

$$p_{H_1}(r/A_0) = \int_0^{+\infty} \sum_{k=1}^{M} \sum_{i=1}^{N} \gamma_i \, \lambda_k \frac{r\,\rho\,J_0(\rho\,r)}{\sqrt{(\rho^2 + \delta_i^2)\,(\rho^2 + \alpha_k^2)}} d\rho \tag{28}$$

which leads to the detection probability formula with the detection threshold θ always given by the resolution of (26) :

$$P_d = 1 - \int_0^{+\infty} \sum_{k=1}^{M} \sum_{i=1}^{N} \frac{\gamma_i \, \lambda_k \, \theta \, J_1(\rho\,\theta)}{\sqrt{(\rho^2 + \delta_i^2)\,(\rho^2 + \alpha_k^2)}} d\rho \tag{29}$$

The relations (25) and (29) are very general and can be easily computed. The figure 2 shows results of pdf and cdf Padé approximation obtained on experimental forest clutter data. The

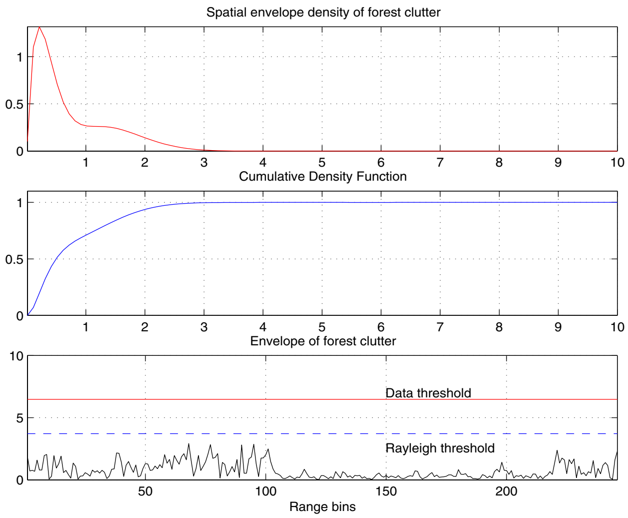

Figure 2: Results of the approximation density obtained on the envelope of experimental data (forest clutter). The detection threshold θ for experimental data is calculated according to equation (26) with $Pfa = 10^{-6}$ and it has to be compared with the one computed for the Rayleigh hypothesis case

figure 3 gives detection performances of an hypothetical non fluctuating target which would be embedded (in phase and amplitude) in such a noise. The curves show the mismatch between the real hypothesis and the Rayleigh hypothesis. The moments have been estimated from the set of complex clutter data y_i according to the classical way :

$$\mu_n = \frac{1}{N} \sum_{i=1}^{N} |y_i|^n \tag{30}$$

The normalized MGF of experimental data takes the form :

$$\begin{aligned} \Phi(u) \quad = \quad & 1 - 0.7652\,u + 0.5\,u^2 - 0.2877\,u^3 + 0.1431\,u^4 \\ & -0.06229\,u^5 + 0.02408\,u^6 - 0.008382\,u^7 \\ & +0.00265\,u^8 - 0.0007672\,u^9 + 0.0002043\,u^{10} \\ & -0.0000503\,u^{11} \end{aligned} \tag{31}$$

and the $[5/6]$ Padé approximation becomes :

$$P^{[5/6]}(u) = \sum_{k=1}^{6} \frac{\lambda_k}{u - \alpha_k} \tag{32}$$

with

$$\begin{aligned} \{\lambda_k\}_{k\in[1,6]} \quad = \quad & \{8.4237 \pm 10.244\,i,\ \ldots \\ & -0.035157 \pm 0.038976\,i,\ \ldots \\ & -8.953 \pm 48.34\,i\} \end{aligned} \tag{33}$$

$$\begin{aligned} \{\alpha_k\}_{k\in[1,6]} \quad = \quad & \{-2.823 \pm 1.9382\,i,\ \ldots \\ & -1.2425 \pm 3.153\,i,\ \ldots \\ & -3.2174 \pm 0.63591\,i\} \end{aligned} \tag{34}$$

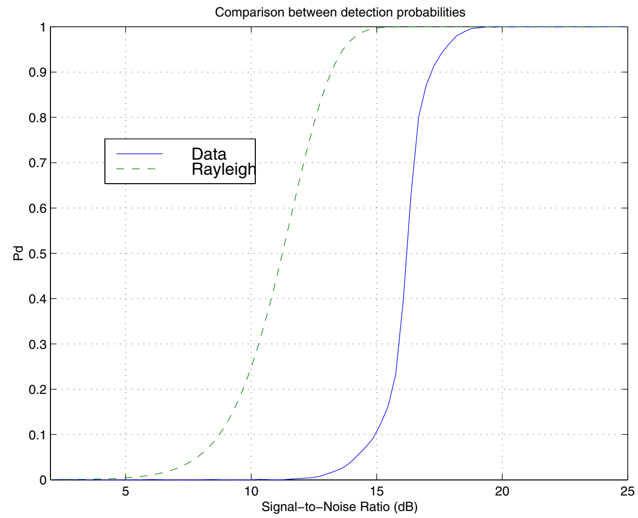

Figure 3: Comparison between detection performances in experimental data (forest clutter) and detection performances in classical Rayleigh noise. The two noise have the same power and the results are shown for a probability of false alarm fixed to $Pfa = 10^{-6}$

5. CONCLUSION

This paper has presented a general method easy to perform for evaluating the performances of target detection from the knowledge of the n-order moments of the envelope of the unwanted noise. This method is based on the Padé approximation of the probability density function which allows to derive simple and general relations of the pair (P_d, P_{fa}). This method has been successfully tested on academic noise (from the true moments and also from the moments estimates) but also on experimental noise data.

6. REFERENCES

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