A ROBUST CHANGE DETECTOR FOR HIGHLY HETEROGENEOUS MULTIVARIATE IMAGES

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Abstract

In this paper, we propose new detectors for Change Detection between two multivariate images. The data is supposed to follow a Compound Gaussian distribution. By using Likelihood Ratio Test (LRT) and Generalised LRT (GLRT) approaches, we derive our detectors. The CFAR behaviour has been studied and the simulations show that they outperform the classic Gaussian Detector when the data is highly heterogeneous.

Introduction

Change Detection (CD) is a classic problem in Remote Sensing. When two images of a same scene at different times are available, the aim is to detect zones on the image corresponding to an alteration in the scene.

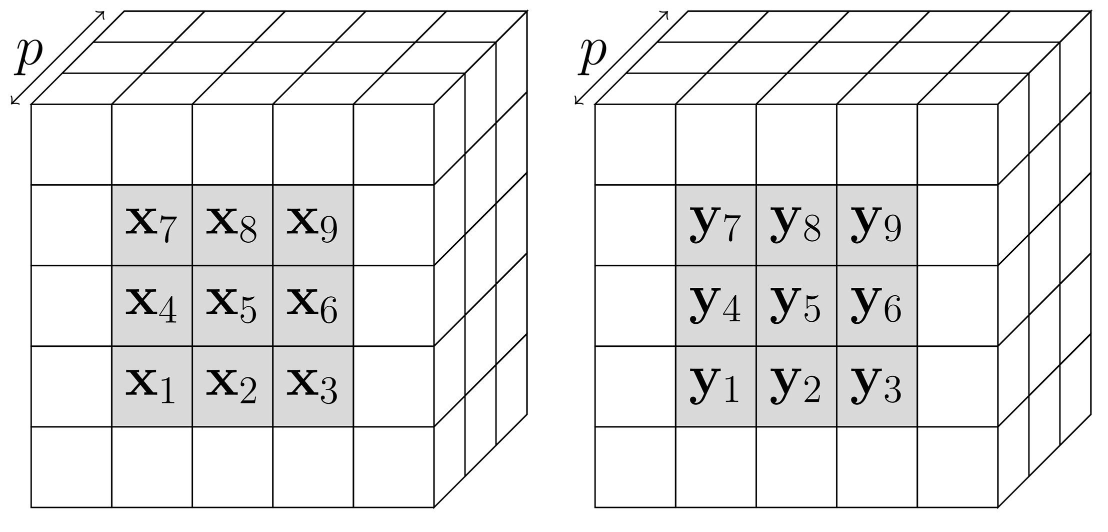

Figure 1: Illustration of local data selection ($N = 9, p = 3$) for detection test. The central pixel is the test pixel.

A classic scheme [1] is to model pixels as **Gaussian** random variables:

$$\forall k \in \{1 \dots N\},\ \mathbf{x}_k \sim \mathbb{C}\mathcal{N}(\mathbf{0}_p, \boldsymbol{\Sigma}_\mathbf{x}),$$
$$\forall k \in \{1 \dots N\},\ \mathbf{y}_k \sim \mathbb{C}\mathcal{N}(\mathbf{0}_p, \boldsymbol{\Sigma}_\mathbf{y}).$$

Change-Detection is done through:

$$\begin{cases} \mathrm{H}_0 : \boldsymbol{\Sigma}_\mathbf{x} = \boldsymbol{\Sigma}_\mathbf{y} \\ \mathrm{H}_1 : \boldsymbol{\Sigma}_\mathbf{x} \neq \boldsymbol{\Sigma}_\mathbf{y} \end{cases}.$$

The GLRT is:

$$\hat{\Lambda}_G = \frac{|(\hat{\boldsymbol{\Sigma}}_\mathbf{x} + \hat{\boldsymbol{\Sigma}}_\mathbf{y})/2|^{2N}}{|\hat{\boldsymbol{\Sigma}}_\mathbf{x}|^N |\hat{\boldsymbol{\Sigma}}_\mathbf{y}|^N} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \lambda, \quad (1)$$

where $\hat{\boldsymbol{\Sigma}}_\mathbf{x} = \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^H}{N}$, $\hat{\boldsymbol{\Sigma}}_\mathbf{y} = \sum_{i=1}^{N} \frac{\mathbf{y}_i \mathbf{y}_i^H}{N}$.

Problem Statement

In heterogeneous images such as high-resolution Synthetic Aperture Radar images, the Gaussian hypothesis is not accurate [2] !

New robust model:

$$\forall k \in \{1 \dots N\},\ \mathbf{x}_k \sim \mathbb{C}\mathcal{N}(\mathbf{0}_p, \tau_k \boldsymbol{\Sigma}_\mathbf{x}),$$
$$\forall k \in \{1 \dots N\},\ \mathbf{y}_k \sim \mathbb{C}\mathcal{N}(\mathbf{0}_p, \theta_k \boldsymbol{\Sigma}_\mathbf{y}).$$

τ_k, θ_k are *deterministic* unknown parameters called texture modelling the heterogeneity.

In the change detection problem, we do not consider the texture parameters:

$$\begin{cases} \mathrm{H}_0 : \boldsymbol{\Sigma}_\mathbf{x} = \boldsymbol{\Sigma}_\mathbf{y} \\ \mathrm{H}_1 : \boldsymbol{\Sigma}_\mathbf{x} \neq \boldsymbol{\Sigma}_\mathbf{y} \end{cases}.$$

Derivation of new statistics

We consider two methods:

- **The 2-step LRT:** we first derive the LRT and plug estimates of the unknown parameters.
  The derivation yields:

$$\hat{\Lambda}_{lrt} = \frac{\left|\hat{\boldsymbol{\Sigma}}_\mathbf{x}\right|^N}{\left|\hat{\boldsymbol{\Sigma}}_\mathbf{y}\right|^N} \exp\left(p \sum_{i=1}^{N} \frac{\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_\mathbf{x}^{-1} \mathbf{y}_i}{\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_\mathbf{y}^{-1} \mathbf{y}_i}\right) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \lambda, \quad (2)$$

where

$$\hat{\boldsymbol{\Sigma}}_\mathbf{x} = \frac{p}{N} \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^H}{\mathbf{x}_i^H \hat{\boldsymbol{\Sigma}}_\mathbf{x}^{-1} \mathbf{x}_i},\ \hat{\boldsymbol{\Sigma}}_\mathbf{y} = \frac{p}{N} \sum_{i=1}^{N} \frac{\mathbf{y}_i \mathbf{y}_i^H}{\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_\mathbf{y}^{-1} \mathbf{y}_i}.$$

$\hat{\boldsymbol{\Sigma}}_\mathbf{x}$ and $\hat{\boldsymbol{\Sigma}}_\mathbf{y}$ are well known Tyler fixed-point estimates.

- **The GLRT:** the unknown parameters are estimated with prior H_0 or H_1.
  The derivation yields:

$$\hat{\Lambda}_{glrt} = \frac{\left|\hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_0}\right|^{2N}}{\left|\hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_1}\right|^N \left|\hat{\boldsymbol{\Sigma}}_\mathbf{y}\right|^N} \prod_{i=1}^{N} \frac{\left(\mathbf{x}_i^H \hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_0}^{-1} \mathbf{x}_i\right)^p \left(\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_0}^{-1} \mathbf{y}_i\right)^p}{\left(\mathbf{x}_i^H \hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_1}^{-1} \mathbf{x}_i\right)^p \left(\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_\mathbf{y}^{-1} \mathbf{y}_i\right)^p} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \lambda$$

where $\hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_0} = \frac{p}{2N} \sum_{i=1}^{N} \left(\frac{\mathbf{x}_i \mathbf{x}_i^H}{\mathbf{x}_i^H \hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_0}^{-1} \mathbf{x}_i} + \frac{\mathbf{y}_i \mathbf{y}_i^H}{\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_0}^{-1} \mathbf{y}_i}\right)$

and $\hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_1} = \frac{p}{N} \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^H}{\mathbf{x}_i^H \hat{\boldsymbol{\Sigma}}_{\mathbf{x},\mathbf{H}_1}^{-1} \mathbf{x}_i}$, $\hat{\boldsymbol{\Sigma}}_\mathbf{y} = \frac{p}{N} \sum_{i=1}^{N} \frac{\mathbf{y}_i \mathbf{y}_i^H}{\mathbf{y}_i^H \hat{\boldsymbol{\Sigma}}_\mathbf{y}^{-1} \mathbf{y}_i}$. (3)

We must impose a trace normalization to ensure *uniqueness* of fixed-point estimates: $\mathrm{Tr}(\hat{\boldsymbol{\Sigma}}) = p$.

CFAR property

- **Texture CFAR:** $\hat{\Lambda}_{lrt}$ and $\hat{\Lambda}_{glrt}$ have this property. Indeed, the textures parameters simplify in the expression of the ratio.
- **Covariance CFAR:** $\hat{\Lambda}_{lrt}$ is **not CFAR** since the trace normalization is not homogeneous in the ratio. $\hat{\Lambda}_{glrt}$ has this property. Indeed, the statistic is invariant by substituting $\mathbf{x}_k \to \boldsymbol{\Sigma}_\mathbf{x}^{-\frac{1}{2}} \mathbf{x}_k$ and $\mathbf{y}_k \to \boldsymbol{\Sigma}_\mathbf{x}^{-\frac{1}{2}} \mathbf{y}_k$ for all $k \in \{1 \dots N\}$.

When using fixed-point estimates in decision statistics, the statistic must be invariant by the normalization constraint !

Simulations

Table 1: Simulation-relevant parameters

α, β	ρ_x, ρ_y	p	N	SNR
shape and scale for Γ-distribution	coefficients for Toeplitz matrices	Size of vector	Number of observations	Signal to Noise Ratio

We generate observations using a Γ-distribution for the texture parameters. The covariance matrices are chosen to be Toeplitz of the form: $(\boldsymbol{\Sigma}_\bullet)(m, n) = \rho_\bullet^{|m-n|}$.

- **CFAR Behaviour:**

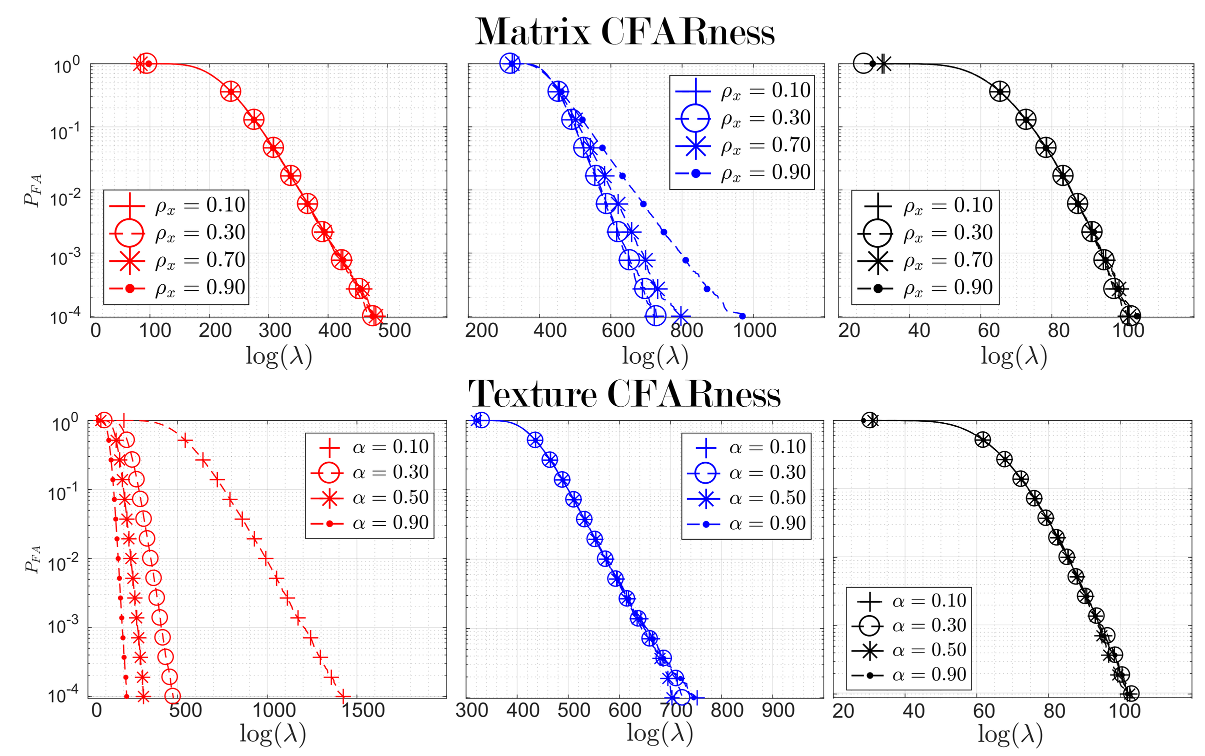

Figure 2: $P_{FA} - \lambda$. Left = $\hat{\Lambda}_G$, Middle = $\hat{\Lambda}_{lrt}$, Right = $\hat{\Lambda}_{glrt}$. $p = 10$, $N = 25$.
Top: $\rho_x = \rho_y$, $\alpha = 0.3$, $\beta = 0.1$. Bottom: $\rho_x = \rho_y = 0.3$, $\beta = 0.1$.

- **ROC plots:**

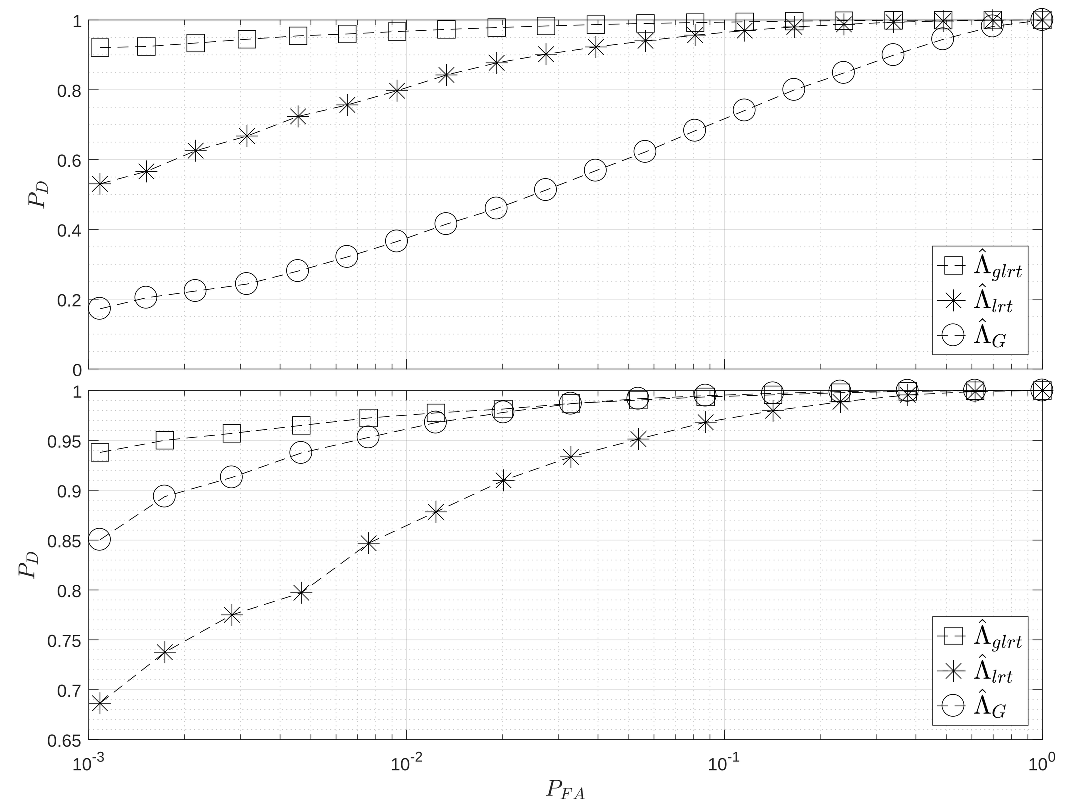

Figure 3: $P_D - P_{FA}$ with $p = 10$, $N = 25$, $\rho_x = 0.1$, $\rho_y = 0.9$, $\alpha = 0.3$, $\beta = 0.1$. Top: SNR = 0 dB. Bottom: SNR= 20 dB. 1000 Monte-Carlo trials.

Conclusion

New detectors for CD on highly heterogeneous multivariate images have been proposed using a robust model. The CFAR behaviour of these new detectors have been studied both theoretically and in simulation. A ROC plot has also been computed to test the performances of the detectors and it shows that the new detectors, outperform the Gaussian one.

References

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