FALSE-ALARM REGULATION FOR TARGET DETECTION IN HYPERSPECTRAL IMAGING

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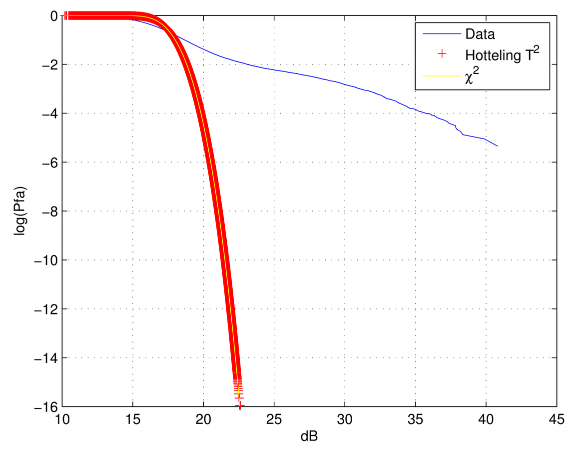
PROBLEMS DESCRIPTION

- **DETECTION OF TARGETS IN HYPERSPECTRAL IMAGES**
  To detect targets (characterized by a given spectral signature p) - Regulation of False Alarm.

The hyperspectral data are positive as they represent radiance or reflectance.

- In the adaptive detection framework, a mean vector and has to be included in the statistical model and estimated jointly with the covariance matrix,
- Some hyperspectral data are proven to be **spatially hetereogeneous** in intensity and/or **cannot be only characterized by Gaussian statistic.**



COMPLEX NORMAL DISTRIBUTION

A m-dimensional vector $\mathbf{x}$ has a complex normal distribution, and if the probability density function exists, it is of the form:

$$f_{\mathbf{x}}(\mathbf{x}) = \pi^{-m}|\boldsymbol{\Sigma}|^{-1}\exp\{-(\mathbf{x}-\boldsymbol{\mu})^H\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\}.$$

covariance matrix — mean vector

The resulting **M**aximum **L**ikelihood **E**stimates are : $\hat{\boldsymbol{\mu}}_{SMV} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i \quad \hat{\boldsymbol{\Sigma}}_{SCM} = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{x}_i-\hat{\boldsymbol{\mu}})(\mathbf{x}_i-\hat{\boldsymbol{\mu}})^H$

COMPLEX WISHART DISTRIBUTION

Let $\mathbf{x}_1, ..., \mathbf{x}_N$ be an IID N-sample, where $\mathbf{x}_i \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. And let $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}_{SMV}$ and $\hat{\mathbf{W}} = N\,\hat{\boldsymbol{\Sigma}}_{SCM}$ referred to as the Wishart matrix. Thus, one has:

- $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{W}}$ are independently distributed;
- $\hat{\boldsymbol{\mu}} \sim \mathcal{CN}\left(\boldsymbol{\mu}, \frac{1}{N}\boldsymbol{\Sigma}\right)$;
- $\hat{\mathbf{W}} \sim \mathcal{CW}(N-1, \boldsymbol{\Sigma})$ is Wishart distributed with $N-1$ degrees of freedom.

TARGET DETECTION SCHEMES - AMF

- Derived under Gaussian hypothesis,
- It is the optimal linear filter in terms of SNR maximization under Gaussian assumption,
- Its false alarm is independent of the covariance matrix, **CFAR-matrix**.

MATCHED FILTER

Known covariance matrix and mean vector

$$\Lambda_{MF} = \frac{|\mathbf{p}^H\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\boldsymbol{\Sigma}^{-1}\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

PFA-THRESHOLD RELATIONSHIP

$$\boxed{PFA_{MF} = \exp(-\lambda)}$$

ADAPTIVE MATCHED FILTER

Unknown covariance matrix and known mean vector

$$\Lambda^{(N)}_{AMF\hat{\boldsymbol{\Sigma}}} = \frac{|\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad \boxed{PFA_{AMF\hat{\boldsymbol{\Sigma}}} = {}_2F_1\left(N-m+1,\, N-m+2;\, N+1;\, -\frac{\lambda}{N}\right)}$$

ADAPTIVE MATCHED FILTER

Unknown covariance matrix and mean vector

$$\Lambda^{(N)}_{AMF\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{|\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Firstly remark that as we jointly estimate the mean and the covariance matrix we lose a degree of freedom.

Let us know consider the AMF replacing $\hat{\boldsymbol{\Sigma}}_{SCM}$ by the Wishart matrix $\hat{\mathbf{W}}_{N-1}$:

$$\Lambda^{(N)}_{AMF\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = N\frac{|\mathbf{p}^H\hat{\mathbf{W}}^{-1}_{N-1}(\mathbf{x}-\hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\hat{\mathbf{W}}^{-1}_{N-1}\mathbf{p})}$$

Since $\hat{\boldsymbol{\mu}} \sim \mathcal{CN}\left(\boldsymbol{\mu}, \frac{1}{N}\boldsymbol{\Sigma}\right)$, one has $\mathbf{x}-\hat{\boldsymbol{\mu}} \sim \mathcal{CN}\left(\mathbf{0}, \frac{N+1}{N}\boldsymbol{\Sigma}\right)$.

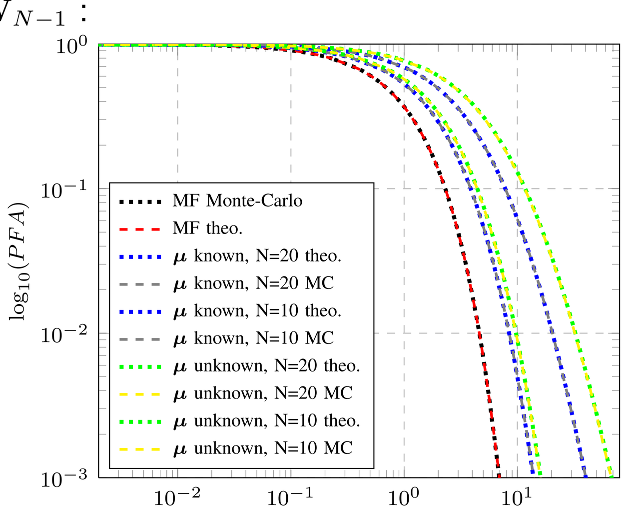
This can be equivalently rewritten as :

$$\mathbf{y} = \sqrt{N/(N+1)}(\mathbf{x}-\hat{\boldsymbol{\mu}}) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$$

When replacing it on the test we obtain:

$$\Lambda^{(N)}_{AMF\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = N\frac{N+1}{N}\frac{|\mathbf{p}^H\hat{\mathbf{W}}^{-1}_{N-1}\mathbf{y}|^2}{(\mathbf{p}^H\hat{\mathbf{W}}^{-1}_{N-1}\mathbf{p})} = \frac{(N+1)}{(N-1)}\Lambda^{(N-1)}_{AMF\,\hat{\boldsymbol{\Sigma}}}$$

$$\boxed{PFA_{AMF\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = {}_2F_1\left(N-m,\, N-m+1;\, N;\, -\frac{\lambda}{N+1}\right)}$$



TARGET DETECTION SCHEMES - ANMF

- Derived under Gaussian hypothesis,
- It is invariant under scale change of the observation vector,
- Its false alarm is independent of the covariance matrix, **CFAR-matrix**.

NORMALIZED MATCHED FILTER

Known covariance matrix and mean vector

$$\Lambda_{NMF} = \frac{|\mathbf{p}^H\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\boldsymbol{\Sigma}^{-1}\mathbf{p})\left((\mathbf{x}-\boldsymbol{\mu})^H\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

PFA-THRESHOLD RELATIONSHIP

$$\boxed{PFA_{NMF} = (1-\lambda)^{(m-1)}}$$

ADAPTIVE NORMALIZED MATCHED FILTER

Unknown covariance matrix and known mean vector

$$\Lambda^{(N)}_{ANMF\hat{\boldsymbol{\Sigma}}} = \frac{|\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p})\left((\mathbf{x}-\boldsymbol{\mu})^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

$$\boxed{PFA_{ANMF\hat{\boldsymbol{\Sigma}}} = (1-\lambda)^{N-m+1}\,{}_2F_1(N-m+2, N-m+1; N+1; \lambda)}$$

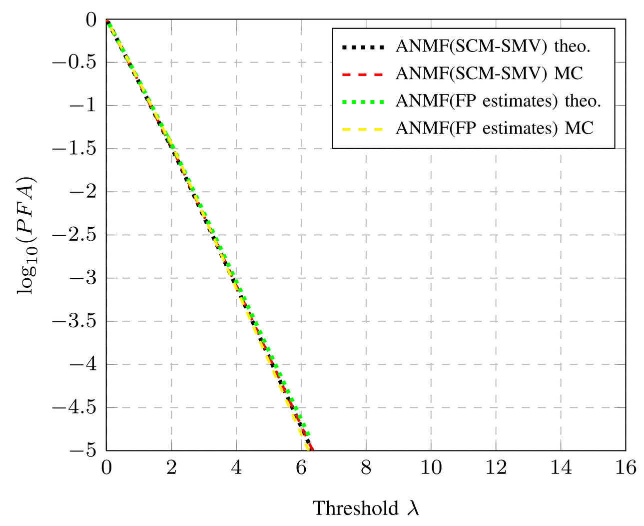
ADAPTIVE NORMALIZED MATCHED FILTER

Unknown covariance matrix and mean vector

$$\Lambda_{ANMF\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{|\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p})\left((\mathbf{x}-\hat{\boldsymbol{\mu}})^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\hat{\boldsymbol{\mu}})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Due to the normalization term $(\mathbf{x}-\hat{\boldsymbol{\mu}})^H\hat{\boldsymbol{\Sigma}}^{-1}(\mathbf{x}-\hat{\boldsymbol{\mu}})$, the correction factor $(N+1)/N$ appears both at the numerator and at the denominator and consequently, it disappears.

Since the detector is homogeneous in terms of covariance matrix, the factor N also disappears.



$$\boxed{PFA_{ANMF\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = (1-\lambda)^{N-m}\,{}_2F_1(N-m+1, N-m; N; \lambda)}$$

ROBUST TARGET DETECTION - ANMF

In order to take into account **heterogeneity** and **non-Gaussianity** for background modeling, the class of **Elliptical Distribution Model** is considered:

$$f_{\mathbf{x}}(\mathbf{x}) = |\boldsymbol{\Sigma}|^{-1}h_m\left((\mathbf{x}-\boldsymbol{\mu})^H\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$

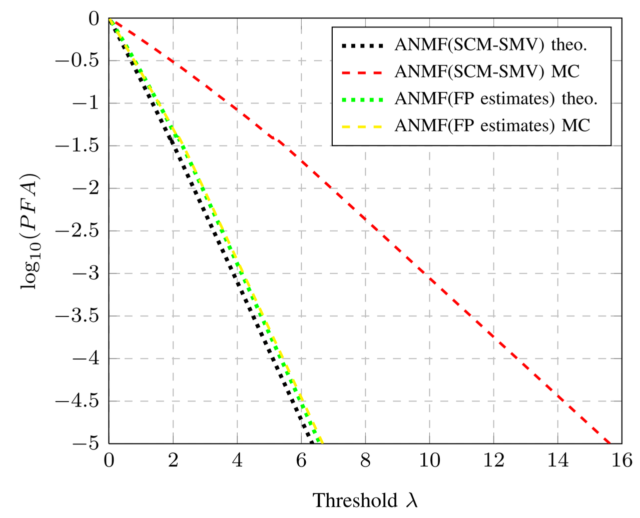
density generator

- Due to its invariance properties ANMF detector provide the best detection results in non-Gaussian environment,
- One has to introduce Robust Estimation procedures to achieve robustness in the detection scheme.

FIXED POINT ESTIMATORS

$$\hat{\boldsymbol{\mu}}_{FP} = \frac{\displaystyle\sum_{i=1}^{N}\frac{\mathbf{x}_i}{(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})^H\hat{\boldsymbol{\Sigma}}^{-1}_{FP}(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})^{1/2}}}{\displaystyle\sum_{i=1}^{N}\frac{1}{(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})^H\hat{\boldsymbol{\Sigma}}^{-1}_{FP}(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})^{1/2}}}, \quad \hat{\boldsymbol{\Sigma}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})^H}{(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})^H\hat{\boldsymbol{\Sigma}}^{-1}_{FP}(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_{FP})}$$

- **This estimate does not depend on the elliptical distribution density generator,**
- **The Fixed Point Matrix Estimate is consistent, unbiased, asymptotically Gaussian and, for N large enough, it behaves as a Wishart matrix with $\frac{m}{m+1}N$ degrees of freedom.**



$$\boxed{PFA_{ANMF-FP} = (1-\lambda)^{\frac{m}{m+1}(N-1)-m+1}\,{}_2F_1\left(\frac{m}{m+1}(N-1)-m+2,\, \frac{m}{m+1}(N-1)-m+1;\, \frac{m}{m+1}(N-1)+1;\, \lambda\right)}$$

CONCLUSIONS

- **The AMF and the ANMF have been analyzed in the case where both the covariance matrix and the mean vector are unknown and need to be estimated,**
- **Closed-form expressions for "PFA-threshold" relationships have been derived under Gaussian assumptions,**
- **Additionally, Kelly detector has been studied for non-zero mean Gaussian distribution but non-closed form for "PFA-threshold" relationship can be obtained,**
- **In Elliptical distributions framework, the joint robust location and scale estimators have been proposed and they provide better false-alarm regulation and an improvement for detection in heterogeneous and/or non-Gaussian background.**