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BORD: bayesian optimum radar detector

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Abstract

We derive the expression of an optimum non-Gaussian radar detector from the non-Gaussian spherically invariant random process (SIRP) clutter model and a bayesian estimator of the SIRP *characteristic density*. SIRP modelizes non-Gaussian process as a complex Gaussian process whose variance, the so-called *texture*, is itself a positive random variable (r.v.). After performing a bayesian estimation of the *texture* probability density function (PDF) from reference clutter cells we derive the so-called bayesian optimum radar detector (BORD) without any knowledge about the clutter statistics. We also derive the asymptotic expression of BORD (in law convergence), the so-called asymptotic BORD, as well as its theoretical performance (analytical threshold expression). BORD performance curves are shown for an unknown target signal embedded in correlated K-distributed and are compared with those of the optimum K-distributed detector. These results show that BORD reach optimal detector performances.

Keywords: SIRP Model; Radar detection; Bayesian estimation

1. Introduction

Coherent radar detection against non-Gaussian clutter has gained many interests in the radar community since experimental clutter measurements made by organizations like MIT [17,18,4] have shown to fit non-Gaussian statistical models. One of the most tractable and elegant non-Gaussian models results in the so-called *spherically invariant random process* (SIRP) which states that many non-Gaussian random processes are the product of a Gaussian random process (called the *speckle*) with a non-negative random variable (r.v.) (the so-called *texture*), so that an SIRP is a compound Gaussian process. This model is the base of many results like Gini et al.'s works [8] in which the optimum detector in the presence of composite disturbance of known statistics modeled as SIRP is derived.

In this paper, a bayesian approach is proposed to determine the probability density function (PDF) of the *texture* from N_{ref} reference clutter cells. We use Bayes' rule and a Monte Carlo estimation given a *non-informative* prior on the *texture*. This approach exploits the SIRP model particularity to describe non-Gaussian processes as compound processes and allows one to derive the expression of the optimum detector called bayesian optimum radar detector

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(BORD). Henceforth, it is no more necessary to have any knowledge about the clutter statistics and BORD deals directly with the received data. In Sections 2 and 3 we briefly recall the formulation of a detection problem and describe how the SIRP model clutter yields to general and particular optimum detector. In Section 4, we explain the bayesian approach used to determine a bayesian estimator to the *texture* PDF and give the expression of the resulting BORD. In this section the asymptotic expression of BORD (the so-called asymptotic BORD) and the analytical performance of the Asymptotic BORD are also derived. Section 5 is devoted to the simulations description to evaluate BORD performance (compared with optimum detectors performance). Conclusions and outlooks are given in Section 6.

2. General relations of detection theory

We consider here the basic problem of detecting the presence (H_1) or absence (H_0) of a complex signal $\mathbf{s}$ in a set of N_{ref} measurements of m-complex vectors $\mathbf{y} = \mathbf{y}_I + j\mathbf{y}_Q$ corrupted by a sum $\mathbf{c}$ of independent additive complex noises (noises+clutter). The problem can be described in terms of a statistical hypothesis test:

$$H_0 : \mathbf{y} = \mathbf{c}, \quad (1)$$

$$H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c}. \quad (2)$$

When present, the target signal $\mathbf{s}$ corresponds to a modified version of the perfectly known transmitted signal $\mathbf{t}$ and can be rewritten as $\mathbf{s} = AT(\underline{\theta})\mathbf{t}$. A is the target complex amplitude and we suppose to have determined all the others parameters ($\underline{\theta}$) which characterize the target (Doppler frequency, time delay, etc). In the following, $\mathbf{p} = T(\underline{\theta})\mathbf{t}$.

The observed vector $\mathbf{y}$ is used to form the likelihood ratio test (LRT) $\Lambda(\mathbf{y})$ which is compared with a threshold η set to a desired false alarm probability (P_{fa}) value:

$$\Lambda(\mathbf{y}) = \frac{p_{\mathbf{y}}(\mathbf{y}/H_1)}{p_{\mathbf{y}}(\mathbf{y}/H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (3)$$

LRT performances follow from the statistics of the data: P_{fa} is the probability of choosing H_1 when the target is absent, and the detection probability (P_{d}) is the probability of choosing H_1 when the target is present, that is,

$$P_{\text{fa}} = \mathbb{P}(\Lambda(\mathbf{y}) > \eta | H_0)$$

and

$$P_{\text{d}} = \mathbb{P}(\Lambda(\mathbf{y}) > \eta | H_1).$$

3. Non-Gaussian clutter case: SIRV and optimum radar detector

In the case of non-Gaussian clutter, detection strategies can be derived if we consider a particular clutter nature, i.e. if an a priori hypothesis is made on the clutter statistics. To model non-Gaussian clutter and derive general detector expressions, we use the SIRP representation [1,8,13,19].

3.1. Description and general expressions

A spherically invariant random vector (SIRV) is a vector issued from a SIRP which modelizes each element of the clutter vector $\mathbf{c}$ as the product of a m-complex Gaussian vector $\mathbf{x}$ (the *speckle*) ($\mathcal{CN}(\mathbf{0}, 2\mathbf{M})$) with a positive r.v. τ (the *texture*), that is $\mathbf{c} = \mathbf{x}\sqrt{\tau}$. The so-formed vector $\mathbf{c}$ is, conditionally to τ, a complex Gaussian random vector ($\mathcal{CN}(\mathbf{0}, 2\tau\mathbf{M})$) with multivariate PDF $p(\mathbf{c}/\tau)$. The PDF of the clutter is then

$$p(\mathbf{c}) = \int_0^{+\infty} \frac{\tau^{-m}}{(2\pi)^m |\mathbf{M}|} \exp\left(-\frac{\mathbf{c}^\dagger \mathbf{M}^{-1}\mathbf{c}}{2\tau}\right) p(\tau)\,d\tau, \quad (4)$$

where $\dagger$ is the transpose conjugate operator, and $|\mathbf{M}|$ is the determinant of the matrix $\mathbf{M}$. This general expression leads, for a known $p(\tau)$, to multivariate PDFs of non-Gaussian random vectors. For example, joint K-distributed PDF is obtained if $p(\tau)$ is a Gamma PDF (see further).

3.2. Optimum SIRV detector

Applied to the detection problem, expression (4) gives $p_{\mathbf{C}}(\mathbf{y}/H_0)$ and $p_{\mathbf{C}}(\mathbf{y}/H_1) = p_{\mathbf{C}}(\mathbf{y} - \mathbf{s}/H_0)$ when the target signal $\mathbf{s}$ is known. The LRT becomes

$$\frac{\int_0^{+\infty} \tau^{-m} \exp(-q_1(\mathbf{y})/2\tau)\,\mathrm{d}\tau}{\int_0^{+\infty} \tau^{-m} \exp(-q_0(\mathbf{y})/2\tau)\,\mathrm{d}\tau} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{5}$$

where

$$q_0(\mathbf{y}) = \mathbf{y}^\dagger \mathbf{M}^{-1}\mathbf{y}, \tag{6}$$

$q_1(\mathbf{y}) = q_0(\mathbf{y}-\mathbf{s})$ for a known signal $\mathbf{s}$ and $\lambda = \ln(\eta)$. When the target complex amplitude A is unknown, ML estimation of A is performed [8]:

$$\hat{A}_{\mathrm{ML}} = \frac{\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{y}}{\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{p}}, \tag{7}$$

and the detection strategy is given by (5) where now [8]

$$q_1(\mathbf{y}) = \mathbf{y}^\dagger \mathbf{M}^{-1}\mathbf{y} - \frac{|\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{y}|^2}{\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{p}}. \tag{8}$$

With (8), expression (5) is called generalized LRT (GLRT).

3.3. Examples: various optimum SIRV detectors

All the following optimum SIRV detectors are given for an unknown amplitude target whose value is estimated is the ML sense. So, the quadratic forms $q_0(\mathbf{y})$ and $q_1(\mathbf{y})$ are, respectively, given by (6) and (8).

3.3.1. OKD: optimum K detector

In the case of K-distributed clutter (size m) with parameters ν and b, *texture* PDF is a Gamma$(\nu, 2/b^2)$ PDF with expression

$$p(\tau) = \frac{\tau^{\nu-1} b^{2\nu}}{\Gamma(\nu)2^\nu} \exp\left(-\frac{\tau b^2}{2}\right). \tag{9}$$

The PDF of $\mathbf{y}$ under H_0 hypothesis is then given by

$$p_{\mathbf{y}}(\mathbf{y}/H_0) = \frac{2b^{\nu+m} q_0(\mathbf{y})^{(\nu-m)/2}}{\pi^m |\mathbf{M}| \Gamma(\nu) 2^{\nu+m}} K_{\nu-m}\left(b\sqrt{q_0(\mathbf{y})}\right), \tag{10}$$

where $K_\nu(.)$ is the modified Bessel function of order ν and $\Gamma(.)$ is the Gamma function. The value of ν determines the spikiness of the distribution. Following the same processes with (5) expression of the so-called optimum K-distributed detector (OKD) becomes $\forall m \geqslant 2$

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{(\nu-m)/2} \frac{K_{\nu-m}(b\sqrt{q_1(\mathbf{y})})}{K_{\nu-m}(b\sqrt{q_0(\mathbf{y})})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{11}$$

For $m = 1$, $q_0(\mathbf{y}) = |y|^2$ (where y is the scalar vector we observe) and expression (11) becomes

$$|y|^{\nu-1} K_{\nu-1}(b|y|) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{2^{\nu-1}\Gamma(\nu)}{2b^{\nu-1}\nu\eta}. \tag{12}$$

3.3.2. OLD: optimum Laplace detector

This detector can be considered as a particular case of OKD since the K-distribution becomes a Laplace PDF when $\nu = 1$. The *texture* PDF is then an exponential PDF with parameter $b^2/2$, with expression

$$p(\tau) = \frac{b^2}{2} \exp\left(-\frac{b^2 \tau}{2}\right), \tag{13}$$

and the resulting expression of the OLD becomes $\forall m \geqslant 2$

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{(1-m)/2} \frac{K_{1-m}(b\sqrt{q_1(\mathbf{y})})}{K_{1-m}(b\sqrt{q_0(\mathbf{y})})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{14}$$

For $m = 1$, $q_0(\mathbf{y}) = |y|^2$ and we just have

$$K_0(b|y|) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2\eta}. \tag{15}$$

3.3.3. OStD: optimum Student-*t* detector

Student-$t(\nu, b)$ marginal PDF is obtained when the *texture* PDF is an inverse gamma PDF $\mathscr{IG}(\nu, 2/b^2)$ whose expression is

$$p(\tau) = \frac{b^{2\nu} \tau^{-\nu-1}}{2^\nu \Gamma(\nu)} \exp\left(-\frac{b^2}{2\tau}\right). \tag{16}$$

The resulting expression of the OStD is then

$$\left(\frac{b^2 + q_0(\mathbf{y})}{b^2 + q_1(\mathbf{y})}\right)^{m+\nu} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{17}$$

3.3.4. OχD: optimum χ detector

As for the other examples given from now, the *texture* PDF of a χ SIRV (the marginal PDF is a χ PDF with ν degrees of freedom and parameter b, σ^2 is the variance) is analytically unknown. It is, however, possible [13] to derive the optimum χ detector for $v \leqslant 1$, which is

$$\frac{\sum_{k=1}^{m}(-1)^{k-1}C_{m-1}^{k-1}(b^2\sigma^2)^{k-1}D_k(q_1(\mathbf{y}),v,b\sigma)}{\sum_{k=1}^{m}(-1)^{k-1}C_{m-1}^{k-1}(b^2\sigma^2)^{k-1}D_k(q_0(\mathbf{y}),v,b,\sigma)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{18}$$

where for $j=0,1$,

$$D_k(q_j(\mathbf{y}),v,b,\sigma) = \prod_{i=1}^{m-1}(v-i)(q_j(\mathbf{y}))^{v-k}\mathrm{e}^{-b^2\sigma^2 q_j(\mathbf{y})}$$

and

$$C_{m-1}^{k-1} = \frac{(m-1)!}{(k-1)!(m-k)!}.$$

3.3.5. OgRD: optimum generalized Rayleigh detector

As said previously, the *texture* PDF of a generalized Rayleigh SIRV (the marginal PDF is a generalized Rayleigh PDF with parameters η and β, σ^2 is the variance) is analytically unknown but the optimum generalized Rayleigh detector is given for $\eta \leqslant 2$ by

$$\left(\frac{q_1(\mathbf{y})}{q_0(\mathbf{y})}\right)^{1-m} \exp\left(\frac{\sigma^\eta}{\beta^\eta}[q_0(\mathbf{y})^{\eta/2} - q_1(\mathbf{y})^{\eta/2}]\right) \times \frac{\sum_{k=1}^{m-1}(D_k(\eta)/k!)(\sigma\sqrt{q_1(\mathbf{y})}/\beta)^{\eta k}}{\sum_{k=1}^{m-1}(D_k(\eta)/k!)(\sigma\sqrt{q_0(\mathbf{y})}/\beta)^{\eta k}} \underset{H_0}{\overset{H_1}{\gtrless}} \xi, \tag{19}$$

where ξ is an threshold notation other than η,

$$D_k(\eta) = \sum_{j=1}^{k}(-1)^j C_k^j \prod_{i=0}^{j-1}\left(\frac{j\eta}{2} - i\right),$$

and $C_k^j = (k)!/(j)!(k-j)!$.

3.3.6. OWD: optimum Weibull detector

For a Weibull SIRV, the *texture* PDF can be written as an integral form depending on the G-Meijer's functions. Its expression can be found in [3,9] and the optimum Weibull detector is given for $0 \leqslant b \leqslant 2$ by

$$\left(\frac{q_0(\mathbf{y})}{q_1(\mathbf{y})}\right)^m \times \frac{\sum_{k=1}^{m-1}(D_k(b)/k!)(a\sigma^b)^k F_1^{kb}(\mathbf{y})\exp(-a\sigma^b F_1^{kb}(\mathbf{y}))}{\sum_{k=1}^{m-1}(D_k(b)/k!)(a\sigma^b)^k F_0^{kb}(\mathbf{y})\exp(-a\sigma^b F_0^{kb}(\mathbf{y}))} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{20}$$

where a, b and σ^2 are, respectively, the two parameters of the marginal SIRV Weibull and its variance. In the OWD expression, we also have

$$F_p^{kb} = \sqrt{q_p(\mathbf{y})}^{kb} \quad \text{for } p = 0,1,$$

$$D_k(b) = \sum_{j=1}^{k}(-1)^j C_k^j \prod_{i=0}^{j-1}\left(\frac{jb}{2} - i\right),$$

and

$$C_k^j = \frac{(k)!}{(j)!(k-j)!}.$$

3.3.7. ORD: optimum rice detector

The Rice marginal PDF of an SIRV, with parameter ρ and variance σ^2, gives rise to the optimum rice detector whose expression is given for $0 < \rho \leqslant 1$ by

$$\exp[K(q_0(\mathbf{y}) - q_1(\mathbf{y}))] \times \frac{\sum_{k=1}^{m}(-1)^k C_{m-1}^k(\rho/2)^k D_k(q_1(\mathbf{y}),\sigma,\rho)}{\sum_{k=1}^{m}(-1)^k C_{m-1}^k(\rho/2)^k D_k(q_0(\mathbf{y}),\sigma,\rho)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{21}$$

where $K = \sigma^2/2(1-\rho^2)$, and for $j = 0,1$

$$D_k(q_j(\mathbf{y})) = \sum_{l=0}^{k} C_k^l I_{k-2l}(K\rho q_j(\mathbf{y})).$$

$I_v(.)$ is the first kind modified Bessel function of order v, and $C_k^l = (k)!/(l)!(k-l)!$.

3.3.8. OCpGD: optimum clutter plus Gaussian detector

When thermal noise is separately accounted in the received data, the general expression (5) has to be changed because the total correlation matrix of the noise plus clutter depends on the *texture* since we have

$$\mathbf{M}_{y/\tau} = \tfrac{1}{2}\mathbb{E}(\mathbf{y}\mathbf{y}^{\dagger}/\tau) = \tau\mathbf{M}_x + \sigma_b^2\mathbf{I}, \tag{22}$$

where $\mathbf{I}$ is the identity matrix. GLRT expression (5) becomes

$$\frac{\int_0^{+\infty}(1/|\tau\mathbf{M}_x + \sigma_b^2\mathbf{I}|)\exp(-\frac{q_1(\mathbf{y},\tau)}{2})\,p(\tau)\,\mathrm{d}\tau}{\int_0^{+\infty}(1/|\tau\mathbf{M}_x + \sigma_b^2\mathbf{I}|)\exp(-\frac{q_0(\mathbf{y},\tau)}{2})\,p(\tau)\,\mathrm{d}\tau} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{23}$$

where now

$$q_0(\mathbf{y},\tau) = \mathbf{y}^{\dagger}(\tau\mathbf{M}_x + \sigma_b^2\mathbf{I})^{-1}\mathbf{y}, \tag{24}$$

and $q_1(\mathbf{y},\tau)$ is deduced from $q_0(\mathbf{y},\tau)$ as in the previous case (known or unknown target amplitude).

This case was introduced by Gini et al. in [8] where the authors consider K-distributed clutter plus thermal noise to derive OKGD (optimum K plus Gaussian detector). This detector cannot be derived in a simple analytical form but theoretical performances are given in an integral form.

From expressions (5) and (23) we can see that SIRV detectors depends on the *texture* PDF. In [9,10] the authors used a Padé approximation to estimate the *texture* PDF and then to derive an adaptive detector. This method depends on the quality of the moment estimation and requires reference clutter cells to estimate the Padé coefficients.

In the next section we propose to use a bayesian estimator of the *texture* PDF which comes from the Bayes'rule and Monte Carlo integration. Then, BORD expression is derived.

4. Bayesian optimum radar detector

4.1. Bayesian study of the problem

As we have said in the previous section, for a known *texture* PDF $p(\tau)$, it is possible to derive the associated detector expression. The idea of a bayesian approach is to determine, from N_{ref} reference clutter cells of size m, $\mathbf{R} = [\mathbf{r}_1,\ldots,\mathbf{r}_{N_{\mathrm{ref}}}]^{\mathrm{T}}$ where $\mathbf{r}_i = [r_i(1),\ldots,r_i(m)]^{\mathrm{T}}$, a bayesian estimator for $p(\tau)$. We write $p(\tau)$ as follows:

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r})\,p(\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{25}$$

Given $\mathbf{r}_{i=1}^{N_{\mathrm{ref}}}$ a Monte Carlo estimation of (25) is

$$\hat{p}_{N_{\mathrm{ref}}}(\tau) = \frac{1}{N_{\mathrm{ref}}}\sum_{i=1}^{N_{\mathrm{ref}}} p(\tau/\mathbf{r}_i). \tag{26}$$

This estimate is unbiased and as the samples $\{\mathbf{r}_i,\ i = 1,\ldots,N_{\mathrm{ref}}\}$ are statistically independent, the strong law of large numbers applies and gives

$$\lim_{N_{\mathrm{ref}}\to+\infty} \hat{p}_{N_{\mathrm{ref}}}(\tau) \overset{\mathrm{a.s.}}{\longrightarrow} p(\tau). \tag{27}$$

To evaluate (26) we have to know the expression of the a posteriori PDF of τ given the N_{ref} reference cells, that is the expression of $p(\tau/\mathbf{r}_i)$. Using the Bayes'rule, we have

$$p(\tau/\mathbf{r}_i) = \frac{p(\mathbf{r}_i/\tau)\,p(\tau)}{p(\mathbf{r}_i)}, \tag{28}$$

and as $p(\tau)$ is unknown (as well as $p(\mathbf{r}_i)$) we replace it by a prior distribution, called $g(\tau)$. This processing step works as an information processor that updates the prior density function $g(\tau)$ into the posterior PDF $p(\tau/\mathbf{r}_i)$. The Bayes'rule can be interpreted as a relevant mechanism to provide a rational solution of how to learn from the reference cells about the quantity of interest τ. By this way, Eq. (26) becomes

$$\hat{p}_{N_{\mathrm{ref}}}(\tau) = \frac{1}{N_{\mathrm{ref}}}\sum_{i=1}^{N_{\mathrm{ref}}} \frac{p(\mathbf{r}_i/\tau)g(\tau)}{p(\mathbf{r}_i)}, \tag{29}$$

where the normalization constant $p(\mathbf{r}_i)$ is obtained by integrating the numerator in (26) over $g(\tau)$ and is given by

$$p(\mathbf{r}_i) = \int_0^{+\infty} p(\mathbf{r}_i/\tau)g(\tau)\,\mathrm{d}\tau. \tag{30}$$

Replacing $\hat{p}_{N_{\mathrm{ref}}}(\tau)$ in (5), the expression of the so-called Bayesian Optimum Radar Detector can be derived.

Other methods are investigated in [7,15] to attain detector adaptivity but only with respect to parameters of $p(\tau)$. The proposed method makes the detector adaptive with respect to the unknown $p(\tau)$ and it is no more necessary to have knowledge about the clutter statistics.

4.2. BORD expression

The $N_{\rm ref}$ reference clutter cells $[\mathbf{r}_1,\ldots,\mathbf{r}_{N_{\rm ref}}]^{\rm T}$ are supposed to be modeled as SIRV and so we have

$$p(\mathbf{r}_i/\tau)=\frac{\tau^{-m}}{(2\pi)^m|\mathbf{M}|}\exp\left(-\frac{Z_i}{2\tau}\right), \tag{31}$$

where $Z_i=\mathbf{r}_i^\dagger\mathbf{M}^{-1}\mathbf{r}_i$. If we consider this likelihood function as a function of τ it is proportional to an inverse gamma PDF, $\mathscr{IG}(m+1,2/Z_i)$ (an inverse gamma PDF is the PDF of the inverse of a Gamma-distributed variable). In (28) a prior distribution is required for the reference clutter *texture* and in this case we could choose an inverse gamma prior, $\mathscr{IG}(a_p,b_p)$ (parameters a_p and b_p would have to be chosen in order to make the prior as non-restrictive as possible). This prior would be the so-called *conjugate prior* because the resulting PDF would belong to the same PDF family as the likelihood function (see [14,16] for more details).

As the clutter statistics is unknown we choose a non-informative prior distribution $g(\tau)=1/\tau$, called Jeffrey's prior which is proportional to the square root of the Fischer's information measure and which is also an asymptotical case of the inverse gamma PDF when parameters $a_p\to 0$ and $b_p\to+\infty$.

With the non-informative prior, the a posteriori PDF of τ given the $N_{\rm ref}$ reference cells can be derived and (28) becomes

$$p(\tau/\mathbf{r}_i)=\frac{\tau^{-m-1}}{(2\pi)^m|\mathbf{M}|\,p(\mathbf{r}_i)}\exp\left(-\frac{Z_i}{2\tau}\right). \tag{32}$$

The normalization constant $p(\mathbf{r}_i)$ is computed as follows:

$$\begin{aligned}p(\mathbf{r}_i)&=\int_0^{+\infty}p(\mathbf{r}_i/\tau)g(\tau)\,\mathrm{d}\tau\\&=\int_0^{+\infty}\frac{\tau^{-m-1}}{(2\pi)^m|\mathbf{M}|}\exp\left(-\frac{Z_i}{2\tau}\right)\mathrm{d}\tau\\&=\frac{\Gamma(m)}{\pi^m|\mathbf{M}|Z_i^m},\end{aligned} \tag{33}$$

and (32) becomes

$$p(\tau/\mathbf{r}_i)=\frac{Z_i^m}{2^m\Gamma(m)}\tau^{-m-1}\exp\left(-\frac{Z_i}{2\tau}\right). \tag{34}$$

This expression is exactly an inverse gamma PDF with parameters m and $2/Z_i$. So, we have

$$\hat{p}_{N_{\rm ref}}(\tau)=\frac{\tau^{-m-1}}{2^m\Gamma(m)N_{\rm ref}}\sum_{i=1}^{N_{\rm ref}}Z_i^m\exp\left(-\frac{Z_i}{2\tau}\right), \tag{35}$$

where we recall that $Z_i=\mathbf{r}_i^\dagger\mathbf{M}^{-1}\mathbf{r}_i$. Replacing (35) in (4) for each observed vector $\mathbf{y}_{\rm obs}$ (size m) and given the $N_{\rm ref}$ reference clutter vectors $\mathbf{r}_{i=1}^{N_{\rm ref}}$ we have to compute the following expression under H_j $(j=0,1)$ to form the GLRT:

$$\begin{aligned}&A_m^{N_{\rm ref}}\sum_{i=1}^{N_{\rm ref}}Z_i^m\int_0^{+\infty}\tau^{-2m-1}\exp\left(-\frac{W_{i,j}(\mathbf{y}_{\rm obs})}{2\tau}\right)\mathrm{d}\tau\\&=2^{2m}A_m^{N_{\rm ref}}\Gamma(2m)\sum_{i=1}^{N_{\rm ref}}\frac{Z_i^m}{(W_{i,j}(\mathbf{y}_{\rm obs}))^{2m}},\end{aligned} \tag{36}$$

where Z_i is given previously and

$$(A_m^{N_{\rm ref}})^{-1}=(2\pi)^m|\mathbf{M}|N_{\rm ref}2^m\Gamma(m),$$

$$W_{i,j}(\mathbf{y}_{\rm obs})=q_j(\mathbf{y}_{\rm obs})+Z_i.$$

The so-called BORD expression becomes

$$\Lambda_{N_{\rm ref}}(\mathbf{y}_{\rm obs})=\frac{\sum_{i=1}^{N_{\rm ref}}\left[\frac{Z_i}{(q_1(\mathbf{y}_{\rm obs})+Z_i)^2}\right]^m}{\sum_{i=1}^{N_{\rm ref}}\left[\frac{Z_i}{(q_0(\mathbf{y}_{\rm obs})+Z_i)^2}\right]^m}\underset{\rm H_0}{\overset{\rm H_1}{\gtrless}}\lambda. \tag{37}$$

Both of the quadratic forms q_0 and q_1 are, respectively, given by (6) and (8).

BORD expression depends only on the reference clutter cells which provide all the necessary information about the clutter statistics. That makes itself “self-adaptive” if the correlation matrix is determined from the reference cells of the clutter. This problem was investigated in [5,7] where the authors use the normalized sample covariance matrix estimator (NSCM). Given that $\mathbf{M}=\mathbf{M}_r/\mathbb{E}(\tau)=\mathbb{E}(\mathbf{r}\mathbf{r}^\dagger)/\mathbb{E}(\mathbf{r}^\dagger\mathbf{r})$, the NSCM estimate of $\mathbf{M}$ is given by

$$\hat{\mathbf{M}}=\frac{m}{N_{\rm ref}}\sum_{k=1}^{N_{\rm ref}}\frac{\mathbf{r}_k\mathbf{r}_k^\dagger}{\mathbf{r}_k^\dagger\mathbf{r}_k}, \tag{38}$$

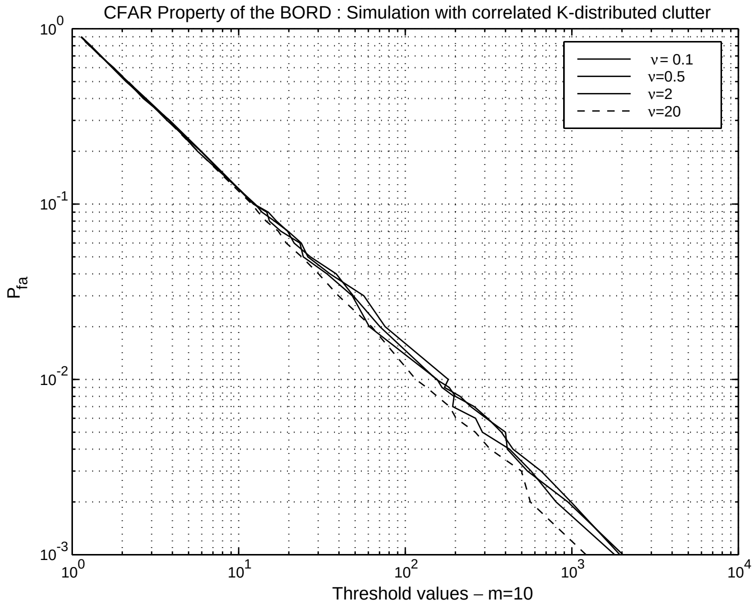

Fig. 1. CFAR property of the BORD with respect to the *texture* PDF shown for correlated K-distributed environments with parameters $v=0.1,\ 0.5,\ 2,\ 20$ for $m=10$.

where $\mathbf{r}_k$ $(k=1,\ldots,N_{\text{ref}})$ are the N_{ref} reference clutter cells of size m.

4.3. CFAR property of the BORD

We can show by simulations that the BORD is CFAR (Constant False Alarm Rate) with respect to the *texture* PDF. It means that whatever the clutter statistics is (dually whatever the *texture* PDF is), BORD probability of false alarm is the same. In other words the BORD law is independent of the clutter statistics. This property is shown in Figs. 1 and 2 where the detection threshold η is plotted against different P_{fa} values. What ever the clutter statistics is (K-distributed with various parameters, Gaussian, Student-t or whose *texture* PDF is a Weibull PDF), the calculation of the detection threshold for a fixed P_{fa} is the same for all.

4.4. Asymptotical result of the BORD: AsBORD

BORD expression comes after a Monte Carlo estimation of (25) given N_{ref} reference clutter vector $\mathbf{r}_{i=1}^{N_{\text{ref}}}$. Given $Z_i=\mathbf{r}_i^{\dagger}\mathbf{M}^{-1}\mathbf{r}_i$, which is a positive r.v. with PDF $p(Z)$, BORD expression can be considered as the Monte Carlo estimation of

$$\frac{\int_0^{+\infty}(Z^m/(q_1(\mathbf{y}_{\text{obs}})+Z)^{2m})\,p(Z)\,\mathrm{d}Z}{\int_0^{+\infty}(Z^m/(q_0(\mathbf{y}_{\text{obs}})+Z)^{2m})\,p(Z)\,\mathrm{d}Z}. \tag{39}$$

Given that $\mathbf{r}=\sqrt{\tau}\mathbf{x}$ where $\mathbf{x}$ is a complex Gaussian vector of size m with covariance matrix $2\mathbf{M}$, we have $Z=\mathbf{r}^{\dagger}\mathbf{M}^{-1}\mathbf{r}=\tau\mathbf{x}^{\dagger}\mathbf{M}^{-1}\mathbf{x}$.

The quadratic form $Q=\mathbf{x}^{\dagger}\mathbf{M}^{-1}\mathbf{x}$ is χ^2_{2m} distributed $(\chi^2_{2m}=\mathscr{G}(m,2))$.

So, Z/τ is $\mathscr{G}(m,2\tau)$ and the PDF of Z is derived by integrating $p(Z/\tau)$ over the prior $g(\tau)$. Finally, we can show that

$$\lim_{N_{\text{ref}}\to+\infty}\Lambda_{N_{\text{ref}}}(\mathbf{y}_{\text{obs}})=\left(\frac{q_0(\mathbf{y}_{\text{obs}})}{q_1\mathbf{y}_{obs})}\right)^m. \tag{40}$$

This asymptotical result, called asymptotic BORD (AsBORD), coincides with the GLRT given, for example, in [15]. It was obtained after replacing the *texture* τ by its two maximum likelihood estimates (the one under H_0 and the other under H_1) in the optimum detection structure (5), where τ was considered

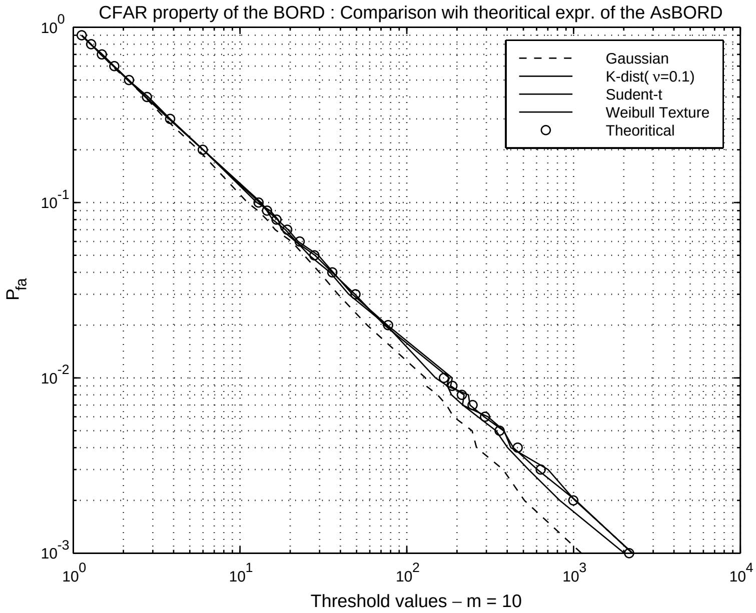

Fig. 2. CFAR property of the BORD with respect to the *texture* PDF shown for different correlated environments: K-distributed clutter with parameter $v = 0.1$, Gaussian clutter, Student-t clutter and unknown clutter whose *texture* PDF is a Weibull PDF. $m = 10$. Comparison with the theoretical expression of the AsBORD threshold.

as an unknown deterministic parameter. This was also obtained after different calculations in [2] and for Gaussian clutter in [11,12]. Moreover, with the NSCM estimate of the correlation matrix this expression is also called adaptive linear quadratic (ALQ) in [5,6].

4.5. CFAR property of the AsBORD

Replacing both of the quadratic forms q_0 and q_1 in (40) and after simple modification, the AsBORD expression can be written as follows:

$$\frac{|\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{x}_{\rm obs}|^2}{(\mathbf{p}^\dagger \mathbf{M}^{-1}\mathbf{p})(\mathbf{x}_{\rm obs}^\dagger \mathbf{M}^{-1}\mathbf{x}_{\rm obs})} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\sqrt[m]{\eta}-1}{\sqrt[m]{\eta}}, \tag{41}$$

where $\mathbf{x}_{\rm obs}$ is the (Gaussian) *speckle* of the observed SIRV.

The left term of (41) is independent of the *texture*. Then AsBORD PDF is statistically independent of the *texture* PDF that makes the AsBORD to be CFAR with respect of the *texture* PDF. In this case it is possible to derive the AsBORD PDF.

4.6. AsBORD PDF

According to (41) the AsBORD expression depends on Gaussian vectors and on a related quadratic form.

Extending the Cochran's theorem to complex Gaussian vectors and after simple computation, the AsBORD PDF is a Beta PDF with parameters 1 and $m-1$ whose expression is [9]

$$p(u) = (m-1)(1-u)^{m-2}. \tag{42}$$

Then the expression of the threshold value depends only on $P_{\rm fa}$ and m, the size of the observed vector [9]:

$$\eta = P_{\rm fa}^{(m/1-m)}. \tag{43}$$

Applying Cochran's theorem implies that the covariance matrix of the observed vector is non-singular. Under this condition AsBORD PDF applies also to the BORD as shown in Fig. 2.

In [9] the AsBORD PDF in the case of real vectors is also derived.

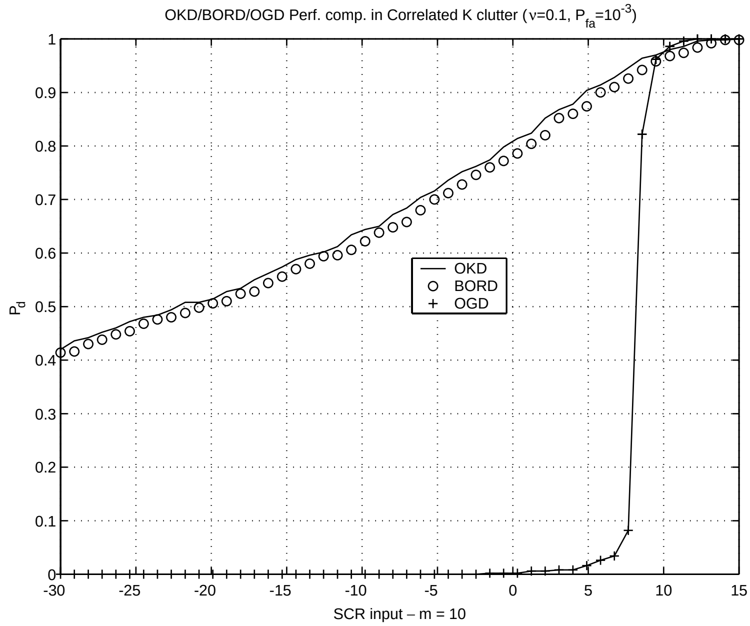

Fig. 3. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($v = 0.1, P_{fa} = 10^{-3},\ m = 10$).

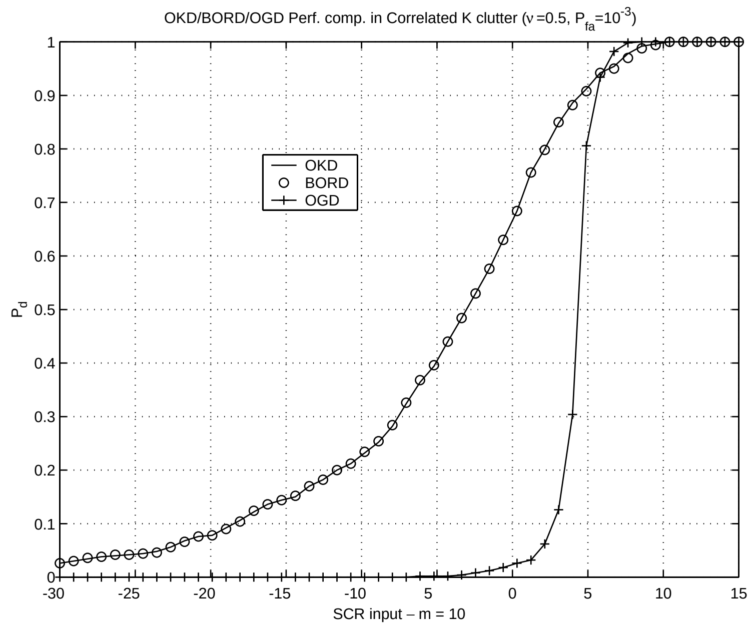

Fig. 4. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($v = 0.5,\ P_{fa} = 10^{-3},\ m = 10$).

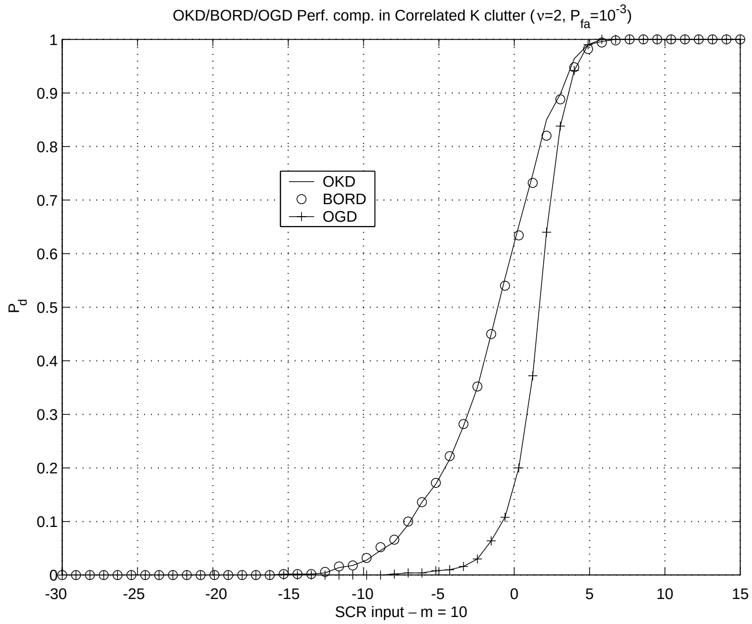

Fig. 5. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($\nu = 2$, $P_{\text{fa}} = 10^{-3}$, $m = 10$).

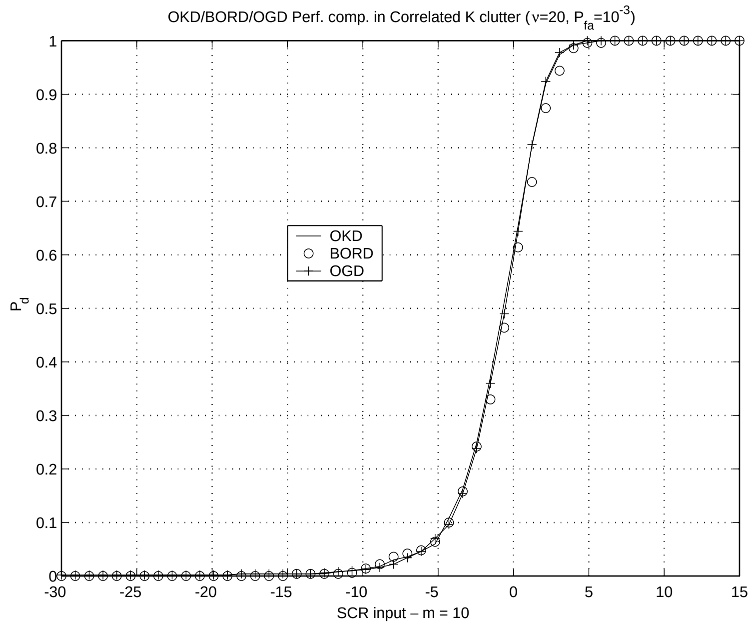

Fig. 6. Performance comparison between OGD, OKD and BORD for an unknown target complex amplitude in correlated K-distributed clutter ($\nu = 20$, $P_{\text{fa}} = 10^{-3}$, $m = 10$).

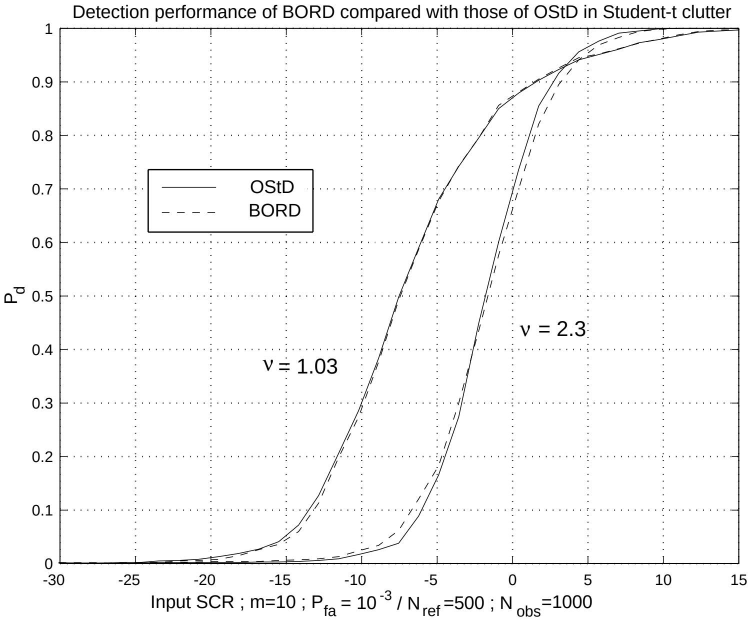

Fig. 7. Performance comparison between OStD and BORD for an unknown target complex amplitude in correlated Student-t clutter ($v = 1.03, 2.3$, $P_{fa} = 10^{-3}$, $m = 10$).

5. Detection performances of BORD

The correlation matrix **M** is considered to be known. **M** values come from a given Gaussian power spectral density and is an $m \times m$ Toeplitz matrix whose first auto-correlation coefficient is $\rho_1 = 0.0098$.

We compare BORD performance with those of optimum detectors such OKD, OGD (optimum Gaussian detector, optimum for Gaussian clutter) or OStD for an unknown target signal embedded in K-distributed clutter or Student-t clutter.

In the case of K-distributed clutter, OKD is optimum and we see that BORD performance reach OKD performance whatever the value of v is. Different values of the shape parameter are tested, $v = 0.1, 0.5, 2, 20$. When $v \to +\infty$ K-PDF tends to a Gaussian PDF which is confirmed in the series of Figs. 3–6.

In the case of Student-t clutter (shown in Fig. 7) the conclusions are the same: BORD reach optimum performances given by those of the optimum detector OStD.

All the curves represent the detection probability P_d versus the signal-to-clutter ratio (SCR) given for one pulse. As $m = 10$ pulses are considered, the total SCR is $10 \log_{10}(m) = 10$ dB more than for one pulse. The detection threshold is previously computed via Monte Carlo simulation for each of the detectors to keep a false alarm rate equal to $P_{fa} = 10^{-3}$.

Once the detection threshold is obtained we evaluate performance with $N_{ref} = 1000$ samples of reference clutter and for $N_{obs} = 500$ samples of observed clutter data (which corresponds in fact to the number of Monte Carlo trials used for the evaluation of the probability of detection) in which an unknown complex target signal for a different SCR is embedded.

6. Conclusions and outlooks

The present paper has addressed a bayesian approach to the determination of the clutter statistics if we consider the clutter vector modeled as a SIRV. By this way a bayesian estimator of the *texture* PDF of the SIRV has been derived from reference clutter cells and the resulting BORD expression depends only on these reference cells. For example, in the case of

CFAR (Constant False Alarm Rate) detector th reference clutter cells are the cells adjacent to the cell under test.

We derive also the asymptotic expression of BORD, called AsBORD, whose PDF is derived in a closed form. With this latter expression we are able to compute the threshold value to set to verify a desired P_{fa} in the case where the correlation matrix is non-singular. Under this assumption, the theoretical threshold expression applies also to BORD. In this paper, we use Monte Carlo simulation to set the threshold value of BORD. In further work we will compare the performances of BORD and AsBORD for experimental ground clutter data and study the applicability of the theoretical threshold expression with respect to the correlation of the data.

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