Off-grid Radar Target Detection with the Normalized Matched Filter: a Monopulse-Based Detection Scheme

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- In real conditions, there is no reason to have $\ = {}_0$. We have mismatch : $\ \neq {}_0$, and performance derived under on-grid model is not met.
- This motivates the search of a robust detection scheme.

The classical Radar detection problem is the following binary Hypothesis Test:

$$\begin{cases} H_0 : r = n \\ H_1 : r = \quad s(\quad) + n \end{cases} ; \text{ where}$$

- r *2* C^N is the observation,
- $s(\quad)$ *2* C^N is the signal echo reflected by a target with parameters (range, angle, Doppler...),
- *2* C is the complex amplitude of the received signal,
- n *2* C^N is the additive noise vector, independent of the source signal. $n \quad \mathcal{CN}(0; \quad ^2 \quad)$.

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- $\mathbf{r} \in \mathbb{C}^N$ is the observation,
- $\mathbf{s}(\theta) \in \mathbb{C}^N$ is the signal echo reflected by a target with parameters θ (range, angle, Doppler...),
- $\alpha \in \mathbb{C}$ is the complex amplitude of the received signal,
- $\mathbf{n} \in \mathbb{C}^N$ is the additive noise vector, independent of the source signal. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$.

Here the signal $\mathbf{s}(\theta)$ follows the general spectral analysis model (angle or Doppler shift in Radar):

$$\mathbf{s}(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{2i\pi\theta}, \ldots, e^{2i\pi(N-1)\theta} \right]^T.$$

with $\theta = 1/N$: grid vectors are orthogonal.

The GLRT is:

$$(r) = \frac{\max_{1} f_{H_1}(r)}{\max_{0} f_{H_0}(r)} \underset{H_0}{\overset{H_1}{\gtrless}} \quad :$$

where

- for $i \in \{0;1\}$, f_{H_i} is the density function of r under H_i and $_i$ are the unknown parameters under H_i,
- guarantees a fixed Probability of False Alarm (PFA).

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When $_1 = f\; ;\quad g$ and $_0 = f\quad g$ with known, the GLRT is the following Normalized Matched Filter (NMF) [Scharf and Lytle, 1971]:

$$t\;(r;\;) = \frac{\left| s(\;)^H\;{}^{-1} r \right|^2}{s(\;)^H\;{}^{-1} s(\;)\quad r^H\;{}^{-1} r} \underset{H_0}{\overset{H_1}{?}} \;:$$

- Mismatch = - $_0$
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- Angle mismatch creates a degradation of the NMF response even without noise
- When uniformly distributed in a cell it can be shownP_D 9 1 [Rabaste et al., 2016]
- Even worse when 6= I

- Extension of the GLRT to off-grid targets:

$$\mathrm{GLRT}(\mathbf{r};\theta_0) = \max_{\theta_c \in [\theta_0 - \Delta/2;\, \theta_0 + \Delta/2]} t(\mathbf{r};\theta_c) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma.$$

The best P_D, no closed form available, threshold unknown, precise approximation can be costly.

- Extension of the GLRT to o -grid targets:

$$\mathrm{GLRT}(r;\ _0) = \max_{c\, 2\, [\ _0 -\ =2;\ _0 +\ =2\ ]} t\ (r;\ _c) \underset{H_0}{\overset{H_1}{?}}\ :$$

  The best P_D, no closed form available, threshold unknown, precise approximation can be costly.
- Existing sub-optimal cost-e cient solutions include
  - Oversampling approximate GLRT, threshold unknown
  - Using DPSS subspace to approximate the cell structure, analytical threshold [Bosse and Rabaste, 2018]
  - Detection with bounded mismatch, not yet suited to low PFA Radar context [Besson, 2006]

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- These solutions do not correct the convergence issue for all and are not always near GLRT.

- Monopulse traditionally used to estimate target parameters from a single pulse [Mosca, 1969].
- The idea is to combine two tests in a function h that carries info about .
- Used with noise, h can give an approximation ^

- Classically in monopulse, the function h is:

$$h_{;\,0}(r) = \frac{t\quad r;\ _0 - \frac{}{2} \quad - \ t\quad r;\ _0 + \frac{}{2}}{t\quad r;\ _0 - \frac{}{2} \quad + \ t\quad r;\ _0 + \frac{}{2}}:$$

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- mismatch: $\ = \ \ - \ \ {}_0$, noise-free function g:

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- Goal: compute $\hat{\ }$ by inverting $g(\ )$ thanks to h applied on noisy signal r.

$$(\ ) = T \left[\ 1 \quad \ldots \quad {}^{N-1} \right]$$

- $g_{;0}$ needs to be invertible. This is not always the case.

Candidate g(:) functions for N=10, $_0 = 0$.

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- We use g_l in the following even with colored Gaussian noise. We note it g.

Candidate g(:) functions for N=10, $_0 = 0$.

- The test procedure is the following, for every γ of the grid:

The test procedure

1. compute $t_l \; r; \; _0 - \frac{}{2}$ and $t_l \; r; \; _0 + \frac{}{2}$;
2. compute $\hat{} = g^{-1}(h_{l; \; 0}(r))$;
3. run the nal tests $t \;\; r; \hat{} + \; _0 \underset{H_0}{\overset{H_1}{?}} \; g$.

- The test procedure is the following, for every $_g$ of the grid:

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3. run the nal tests $t \quad r; \hat{} + \;_0 \; \underset{H_0}{\overset{H_1}{?}} \;_g$.

- the statistic of $t \quad r; \hat{} + \;_0$ depends on the non-independent random variables and $\hat{}$ =) no closed form available for $_g$
- $_g$ is approximated with Monte Carlo simulations

Let us describe some properties of this approach:

- Only 2N tests are run for the whole spectral space, and the rest of the computations are simply lookup table operations,
- When the SNR tends to infinity, ^ = and the Probability of Detection (PD) tends to 1,
- When = I, ^ is an approximate MLE and our test is an approximate GLRT [Mosca, 1969],
- When $\neq$ I, our test is still close to the GLRT in term of performance.

We compare our scheme to detectors of similar cost:

- An oversampled NMF with 2 tests per cell,
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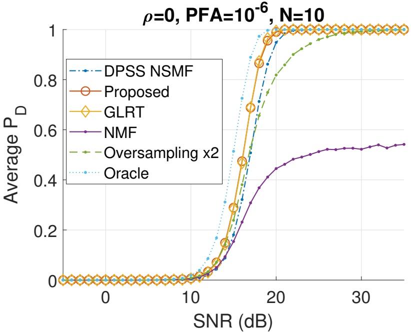
We also compare it to :

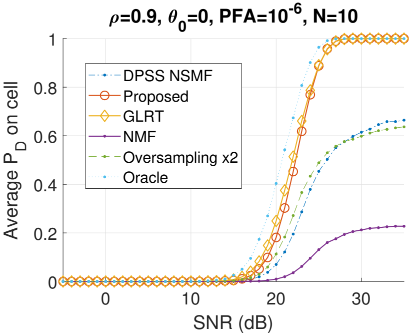
- The classical NMF
- An approximate GLRT using 50 tests per cell
- The Oracle detector, which knows where the target is and as such is the best detector possible

- Target parameter drawn at random uniformly.
- Our detector converges to 1 asymptotically and outperforms other detectors in the same computational range.



P_D of the detectors under white noise, for a P_{FA} of 10^{-6}, $N = 10$.

- Target parameter drawn at random uniformly in [$_0$ - =2; $_0$ + =2].
- In this cell, our detector stays close to the GLRT and does greatly better than the other detectors, which do not converge to 1.



P_D of the detectors with = 0:9, $_0$ = 0 for a P_{FA} of 10^{-6}, N = 10.

We introduced a new detector that approximates GLRT under white noise for off-grid targets.

- simply based on the well-known monopulse procedure, classically used in array processing.

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- simply based on the well-known monopulse procedure, classically used in array processing.
- performance is close to GLRT while being cost-efficient
- Future works will investigate the performance of our detector under adaptive context, other noise models, and PFA-threshold relationship.

Thank You For Listening !

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