Off-grid Radar Target Detection with the Normalized Matched Filter: a Monopulse-Based Detection Scheme

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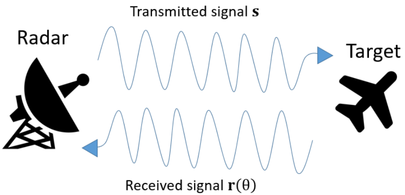
Context: The Radar detection problem

- Primary goal of Radar systems: detect targets.
- Emit signal, and search for echoes in received signal.



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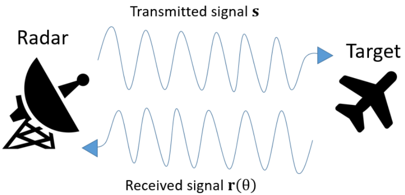

- Received signal depends on unknown target parameters θ.
- For practical reasons, tests are run for fixed values of parameters θ_0 in a Grid $G = \{k\Delta, k \in [0\,..\,N-1]\}$, with N the number of samples and Δ the sampling interval. Cell : $[\theta_0 - \Delta/2, \theta_0 + \Delta/2]$.

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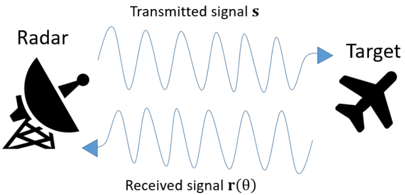

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- In real conditions, there is no reason to have $\theta = \theta_0$. We have mismatch : $\theta \neq \theta_0$, and performance derived under on-grid model is not met.
- This motivates the search of a robust detection scheme.

Contents

The Radar detection problem

The classical Radar detection problem is the following binary Hypothesis Test:

$$\begin{cases} H_0 : \mathbf{r} = \mathbf{n} \\ H_1 : \mathbf{r} = \alpha\, \mathbf{s}(\theta) + \mathbf{n} \end{cases}, \text{ where}$$

- $\mathbf{r} \in \mathbb{C}^N$ is the observation,
- $\mathbf{s}(\theta) \in \mathbb{C}^N$ is the signal echo reflected by a target with parameters θ (range, angle, Doppler...),
- $\alpha \in \mathbb{C}$ is the complex amplitude of the received signal,
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Here the signal $\mathbf{s}(\theta)$ follows the general spectral analysis model (angle or Doppler shift in Radar):

$$\mathbf{s}(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{2i\pi\theta}, \dots, e^{2i\pi(N-1)\theta}\right]^T .$$

with $\Delta = 1/N$: grid vectors are orthogonal.

The Generalized Likelihood Ratio Test

The GLRT is:

$$\Lambda(\mathbf{r}) = \frac{\max\limits_{\boldsymbol{\lambda}_1} f_{H_1}(\mathbf{r})}{\max\limits_{\boldsymbol{\lambda}_0} f_{H_0}(\mathbf{r})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

where

- for $i \in \{0, 1\}$, f_{H_i} is the density function of $\mathbf{r}$ under H_i and λ_i are the unknown parameters under H_i,
- η guarantees a fixed Probability of False Alarm (PFA).

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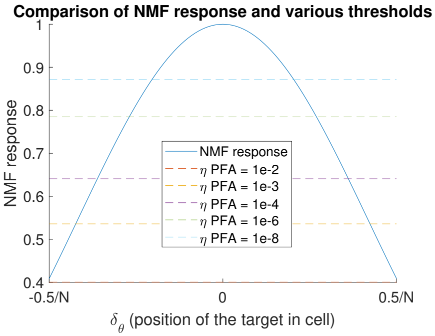
When $\lambda_1 = \{\sigma, \alpha\}$ and $\lambda_0 = \{\sigma\}$, with θ known, the GLRT is the following Normalized Matched Filter (NMF) [Scharf and Lytle, 1971]:

$$t_{\Gamma}(\mathbf{r}, \theta) = \frac{\left|\mathbf{s}(\theta)^H \boldsymbol{\Gamma}^{-1} \mathbf{r}\right|^2}{\left(\mathbf{s}(\theta)^H \boldsymbol{\Gamma}^{-1} \mathbf{s}(\theta)\right) \left(\mathbf{r}^H \boldsymbol{\Gamma}^{-1} \mathbf{r}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

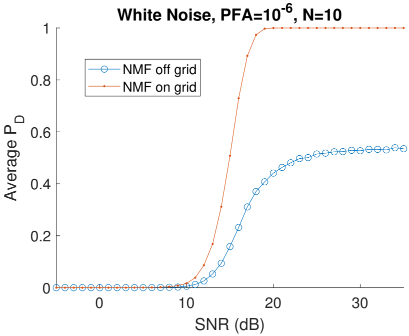
Impact of the off-grid target on NMF

- Mismatch $\delta = \theta - \theta_0$
- Angle mismatch creates a degradation of the NMF response even without noise



Impact of the off-grid target on NMF

- Mismatch $\delta = \theta - \theta_0$
- Angle mismatch creates a degradation of the NMF response even without noise
- When θ uniformly distributed in a cell it can be shown $P_D \nrightarrow 1$ [Rabaste et al., 2016]
- Even worse when $\boldsymbol{\Gamma} \neq \mathbf{I}$



Existing Solutions

- Extension of the GLRT to off-grid targets:

$$\mathrm{GLRT}(\mathbf{r}, \theta_0) = \max_{\theta_c \in [\theta_0 - \Delta/2, \theta_0 + \Delta/2]} t_\Gamma(\mathbf{r}, \theta_c) \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

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- Existing sub-optimal cost-efficient solutions include
  - Oversampling approximate GLRT, threshold unknown
  - Using DPSS subspace to approximate the cell structure, analytical threshold [Bosse and Rabaste, 2018]
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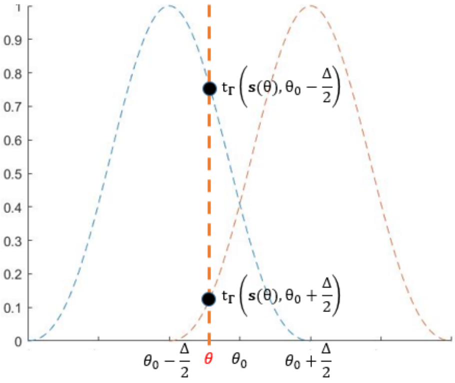
- These solutions do not correct the convergence issue for all Γ and are not always near GLRT.

Proposed Monopulse Inspired Scheme

- Monopulse traditionally used to estimate target parameters from a single pulse [Mosca, 1969].
- The idea is to combine two tests in a function h that carries info about θ.
- Used with noise, h can give an approximation $\hat{\delta}$



Monopulse Functions

- Classically in monopulse, the function h is:

$$h_{\Gamma,\theta_0}(\mathbf{r}) = \frac{t_\Gamma\left(\mathbf{r}, \theta_0 - \frac{\Delta}{2}\right) - t_\Gamma\left(\mathbf{r}, \theta_0 + \frac{\Delta}{2}\right)}{t_\Gamma\left(\mathbf{r}, \theta_0 - \frac{\Delta}{2}\right) + t_\Gamma\left(\mathbf{r}, \theta_0 + \frac{\Delta}{2}\right)}.$$

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- mismatch: $\delta = \theta - \theta_0$, noise-free function g:

$$g_{\Gamma,\theta_0}(\delta) = h_{\Gamma,\theta_0}(\mathbf{s}(\theta_0 + \delta))$$

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- mismatch: $\delta = \theta - \theta_0$, noise-free function g:

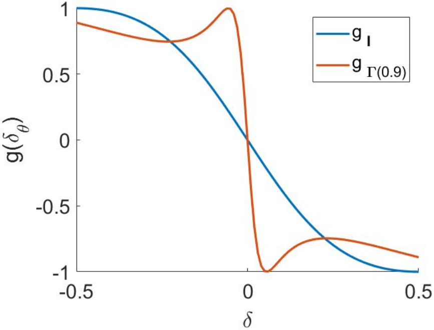
$$g_{\Gamma,\theta_0}(\delta) = h_{\Gamma,\theta_0}(\mathbf{s}(\theta_0 + \delta))$$

- Goal: compute $\hat{\delta}$ by inverting $g(\delta)$ thanks to h applied on noisy signal $\mathbf{r}$.

Choice of g

$$\Gamma(\rho) = \mathcal{T}\left(\begin{bmatrix}1 & \rho & \dots \rho^{N-1}\end{bmatrix}\right)$$

- g_{Γ,θ_0} needs to be invertible. This is not always the case.

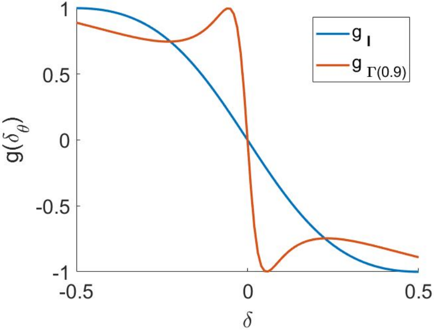


Candidate $g(.)$ functions for N=10, $\theta_0 = 0$.

Choice of g

$$\Gamma(\rho) = \mathcal{T}\left(\left[1 \quad \rho \quad \ldots \rho^{N-1}\right]\right)$$

- g_{Γ,θ_0} needs to be invertible. This is not always the case.
- We use $g_{\mathbf{I}}$ in the following even with colored Gaussian noise. We note it g.



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The Procedure

- The test procedure is the following, for every θ_0 of the grid:

The test procedure

1. compute $t_{\mathbf{I}}\left(\mathbf{r}, \theta_0 - \frac{\Delta}{2}\right)$ and $t_{\mathbf{I}}\left(\mathbf{r}, \theta_0 + \frac{\Delta}{2}\right)$;
2. compute $\hat{\delta} = g^{-1}\left(h_{\mathbf{I},\theta_0}(\mathbf{r})\right)$;
3. run the final tests $t_{\Gamma}\left(\mathbf{r}, \hat{\delta} + \theta_0\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta_g$.

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- the statistic of $t_{\mathbf{\Gamma}}\left(\mathbf{r}, \hat{\delta} + \theta_0\right)$ depends on the non-independent random variables $\mathbf{r}$ and $\hat{\delta} \implies$ no closed form available for η_g
- η_g is approximated with Monte Carlo simulations

Properties of this approach

Let us describe some properties of this approach:

- Only $2N$ tests are run for the whole spectral space, and the rest of the computations are simply lookup table operations,
- When the SNR tends to infinity, $\widehat{\theta} = \theta$ and the Probability of Detection (PD) tends to 1,
- When $\mathbf{\Gamma} = \mathbf{I}$, $\widehat{\theta}$ is an approximate MLE and our test is an approximate GLRT [Mosca, 1969],
- When $\mathbf{\Gamma} \neq \mathbf{I}$, our test is still close to the GLRT in term of performance.

Detector Comparison

We compare our scheme to detectors of similar cost:

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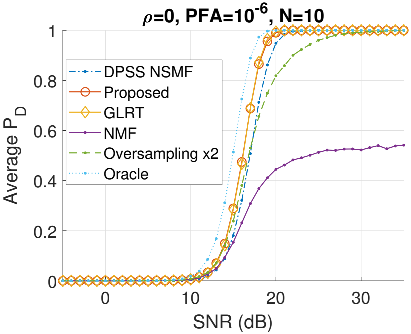
We also compare it to :

- The classical NMF
- An approximate GLRT using 50 tests per cell
- The Oracle detector, which knows where the target is and as such is the best detector possible

Simulation under white noise

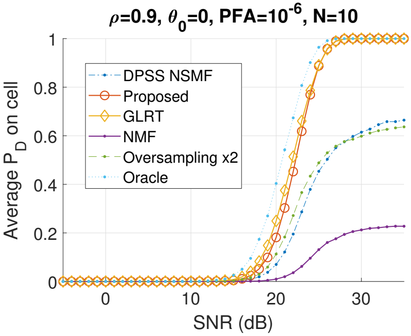
- Target parameter θ drawn at random uniformly.
- Our detector converges to 1 asymptotically and outperforms other detectors in the same computational range.



P_D of the detectors under white noise, for a P_{FA} of 10^{-6},$N = 10$.

Simulation under colored noise

- Target parameter θ drawn at random uniformly in $[\theta_0 - \Delta/2, \theta_0 + \Delta/2]$.
- In this cell, our detector stays close to the GLRT and does greatly better than the other detectors, which do not converge to 1.



P_D of the detectors with $\rho = 0.9$, $\theta_0 = 0$ for a P_{FA} of 10^{-6},$N = 10$.

Conclusions

We introduced a new detector that approximates GLRT under white noise for off-grid targets.

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- simply based on the well-known monopulse procedure, classically used in array processing.
- performance is close to GLRT while being cost-efficient
- Future works will investigate the performance of our detector under adaptive context, other noise models, and PFA-threshold relationship.

The End

Thank You For Listening !

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