

SPATIO-TEMPORAL ADAPTIVE DETECTOR IN NON-HOMOGENEOUS AND LOW-RANK CLUTTER

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ABSTRACT

Reducing the number of secondary data used to estimate the Clutter Covariance Matrix (CCM) for Space Time Adaptive Processing (STAP) techniques is still an active research topic. Low rank CCM estimates have already been proposed but only for homogeneous and Gaussian clutter. We propose in this paper to extend the low-rank CCM methods for heterogeneous and/or non-Gaussian clutter. We derive a new detector based on low-rank techniques and exploiting properties of the Normalized Sample Covariance Matrix (NSCM). This detector is shown to exhibit a smaller SNR loss than classical STAP detectors. Moreover, the new detector has a texture-CFAR property with respect to non-Gaussian SIRV model and has more robust behavior when some targets are present in the secondary data. We also give experimental comparison results between the classical STAP detectors and the new one for STAP data.

Index Terms— STAP, non-homogeneous and low rank clutter, CFAR detector, Normalized Sample Covariance Matrix.

1. INTRODUCTION

Space Time Adaptive Processing (STAP) is a recent technique [1] used in airborne phased array radar to detect moving target embedded in an interference background such as jamming or strong clutter. While conventional radars are capable of detecting targets both in the time domain related to target range and in the frequency domain related to target velocity, STAP uses an additional domain (space) related to the target angular localization. The consequence is a two-dimensional adaptive filtering technique which uses jointly temporal and spatial dimensions to suppress interference and to improve target detection. Recently, STAP can be jointly used with High Resolution Synthetic Aperture Radar (SAR) imaging waveforms for a better classification of the moving target. But in this case, the widely used hypothesis of a Gaussian noise is not valid anymore and detection performance

significantly decreases. For that purpose, non-Gaussian models for the clutter have to be considered. In the literature of radar detection and estimation, the Spherically Invariant Random Vectors (SIRV) [2] are generally used for their statistical properties and for their good fitting with real data [3]. This modeling includes classical distributions as for example the Gaussian distribution, the K-distribution or the Weibull distribution.

One of the main challenging problem in STAP detection is to estimate the Clutter Covariance Matrix (CCM); better the accuracy of the estimate, better the detection performance. The CCM is estimated from signal-free and independent data, called the secondary data. Under non-Gaussian clutter assumptions, this raises several difficulties. First, the heterogeneity of the clutter deteriorates the CCM estimation accuracy. Indeed, if the secondary data do not result from the same parametrized distribution, i.e. the same covariance matrix, this estimation process makes no sense. On the other hand, the secondary data may contain parts of the target (e.g. sidelobes) present in the range cell under study, even if this problem can be partly avoided with guard cells. These two problems show the necessity of a robust CCM estimator and, as a consequence, the robustness of the STAP detector. This is the purpose of this paper.

In this paper, to overcome the problem of clutter heterogeneity, we propose to use the Normalized Sample Covariance Matrix (NSCM) [4] which is invariant to the clutter power variations (texture), while for the problem of target contamination, we propose a Low Rank (LR) [5] approach which requires fewer secondary data for the CCM estimation. The combination of these two techniques leads to a new improved detector, called the Low Rank-Normalized Sample Covariance Matrix Test (LR-NSCMT). Then, the statistical analysis of this detector provides the Constant False Alarm Rate (CFAR) properties with respect to the texture and, asymptotically, in term of the CCM. Moreover, the LR-NSCMT is applied to realistic STAP data (built from a true Very High Resolution SAR image) and its robustness to target contamination is validated.

The paper is organized as follows: Section 2 gives the

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problem statement while Sections 3 and 4 contain the main contributions. First, we derive the LR-NSCMT and we analyze its statistical properties and then, the detector is applied to real STAP data. Finally, Section 5 concludes this work.

2. PROBLEM STATEMENT

The problem of STAP [1] is considered when an airborne radar is used to detect a moving target. We focus here without loss of generality on GMTI (Ground Target Moving Indicator) applications for which illumination is made across the flying path. Typically, the radar receiver consists of an array of N antenna elements collecting M pulses in a coherent processing interval. We are interested in detecting a complex signal \mathbf{s} corrupted by an additive SIRV clutter \mathbf{c} in a NM -dimensional complex vector \mathbf{y} . The complex signal is parameterized by the target characteristics: the target amplitude α_0 , the normalized Doppler frequency f_0 and the azimuthal angle θ_0 of the target. For a given target range bin under test, the detection scheme can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \dots, N_s \\ H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \dots, N_s \end{cases}, \quad (1)$$

where \mathbf{y}_i are the N_s signal-free independent measurements, i.e. the secondary data, used to estimate the CCM.

Under the hypothesis H_1 , it is assumed that the observed data consists in the sum of a signal $\mathbf{s} = \alpha_0 \mathbf{p}(\theta_0, f_0)$ and clutter \mathbf{c} , where $\mathbf{p}(\theta_0, f_0)$ is the classical complex steering vector and α_0 is the signal complex amplitude. The parameters f_0 , θ_0 and α_0 are unknown.

The clutter is modeled in this paper as a SIRV, a non-homogeneous Gaussian process with random power: its randomness is induced by spatial variation in the radar backscattering. A SIRV [2] is the product of the square root of a positive random variable τ (called the texture) with unknown Probability Density Function (PDF) $p(\tau)$, and a NM -dimensional independent complex Gaussian vector \mathbf{x} (called the *speckle*), with zero-mean and covariance matrix $\mathbf{M} = E(\mathbf{x}\mathbf{x}^H)$ (H denotes the conjugate transpose operator):

$$\mathbf{c} = \sqrt{\tau}\mathbf{x} \quad (2)$$

Note that the covariance matrix is normalized according to $\text{Tr}(\mathbf{M}) = NM$ [6] for identifiability considerations.

Under hypotheses H_0 and H_1 , the observations PDF depends on several unknown quantities: the amplitude α_0 , the target parameters θ_0 and f_0 , the texture PDF $p(\tau)$ and the covariance matrix \mathbf{M} . Therefore, a Generalized Likelihood Ratio Test (GLRT) is usually developed. The major difficulty comes from the estimation of the texture Probability Density Function (PDF). When \mathbf{M} is known, this problem was solved in a different ways in [7, 8] (BORD) and [9] (GLRT - Linear Quadratic):

$$\Lambda(\mathbf{M}^{-1}, \theta, f) = \max_{\theta, f} \frac{|\mathbf{p}(\theta, f)^H \mathbf{M}^{-1} \mathbf{y}|^2}{(\mathbf{p}(\theta, f)^H \mathbf{M}^{-1} \mathbf{p}(\theta, f))(\mathbf{y}^H \mathbf{M}^{-1} \mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \quad (3)$$

This test has interesting properties: it is CFAR with regards to the texture distribution and a closed-form relationship between the Probability of False Alarm (PFA) and the threshold λ has been derived.

But, in practice, \mathbf{M} is generally unknown and has to be estimated from the secondary data \mathbf{y}_i . The CCM estimate is denoted $\hat{\mathbf{M}}$. Then, test (3) may be rewritten in its adaptive version by replacing \mathbf{M} by its estimate $\hat{\mathbf{M}}$. In this case, and for Gaussian clutter, it is well known that the Reed-Mallett-Brennan's rule ensures a SNR Loss equal to 3 dB for $N_s = 2NM$.

Let λ_n with $1 \leq n \leq NM$ be the CCM eigenvalues. The CCM is well known to exhibit a low rank $r \ll NM$ provided by Brennan's rule [10]. This low rank assumption will be made in the following:

$$\lambda_1, \dots, \lambda_r \gg \lambda_{r+1} = \dots = \lambda_{NM} \quad (4)$$

In this low-rank clutter STAP context [5], let us denote by $\hat{\mathbf{\Pi}}_c$ the estimated projector onto the clutter subspace, i.e. the subspace spanned by the eigenvectors associated to the r greatest eigenvalues. The inverse CCM may then be approximated as follows (up to a scal factor):

$$\hat{\mathbf{M}}^{-1} \propto \mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_c \quad (5)$$

By substituting $\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_c$ for $\hat{\mathbf{M}}^{-1}$ in Eq. (3), we obtain a new test $\hat{\Lambda}(\hat{\mathbf{\Pi}}_c, \theta, f) = \Lambda(\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_c, \theta, f) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda'$.

This new test has several advantages: the first one is that the SNR loss has been shown to be roughly equal to 3 dB when $N_s = 2r$ in the case of homogeneous Gaussian secondary data [11, 12]. The second one concerns the possibility for the data \mathbf{y}_i to be corrupted by the signal without a strong decrease of performance when the signal to clutter ratio is low [13]. This allows to have firstly a significant reduction of the required number of secondary data and, secondly, robustness to data corrupted by a target.

This paper proposes a new method to estimate the projector $\mathbf{\Pi}_c$ in the case of a non-homogeneous clutter.

3. ESTIMATION OF $\mathbf{\Pi}_c$

Let us first recall the common way to estimate the projector $\mathbf{\Pi}_c$ in the case of homogeneous clutter. The Sample Covariance Matrix (SCM) is computed from secondary data \mathbf{y}_i :

$$\hat{\mathbf{M}}_{SCM} = \frac{1}{N_s} \sum_{i=1}^{N_s} \mathbf{y}_i \mathbf{y}_i^H \quad (6)$$

The Eigenvalue Value Decomposition (EVD) of $\hat{\mathbf{M}}_{SCM}$ is next performed:

$$\hat{\mathbf{M}}_{SCM} = (\mathbf{U}_r \ \mathbf{U}_0) \begin{pmatrix} \Sigma_r & \mathbf{0} \\ \mathbf{0} & \Sigma_0 \end{pmatrix} (\mathbf{U}_r \ \mathbf{U}_0)^H, \quad (7)$$

where \mathbf{U}_r and \mathbf{U}_0 are respectively two $(NM \times r)$ and $NM \times (NM - r)$ unitary matrices, $\mathbf{\Sigma}_r = \text{diag}\{\hat{\lambda}_1, \dots, \hat{\lambda}_r\}$ and $\mathbf{\Sigma}_0 = \text{diag}\{\hat{\lambda}_{r+1}, \dots, \hat{\lambda}_{NM}\}$ are the matrices of the estimated eigenvalues. From the low rank clutter assumption, we have the following eigenvalue property: $\hat{\lambda}_1, \dots, \hat{\lambda}_r \gg \hat{\lambda}_{r+1}, \dots, \hat{\lambda}_{NM}$. Next, we obtain the estimated projector onto the clutter subspace [13]:

$$\hat{\mathbf{\Pi}}_{cSCM} = \mathbf{U}_r \mathbf{U}_r^H \quad (8)$$

The resulting Low Rank Sample Covariance Matrix Test (LR-SCMT) becomes:

$$\hat{\Lambda}_{LR-SCMT}(\hat{\mathbf{\Pi}}_{cSCM}, \theta, f) = \Lambda(\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_{cSCM}, \theta, f) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda'' \quad (9)$$

In the case of non-homogeneous clutter, it is well-known that the SCM is not a good estimate of the clutter covariance matrix. As a consequence, the same conclusion holds for the estimate $\hat{\mathbf{\Pi}}_{cSCM}$ of $\mathbf{\Pi}_c$. In heterogeneous clutter modeled by SIRV processes, the NSCM is a very interesting alternative to $\hat{\mathbf{M}}_{NSCM}$:

$$\hat{\mathbf{M}}_{NSCM} = \frac{NM}{N_s} \sum_{i=1}^{N_s} \frac{\mathbf{y}_i \mathbf{y}_i^H}{\mathbf{y}_i^H \mathbf{y}_i} \quad (10)$$

The statistical properties of $\hat{\mathbf{M}}_{NSCM}$ have been studied in detail in [14]. In this paper it was shown that, despite its bias, $E(\hat{\mathbf{M}}_{NSCM})$ has the same eigenvectors as the CCM \mathbf{M}_c . The associated eigenvalues are different, but their ordering and their multiplicity are identical. Therefore, the projector $\hat{\mathbf{\Pi}}_{cNSCM}$ built from the NSCM (10) using steps (7) and (8) is a good candidate for estimating $\mathbf{\Pi}_c$ in the case of non-homogeneous clutter. Moreover, this estimate is easily shown to be consistent in our SIRV framework:

$$\hat{\mathbf{\Pi}}_{cNSCM} \xrightarrow{P} \mathbf{\Pi}_c, \quad (11)$$

when $N_s \rightarrow \infty$. This new estimate of the projector leads to a detection test that we call the Low Rank Normalized Sample Covariance Matrix Test (LR-NSCMT):

$$\hat{\Lambda}_{LR-NSCMT}(\hat{\mathbf{\Pi}}_{cNSCM}, \theta, f) = \Lambda(\mathbf{I}_{NM} - \hat{\mathbf{\Pi}}_{cNSCM}, \theta, f) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda''' \quad (12)$$

This new detector is texture-CFAR and also asymptotically M-CFAR thanks to the consistency of $\hat{\mathbf{\Pi}}_{cNSCM}$ (Eq. (11)).

In the next section, the performance of this detector are compared on experimental STAP data to those of classical detection schemes.

4. RESULTS ON STAP DATA

For non-homogeneous clutter, performance of $\hat{\Lambda}_{LR-NSCMT}$ would be better than $\hat{\Lambda}_{SCMT}$ and $\hat{\Lambda}_{LR-SCMT}$ ones, but another interested advantage of this new detector can be pointed out: its excellent behavior when the secondary data are contaminated by the target.

The STAP data are provided by the french DGA/CELAR's simulator that allows to synthesize, in side looking configuration, STAP datacubes from very high resolution RAMSES Synthetic Aperture Radar (SAR) [15]. The number of sensors is $N = 4$ and the number of coherent pulses can be up to $M = 64$. The center frequency and the bandwidth are respectively equal to $f_0 = 10$ GHz and $B = 5$ MHz. The radar velocity is given by $V = 100$ m/s. The inter-element spacing is $d = 0.3$ m and the pulse repetition frequency is $f_r = 1$ kHz. The value of parameter β is therefore equal to $2/3$ and the estimated clutter rank is here equal to $r = 46$ (for $M = 64$) in comparison to the full size of clutter covariance matrix, $MN = 256$. For this particular STAP datacube, clutter statistic is closer to Gaussian's one than SIRV's one. In this scenario, three targets are present: (4 m/s, 0 deg, bin 216), (4 m/s, 0 deg, bin 256) and (-4 m/s, 0 deg, bin 296). This allows to compare performance of the three detectors $\hat{\Lambda}_{SCMT}$, $\hat{\Lambda}_{LR-SCMT}$ and $\hat{\Lambda}_{LR-NSCMT}$ for the range bin 256 under test when the secondary data are contaminated by the targets located in range bins 216 and 296.

Figure 1 shows the results of $\hat{\Lambda}_{SCMT}$ (result of $\hat{\Lambda}_{NSCMT}$ is quite the same), $\hat{\Lambda}_{LR-SCMT}$ and $\hat{\Lambda}_{LR-NSCMT}$ when the targets in range bins 216 and 296 are removed from secondary data. As the number of secondary data is $N_s = 410 < 2NM$, performance of $\hat{\Lambda}_{SCMT}$ detector is poor but the target can still be detected. Results of both low rank detectors are much better. As clutter is homogeneous, their results are the same. This is the first advantage of low rank detectors which need less secondary data for an equivalent performance ($2r \ll 2NM$).

Figure 2 shows the results of $\hat{\Lambda}_{SCMT}$, $\hat{\Lambda}_{LR-SCMT}$ and $\hat{\Lambda}_{LR-NSCMT}$ when all secondary data are considered. First, it can be noticed that the target response is not detected anymore by $\hat{\Lambda}_{SCMT}$ due to the presence of the target contamination in the secondary data. Performance of $\hat{\Lambda}_{LR-SCMT}$ is degraded: in comparison to the previous $\hat{\Lambda}_{LR-SCMT}$ performance without contamination, there is a loss of 6 dB but the target of interest is still detected. Actually, a part of the target in secondary data is estimated in the orthogonal clutter subspace. Finally, performance of $\hat{\Lambda}_{LR-NSCMT}$ is better (no dB loss) and close to the previous one when secondary data are not suffering from target contamination. This is the second advantage of the combination of low rank and the NSCM.

5. CONCLUSION

In this paper, we developed a new low-rank STAP detector for non-homogeneous clutter. This detector is based on statistical properties of the Normalized Sample Covariance Matrix. We showed that this detector has CFAR properties in term of texture and, asymptotically, in term of the CCM. When tested on experimental STAP data obtained from a real clutter, this detector has been shown to provide increased performance compared to conventional full rank detectors. It also exhibits

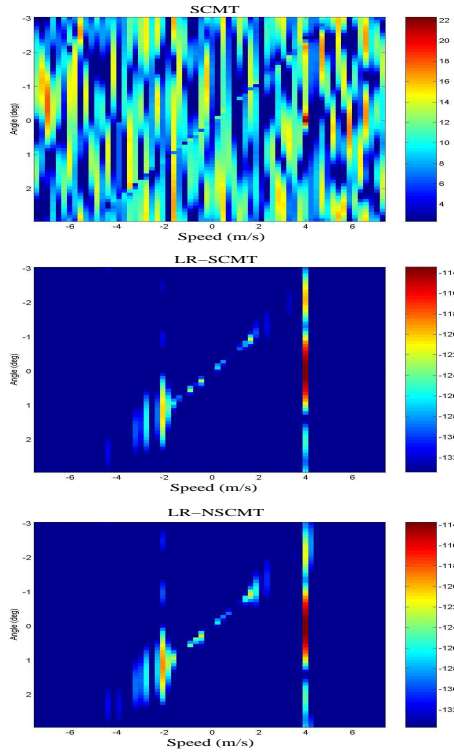


Fig. 1. Comparison of the three detectors $\hat{\Lambda}_{SCMT}$ (top), $\hat{\Lambda}_{LR-SCMT}$ (middle) and $\hat{\Lambda}_{LR-NSCMT}$ (bottom) without target contamination.

robustness to target contamination compared to existing low rank detection schemes.

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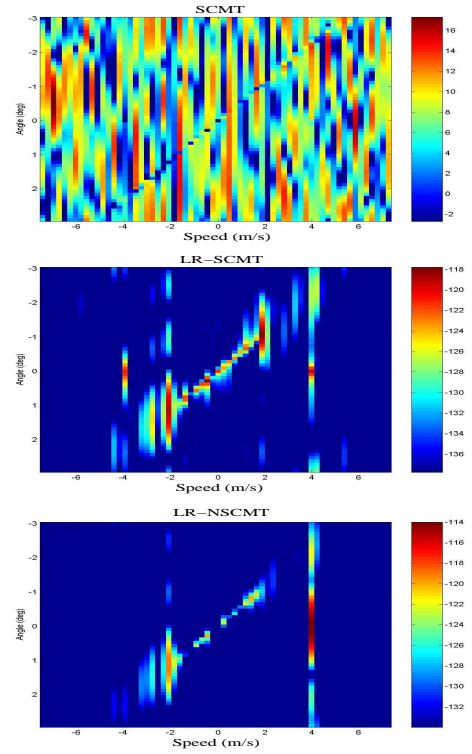


Fig. 2. Comparison of the three detectors $\hat{\Lambda}_{SCMT}$ (top), $\hat{\Lambda}_{LR-SCMT}$ (middle) and $\hat{\Lambda}_{LR-NSCMT}$ (bottom) with target contamination.