ADAPTIVE MIMO RADAR DETECTION IN NON-GAUSSIAN AND HETEROGENEOUS CLUTTER CONSIDERING FLUCTUATING TARGETS

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ABSTRACT

Previously, the Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) has been extended to the Multiple-Input Multiple-Output (MIMO) case where all transmit-receive subarrays are considered jointly as a system such that only one detection threshold is used. The new MIMO detector is Constant False Alarm Rate (CFAR) with respect to the clutter power fluctuations. In this paper, the adaptive version of this detector is considered, as well as a fluctuating target model similar to that of the Swerling Target. The degradation of the detection performance due to the estimation of the covariance matrix and the fluctuation of the target is studied through simulations for both the well-known Optimum Gaussian Detector (OGD) and the new MIMO detector under Gaussian and non-Gaussian clutter.

***Index Terms*—** MIMO Radar, Non-Gaussian Clutter, SIRV, Detection Performance, Fluctuating Targets

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) is a technique used in communications which has recently been adopted for radar applications [1]. In the context of radar, a MIMO radar is one where both the transmit and receive elements are sufficiently separated so as to provide spatial diversity which can be used to improve the probability of detection and resolutions. On top of that, each transmit element sends a different (orthogonal) waveform and this waveform diversity increases the separation between clutter and target returns and hence enhancing the suppression of the clutter.

We consider here non-Gaussian heterogeneous and impulsive clutter since experimental clutter measurements [2] have been found to fit non-Gaussian statistical models. Non-Gaussian models also better reflect the clutter power fluctuations (called texture) from one sub-system to another in the case where these sub-systems are widely separated. The Spherically Invariant Random Vector (SIRV) [3] is chosen as the clutter model as it can model different non-Gaussian clutter depending on the distribution of the texture. It also has a Gaussian kernel which means that certain classical results can still be applied.

A new MIMO non-Gaussian detector has been derived previously in [4] for non-fluctuating targets. In this paper, we consider a fluctuating target model similar to that of Swerling target. This is more realistic as different target aspects are seen by each sub-system due to the wide separation between the sub-systems. The detection performance of the detector when the target is fluctuating is compared to that when the target is stationary.

We also consider the adaptive version of this new detector where the covariance matrix is unknown and has to be estimated. This is done by using the Fixed Point Estimate (FPE) [5, 6, 7] as the classical Sample Covariance Matrix (SCM) is no longer the Maximum Likelihood (ML) estimate under non-Gaussian clutter. This adaptive detector is shown to be Constant False Alarm Rate (CFAR) with respect to the texture. Moreover, it is matrix-CFAR as it does not depend on the unknown covariance matrix. The detection performance now depends on an additional parameter, L_r: the number of secondary data to obtain the FPE.

This paper is organized as follows. Firstly, we consider a general signal model for MIMO (Section 2). Section 3 presents the adaptive version of the MIMO non-Gaussian detector and also the estimation of the covariance matrix. The effects on detection performance due to the fluctuation of the target and the estimation of the covariance matrix are then analyzed through Monte-Carlo simulations under both Gaussian and non-Gaussian clutter (Section 4). Finally, conclusions are presented in Section 5.

2. SIGNAL MODEL

In this section, we consider a target located at (x, y). Let there be $\tilde{N}$ transmit subarrays and $\tilde{M}$ receive subarrays. The n-th transmit and m-th receive subarray contain N_n and M_m elements respectively, for $n = 1, \ldots, \tilde{N}$ and $m = 1, \ldots, \tilde{M}$. The configuration can be seen in Fig. 1.

Let $\mathbf{v}(\theta_{t,n})$ and $\mathbf{a}(\theta_{r,m})$ be the steering vectors and $\theta_{t,n}$ and $\theta_{r,m}$ be the angular location for the target for the n-th transmit and m-th receive subarray respectively. Assuming

Thanks to DSO National Laboratories (Singapore) for funding.

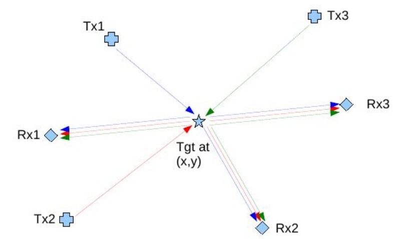


Fig. 1. The configuration where $\tilde{N} = \tilde{M} = 3$.

that orthogonal waveforms are transmitted such that the received signal after matched filtering can be expressed as:

$$\begin{aligned} \mathbf{y}_{m,n} &= \mathrm{B}(m,n)\mathbf{a}(\theta_{r,m}) \otimes \mathbf{v}(\theta_{t,n}) + \mathbf{z}_{m,n} \\ &= \mathrm{B}(m,n)\mathbf{p}_{m,n} + \mathbf{z}_{m,n} \end{aligned} \quad (1)$$

where $\mathbf{B}$ is the $\tilde{M}$x$\tilde{N}$ matrix containing the RCS of the target. $\mathbf{p}_{m,n}$ is the $M_m N_n$x1 bistatic angular steering vector which is equal to $\mathbf{a}(\theta_{r,m}) \otimes \mathbf{v}(\theta_{t,n})$ and $\otimes$ stands for the Kronecker product.

2.1. Clutter Model

$\mathbf{z}_{m,n}$ is a $M_m N_n$x1 vector containing the clutter returns and it is modelled by SIRV:

$$\mathbf{z}_{m,n} = \sqrt{\tau_{m,n}}\mathbf{x}_{m,n}, \quad (2)$$

where $\tau_{m,n}$ (*texture*) is a positive random variable which models the variation in power that arises from the spatial variation in the backscattering of the clutter. $\mathbf{x}_{m,n}$ is a complex circular Gaussian vector with zero mean and covariance matrix $\mathbf{M}_{m,n}$, denoted by $\mathcal{CN}(\mathbf{0}, \mathbf{M}_{m,n})$.

2.2. Target Model

Consider the well-known Swerling I target model where the RCS of the target fluctuates from scan to scan according to a Gaussian distribution. In the MIMO configuration where the subarrays are widely separated, the RCS of the target seen by each subarray pair can also be represented by a Gaussian distribution, i.e. each element of $\mathbf{B}$ is independent and identically distributed (i.i.d.) and $\mathrm{B}(m,n) \sim \mathcal{CN}(0, \sigma_t^2)$ where σ_t^2 is the power of the target. This is consistent with the target model given in [8]. Hence the MIMO-version of Swerling I target is one which is modeled as a Gaussian process which fluctuates independently from subarray to subarray and also from scan to scan.

3. ADAPTIVE MIMO NON-GAUSSIAN DETECTOR

In [4], a new MIMO non-Gaussian detector has been derived and is given by:

$$\prod_{m,n}\left[\frac{1}{1 - \frac{|\mathbf{p}_{m,n}^{\dagger}\mathbf{M}_{m,n}^{-1}\mathbf{y}_{m,n}|^2}{(\mathbf{p}_{m,n}^{\dagger}\mathbf{M}_{m,n}^{-1}\mathbf{p}_{m,n})(\mathbf{y}_{m,n}^{\dagger}\mathbf{M}_{m,n}^{-1}\mathbf{y}_{m,n})}}\right]^{M_m N_n} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta, \quad (3)$$

where $\mathbf{M}_{m,n}$ is the covariance matrix for the m-n receive-transmit pair. The adaptive version of this detector is obtained by replacing the true covariance matrix by an estimate.

Proposition 3.1
The adaptive version of the non-Gaussian detector given in Eqn. (3) is texture- and matrix-CFAR when the covariance matrices are estimated using the FPE [5, 6, 7] which is defined as the unique solution of the equation:

$$\hat{\mathbf{M}}_{m,n} = \frac{N_n M_m}{L_r}\sum_{l=1}^{L_r}\frac{\mathbf{y}_{m,n}(l)\mathbf{y}_{m,n}^{\dagger}(l)}{\mathbf{y}_{m,n}^{\dagger}(l)\hat{\mathbf{M}}_{m,n}^{-1}\mathbf{y}_{m,n}(l)}. \quad (4)$$

where $\mathbf{y}_{m,n}(l)$ are the secondary data and L_r is the number of secondary data used to estimate the FPE.

Proof 3.1
The secondary data is assumed to contain only clutter returns as modeled by Eqn. (2). Due to the normalizing term in the denominator, the contribution of the texture in the FPE cancels out and the FPE does not depend on the texture of the clutter, i.e. it is texture-CFAR [5, 6].

On top of that, the FPE can be computed according to an iterative algorithm which converges towards the FPE irregardless of the choice of the initial matrix [9]. i.e. it is also matrix-CFAR.

The original detector derived in [4] has been shown to be both texture- and matrix-CFAR. Consequently, when its adaptive version is used with the FPE, it remains texture- and matrix-CFAR.

Moreover, the FPE is the ML estimate when the texture is deterministic but unknown [7] and the approximate ML estimate when the texture is a random variable [5, 6].

4. SIMULATION RESULTS

To study the effect of the fluctuating target model on the detection performance, as well as the estimation of covariance matrix, Monte-Carlo simulations are carried out. $\tilde{M} = 3$ receive subarrays are considered, each containing 4 elements. $\tilde{N} = 2$ transmit subarrays, containing 3 elements each, are considered.

Due to space constraints, we consider only 2 types of clutter: Gaussian and K-distributed (Gamma-textured). In order to keep the clutter power, σ^2, constant, the two parameters of the gamma distribution are set such that the statistical mean of the texture remains the same and is equal to σ^2 which is the clutter power for each element. In this simulation, $\sigma^2 = 1$. The parameters used to simulate the texture are shown in Table 1. The parameters are chosen such that there is one instance of impulsive clutter and one which is more similar to the Gaussian case.

Texture distribution	a	b
Gamma	0.5	$\sigma^2/a = 2$
Gamma	2	$\sigma^2/a = 0.5$

Table 1. Texture parameters used for Monte-Carlo simulations.

4.1. Effects of Fluctuating Target

The covariance matrices are assumed to be known so as to better study the effect of the fluctuations of the target returns. The fluctuating target model described in Sec. 2.2 is considered. For comparison, we include the case of a stationary isotropic target.

For comparison, the MIMO Optimum Gaussian Detector (OGD) which is optimum under Gaussian clutter is considered [10, 11]:

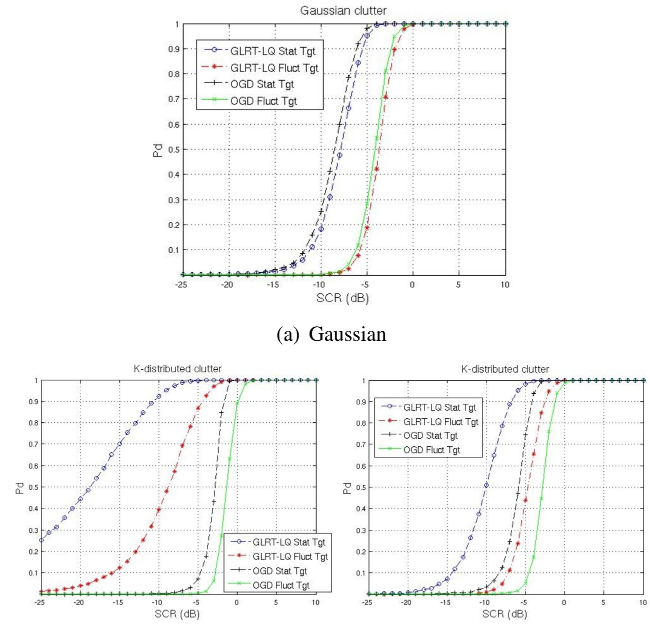
$$\sum_m \sum_n \frac{|\mathbf{p}_{m,n}^\dagger \mathbf{M}_{m,n}^{-1} \mathbf{y}_{m,n}|^2}{\mathbf{p}_{m,n}^\dagger \mathbf{M}_{m,n}^{-1} \mathbf{p}_{m,n}} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta_1. \tag{5}$$

In Fig. 2, we see the detection performance of the MIMO GLRT-LQ and MIMO OGD for the case where the target is stationary and fluctuating under Gaussian and non-Gaussian clutter. Under Gaussian clutter, both MIMO GLRT-LQ and MIMO OGD are almost equally affected by the fluctuations of the target. However, under non-Gaussian clutter, MIMO GLRT-LQ is more affected than MIMO OGD when the clutter is impulsive and the difference becomes small when the clutter is less impulsive. The detection performance of MIMO GLRT-LQ remains better under non-Gaussian clutter.

4.2. Effects of Estimation of Covariance Matrix

In order to study the effect of the estimation of the covariance matrix, we consider that the target is stationary. Here, we consider 2 cases: $L_r = 2L$ and $L_r = 20L$ to compute $\hat{\mathbf{M}}_i$ using the Fixed Point algorithm, and compare them to the case where $\mathbf{M}_i$ are known. As expected, when L_r is large, the detection performance of the adaptive detector tends towards that of the detector where the covariance matrices are known (Fig. 3).

For comparison, we consider the MIMO Adaptive Matched Filter (AMF), the adaptive version of the MIMO OGD where the classical SCM replaces $\mathbf{M}_{m,n}$ in Eqn. (5). In Fig. 4,



(a) Gaussian

(b) K-distributed, a = 0.5 (c) K-distributed, a = 2

Fig. 2. P_d against SCR for Monte-Carlo simulations, considering both the stationary and fluctuating targets and both MIMO GLRT-LQ and MIMO OGD. $P_{fa} = 0.001$.

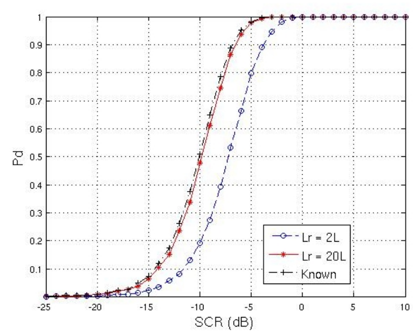
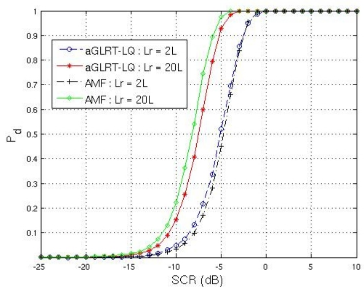


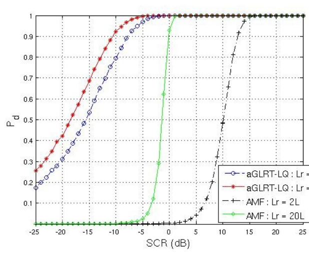
Fig. 3. P_d against SCR for Monte-Carlo simulations for K-distributed clutter, $a = 2$, considering the adaptive MIMO GLRT-LQ with the FPE.

we have the detection performance, under Gaussian and non-Gaussian clutter, of both the adaptive MIMO GLRT-LQ using FPE and the MIMO AMF using SCM.

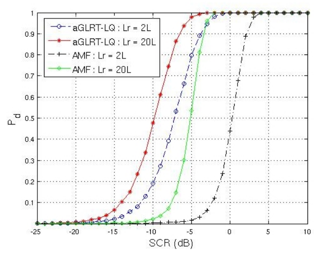
Under Gaussian clutter, the estimation of the covariance matrix does not affect the performance of the adaptive MIMO GLRT-LQ much and it remains comparable to that of the MIMO AMF. Under non-Gaussian clutter, the adaptive MIMO GLRT-LQ performs much better than the MIMO AMF when the clutter is impulsive. When the clutter is less impulsive, the detection performance of both detectors are more similar but the adaptive MIMO GLRT-LQ still works better. Moreover, the MIMO AMF is more affected by the estimation of the covariance matrix as the SCM is no longer the ML estimate under non-Gaussian clutter.



(a) Gaussian



(b) K-distributed, $a = 0.5$



(c) K-distributed, $a = 2$

Fig. 4. P_d against SCR for Monte-Carlo simulations, considering both the adaptive MIMO GLRT-LQ with FPE and the MIMO AMF with SCM. $P_{fa} = 0.001$.

5. CONCLUSIONS

We consider here the MIMO non-Gaussian detector derived previously where all transmit-receive subarrays are considered jointly as a system such that only one detection threshold is used. With this background, we have further analyzed this detector to include a fluctuating target model similar to that of the Swerling I target, as well as the adaptive version of the detector. The degradation of the detection performance due to the fluctuations of the target and the estimation of the covariance matrix is studied through simulations for both the new MIMO detector and the MIMO OGD under Gaussian and non-Gaussian clutter.

The main conclusion is that it is always preferable to use the adaptive GLRT-LQ with the FPE, whatever the clutter distribution, because of the robustness of these tools with respect to the covariance matrix and the texture. Even in the case where the clutter is Gaussian for all subarrays, the covariance matrix and clutter power for each subarray is expected to be different. Hence it is still better to use the new non-Gaussian detector.

6. REFERENCES

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