Radar Detection Schemes for Joint Temporal and Spatial Correlated Clutter Using Vector ARMA Models

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1. Abstract

Adaptive radar detection and estimation schemes are often based on the independence of the training data used for building estimators and detectors. This paper relaxes this constraint and deals with the problem of deriving detection and estimation schemes for joint spatial and temporal correlated radar measurements. To model these correlations, we use the Vector ARMA (VARMA) methodology. The matrix parameters of the VARMA model are estimated by likelihood maximisation in Gaussian and non-Gaussian environments. These matrix estimates are used to bluid Adaptive Radar Detectors, like Adaptive Normalized Matched Filter (ANMF). Their performances are analyzed through simulated datasets. We show that taking into account the spatial covariances may increase the performances significantly compared to classical procedures which ignore the spatial correlations.

2. VARMA(0,1) radar clutter model

A multivariate signal $(\boldsymbol{y}_k)_{k\in\mathbb{Z}}$ is an m-variate VARMA(p,q) model if

$$\boldsymbol{y}_k - \sum_{i=1}^{p} \boldsymbol{\Phi}_i \, \boldsymbol{y}_{k-i} = \boldsymbol{c}_k + \sum_{i=1}^{q} \boldsymbol{\Theta}_i \, \boldsymbol{c}_{k-i},$$

where $(\boldsymbol{c}_k)_{k\in\mathbb{Z}}$ are IID m-variate zero-mean vectors with non-degenerate covariance matrix $\boldsymbol{\Sigma}_c = \mathbb{E}\left(\boldsymbol{c}_k\, \boldsymbol{c}_k^H\right)$ characterizing the temporal dependence of its components. Here, we choose $p = 0$ and $q = 1$, which means that $\boldsymbol{y}_k$ is correlated only with $\boldsymbol{y}_{k-1}$.

The radar detection problem can be stated as a classical binary hypothesis testing [1]

$$\begin{aligned} H_0 &: \boldsymbol{y} = \boldsymbol{c} \qquad & \boldsymbol{y}_k = \boldsymbol{c}_k, k = 1, \ldots, K \\ H_1 &: \boldsymbol{y} = \alpha\, \boldsymbol{p} + \boldsymbol{c} \qquad & \boldsymbol{y}_k = \boldsymbol{c}_k, k = 1, \ldots, K. \end{aligned}$$

The detection test performance is analyzed through the false alarm probability P_{fa} and the probability of detection P_d. These probabilities are computed through the Normalized Matched Filter (GLRT in partially homogeneous Gaussian clutter, approximated GLRT in non-Gaussian CES clutter) [2]

$$\Lambda_{\text{ANMF}} = \frac{\left|\boldsymbol{p}^H \hat{\boldsymbol{M}}^{-1} \boldsymbol{y}\right|^2}{\left(\boldsymbol{p}^H \hat{\boldsymbol{M}}^{-1} \boldsymbol{p}\right)\left(\boldsymbol{y}^H \hat{\boldsymbol{M}}^{-1} \boldsymbol{y}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda,$$

where $\hat{\boldsymbol{M}}$ stands for any estimator of the covariance matrix $\boldsymbol{M}$ of $\boldsymbol{y}$ which satisfies

$$\boldsymbol{M} = \boldsymbol{\Sigma}_c + \boldsymbol{\Theta}_1 \, \boldsymbol{\Sigma}_c \, \boldsymbol{\Theta}_1^H.$$

3. Gaussian Estimators

Independent observations: The likelihood function $L_1(\boldsymbol{Y}, \boldsymbol{M})$ is given by

$$L_1(\boldsymbol{Y};\boldsymbol{M}) = \frac{1}{\pi^{mK}|\boldsymbol{M}|^K} \exp\left(-\sum_{k=1}^{K} \boldsymbol{y}_k^H \, \boldsymbol{M}^{-1} \, \boldsymbol{y}_k\right).$$

The maximum of $L_1(\boldsymbol{Y};\boldsymbol{M})$ with respect to $\boldsymbol{M}$ is the Sample Covariance Matrix [1]

$$\hat{\boldsymbol{M}}_{\text{SCM}} = \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{y}_k \, \boldsymbol{y}_k^H.$$

VARMA(0,1) observations: The parameters can be estimated by conditional or exact likelihood maximisation [3]. The conditional likelihood function is

$$L_2(\boldsymbol{Y};\boldsymbol{\Theta}_1,\boldsymbol{\Sigma_c}) = \frac{1}{\pi^{mK}\,|\boldsymbol{\Sigma}_c|^K} \exp\left(-\sum_{k=1}^{K} \boldsymbol{c}_k^H \, \boldsymbol{\Sigma}_c^{-1} \, \boldsymbol{c}_k\right).$$

The exact likelihood function is

$$L_3(\boldsymbol{Y};\boldsymbol{\Theta}_1,\boldsymbol{\Sigma_c}) = \frac{\exp\left(-(\tilde{\boldsymbol{Y}} - \tilde{\boldsymbol{X}}\hat{\boldsymbol{c}}_0)^H(\tilde{\boldsymbol{Y}} - \tilde{\boldsymbol{X}}\hat{\boldsymbol{c}}_0)\right)}{\pi|\boldsymbol{\Sigma}_c|^K \left|\tilde{\boldsymbol{X}}^H\tilde{\boldsymbol{X}}\right|},$$

where $\hat{c}_0$, $\tilde{\boldsymbol{X}}$ and $\tilde{\boldsymbol{Y}}$ depend only on the observations and $\boldsymbol{\Sigma}_c$ and $\boldsymbol{\Theta}_1$. As the sample size tends to infinity, the conditional MLE's and the exact MLE's both converge almost surely to the true values $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Sigma}_c$. But their finite sample behaviours may be different, especially in the case of small datasets.

4. Compound Gaussian estimators

The random vector $\boldsymbol{c}_k$ is compound Gaussian if $\boldsymbol{c}_k = \sqrt{\tau_k}\boldsymbol{x}_k$ where $\tau_k \geq 0$ is the random texture, $\boldsymbol{x}_k$ is a Gaussian vector, and τ_k is independent of $\boldsymbol{x}_k$.

Independent observations: The conditional PDF of $\boldsymbol{y}_k$ conditionally to τ_k is

$$p(\boldsymbol{y}_k|\tau_k) = \frac{1}{\pi^m|\boldsymbol{M}|\tau_k^m} \exp\left(-\frac{\boldsymbol{y}_k^H \boldsymbol{M}^{-1} \boldsymbol{y}_k}{\tau_k}\right).$$

The value of τ_k that maximizes $p(\boldsymbol{y}_k|\tau_k)$ is $\hat{\tau}_k = \boldsymbol{c}_k^H \boldsymbol{\Sigma}_c^{-1} \boldsymbol{c}_k / m = \boldsymbol{y}_k^H \boldsymbol{M}^{-1} \boldsymbol{y}_k / m$, [4]. The likelihood function is

$$L_4(\boldsymbol{Y};\boldsymbol{M}) = \prod_{k=1}^{K} \frac{m^m \exp(-m)}{\pi^m|\boldsymbol{M}|(\boldsymbol{y}_k^H \boldsymbol{M}^{-1} \boldsymbol{y}_k)^m}.$$

The covariance matrix maximizing $L_4(\boldsymbol{Y};\boldsymbol{M})$ is the Tyler's estimate [5]

$$\hat{\boldsymbol{M}}_{\text{FP}} = \frac{m}{K} \sum_{k=1}^{K} \frac{\boldsymbol{y}_k \boldsymbol{y}_k^H}{\boldsymbol{y}_k^H \hat{\boldsymbol{M}}_{\text{FP}}^{-1} \boldsymbol{y}_k}.$$

VARMA(0,1) observations: The conditional likelihood function of $\boldsymbol{Y}$ is

$$L_5(\boldsymbol{Y};\boldsymbol{\Theta}_1,\boldsymbol{\Sigma_c}) = \prod_{k=1}^{K} \frac{m^m \exp(-m)}{\pi^m|\boldsymbol{\Sigma}_c|(\boldsymbol{c}_k^H \boldsymbol{\Sigma}_c^{-1} \boldsymbol{c}_k)^m}.$$

5. Experimental Results

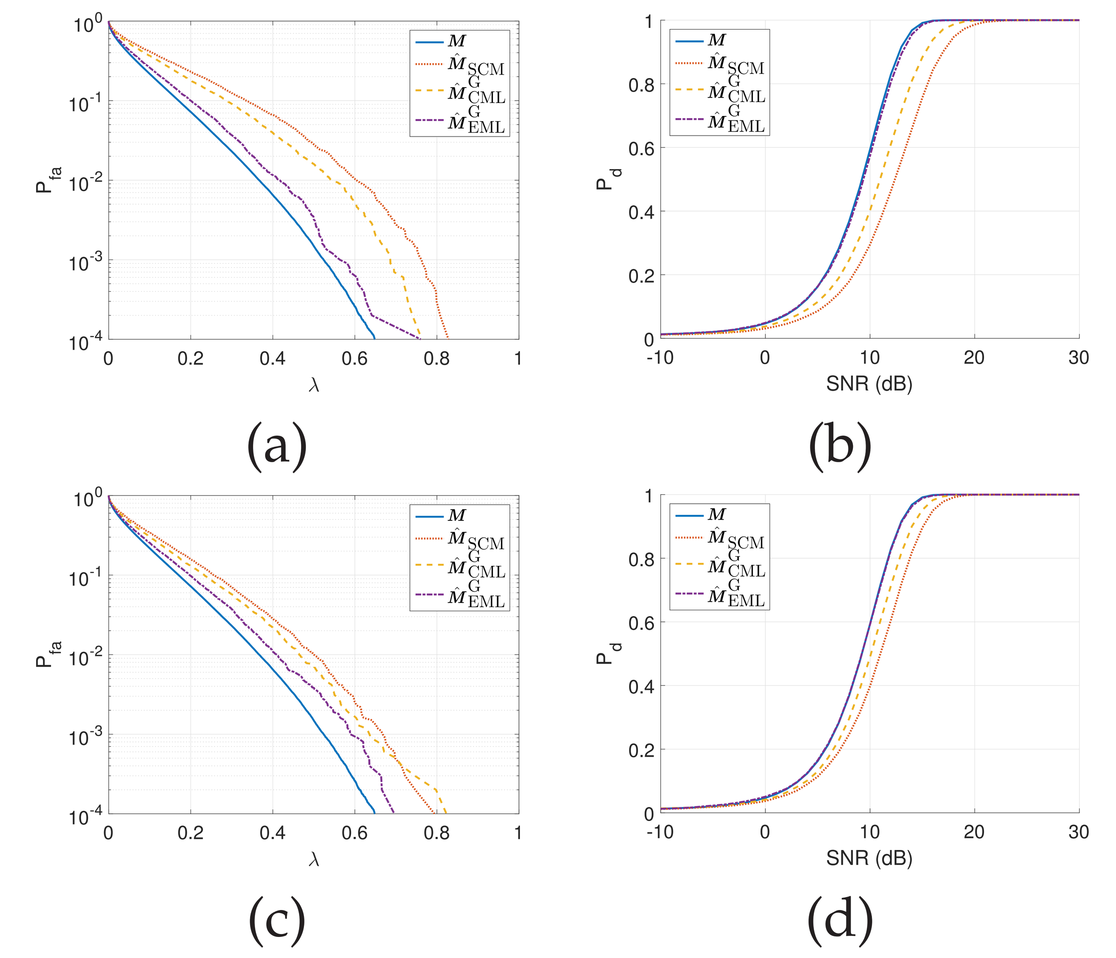

Figure 1: $P_{fa} - \lambda$ plot and the corresponding P_d-SNR relationship for $P_{fa} = 10^{-2}$ for spatially VARMA(0,1) correlated Gaussian clutter ($m = 16$, $\rho = 0.5$, $\boldsymbol{\Theta_1} = 0.9\boldsymbol{I_m}$). (a), (b) $K = 32$. (c), (d) $K = 48$.

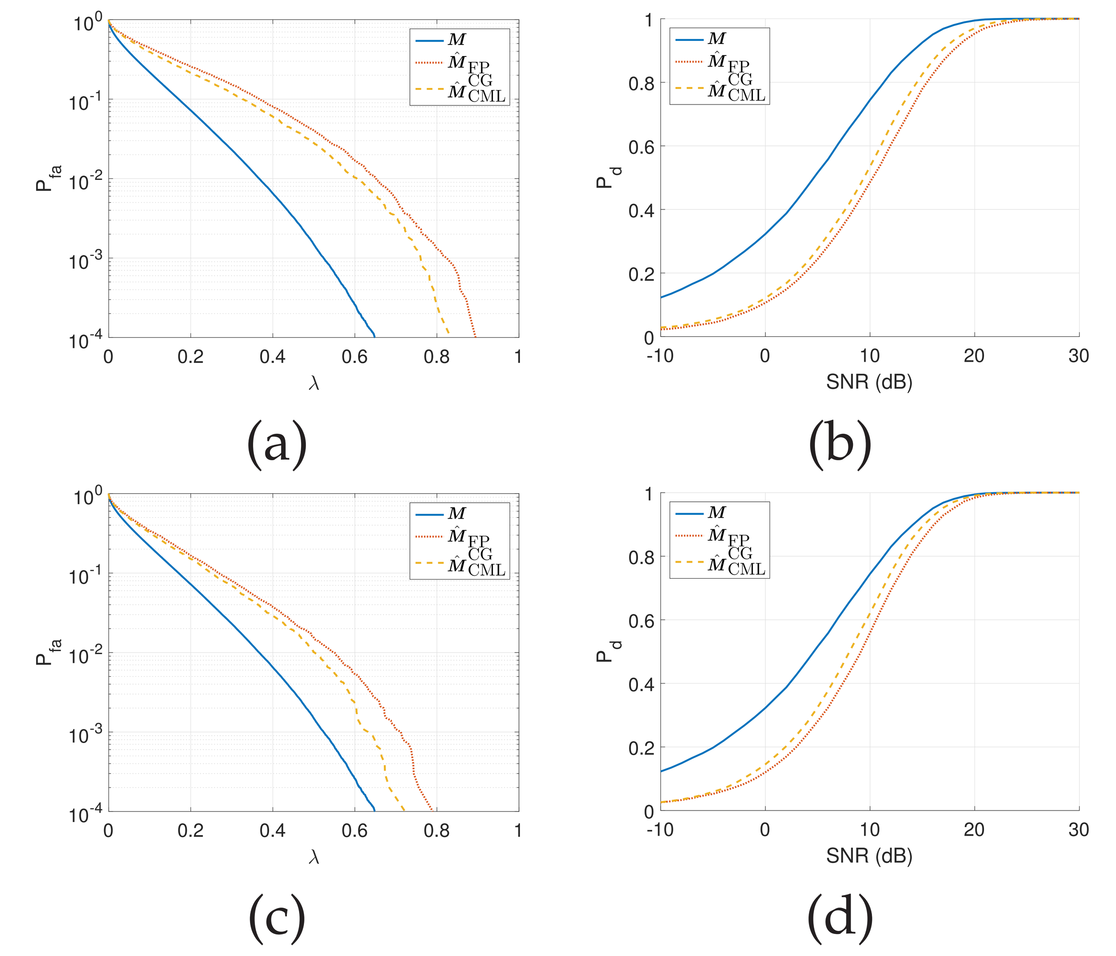

Figure 2: $P_{fa} - \lambda$ plot and the corresponding P_d-SNR relationship for $P_{fa} = 10^{-2}$ for VARMA(0,1) spatially correlated compound Gaussian clutter ($\nu = 0.5$, $m = 16$, $\rho = 0.5$, $\boldsymbol{\Theta_1} = 0.9\boldsymbol{I_m}$). (a), (b) $K = 32$. (c), (d) $K = 48$.

6. Conclusion

We proved that the spatial correlation characterizing the clutter secondary data could be exploited to enhance the performance of the radar detection. This enhancement takes place in simulated Gaussian distributed data through a maximization of both conditional and exact likelihood functions to estimate the data covariance matrix. In the case of non Gaussian distributed clutter, the conditional ML gives a considerable improvement of the quality of detection with respect to the classical Tyler's estimator.

References

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