Out-of-Distribution Radar Detection in Compound Clutter and Thermal Noise through Variational Autoencoders

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Context & Objectives

Radar target detection faces challenges with complex noise environments. Traditional methods perform well in Gaussian noise but struggle with real-world compound Gaussian noise plus thermal noise. We propose a detector based on Variational Autoencoders (VAEs) to handle these complex structures by leveraging Out-Of-Distribution (OOD) detection capabilities.

Our approach trains a VAE exclusively on noise-only data to learn the underlying probability distribution, then exploits its ability to identify samples that deviate from this learned distribution.

Classical Detection Methods

Matched Filter (MF):

$$\Lambda_{\text{MF}}(\mathbf{z}) = \frac{|\mathbf{p}^H \mathbf{\Sigma}^{-1}\mathbf{z}|^2}{\mathbf{p}^H \mathbf{\Sigma}^{-1}\mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Optimal in homogeneous Gaussian noise environments.

Adaptive Matched Filter (AMF-SCM):

$$\Lambda_{\text{AMF}-\text{SCM}}(\mathbf{z}) = \frac{|\mathbf{p}^H \hat{\mathbf{\Sigma}}_{\text{SCM}}^{-1}\mathbf{z}|^2}{\mathbf{p}^H \hat{\mathbf{\Sigma}}_{\text{SCM}}^{-1}\mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Uses Sample Covariance Matrix $\hat{\mathbf{\Sigma}}_{\text{SCM}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{z}_k \mathbf{z}_k^H$.

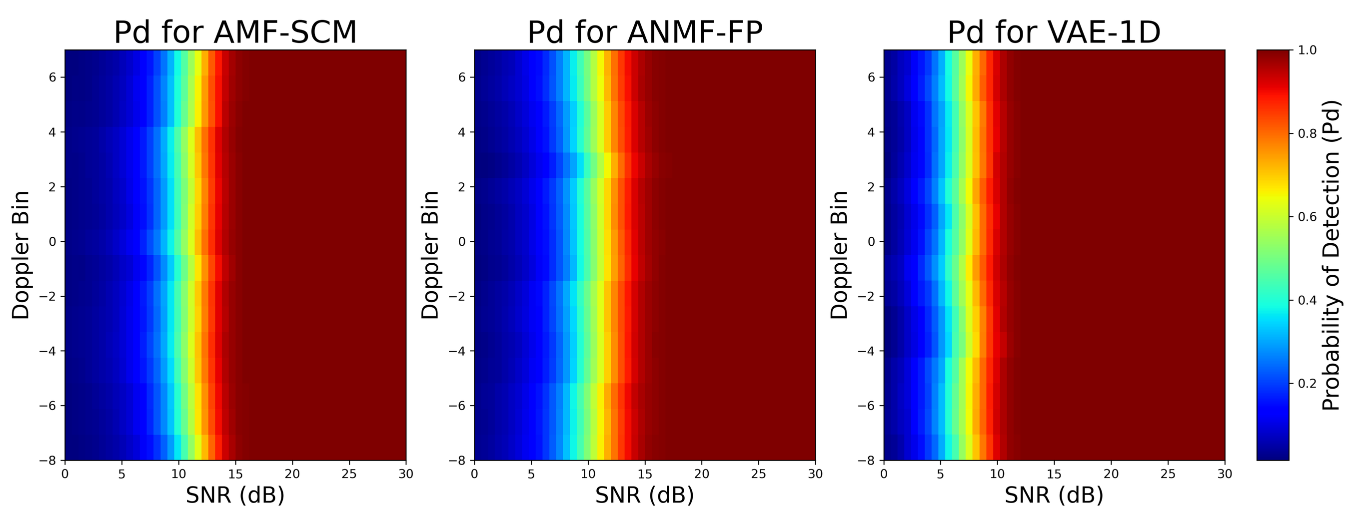
Adaptive Normalized Matched Filter (ANMF-FP):

$$\Lambda_{\text{ANMF}-\text{FP}}(\mathbf{z}) = \frac{|\mathbf{p}^H \hat{\mathbf{\Sigma}}_{\text{FP}}^{-1}\mathbf{z}|^2}{(\mathbf{p}^H \hat{\mathbf{\Sigma}}_{\text{FP}}^{-1}\mathbf{p})(\mathbf{z}^H \hat{\mathbf{\Sigma}}_{\text{FP}}^{-1}\mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

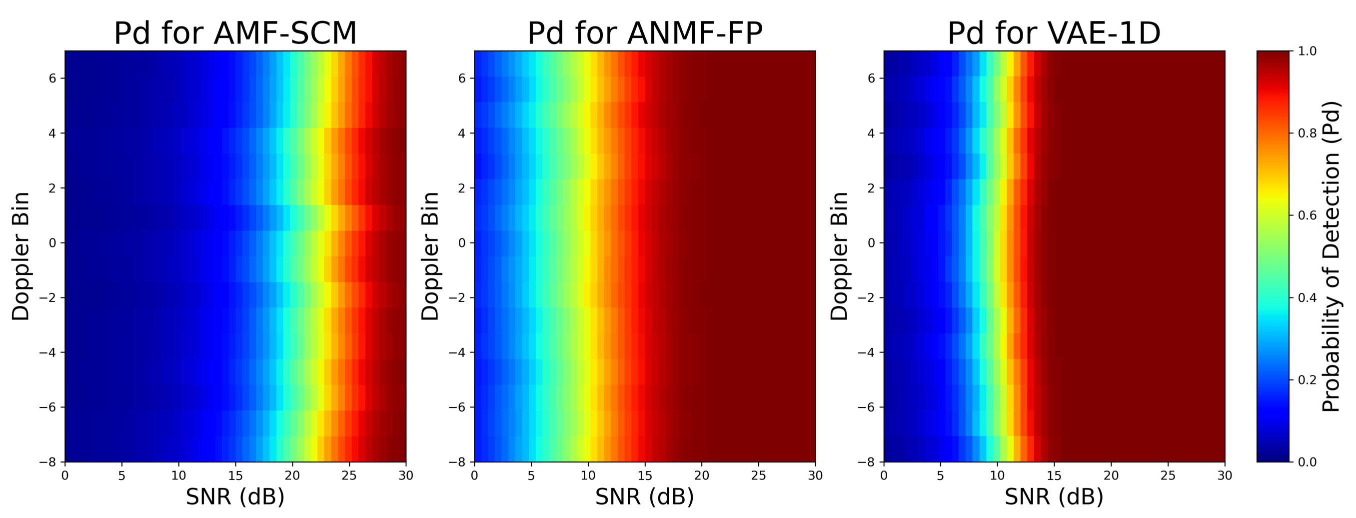
With Tyler's estimator: $\hat{\mathbf{\Sigma}}_{\text{FP}} = \dfrac{m}{K}\displaystyle\sum_{k=1}^{K} \frac{\mathbf{z}_k \mathbf{z}_k^H}{\mathbf{z}_k^H \hat{\mathbf{\Sigma}}_{\text{FP}}^{-1}\mathbf{z}_k}$

Effective in compound Gaussian clutter but struggles with thermal noise.

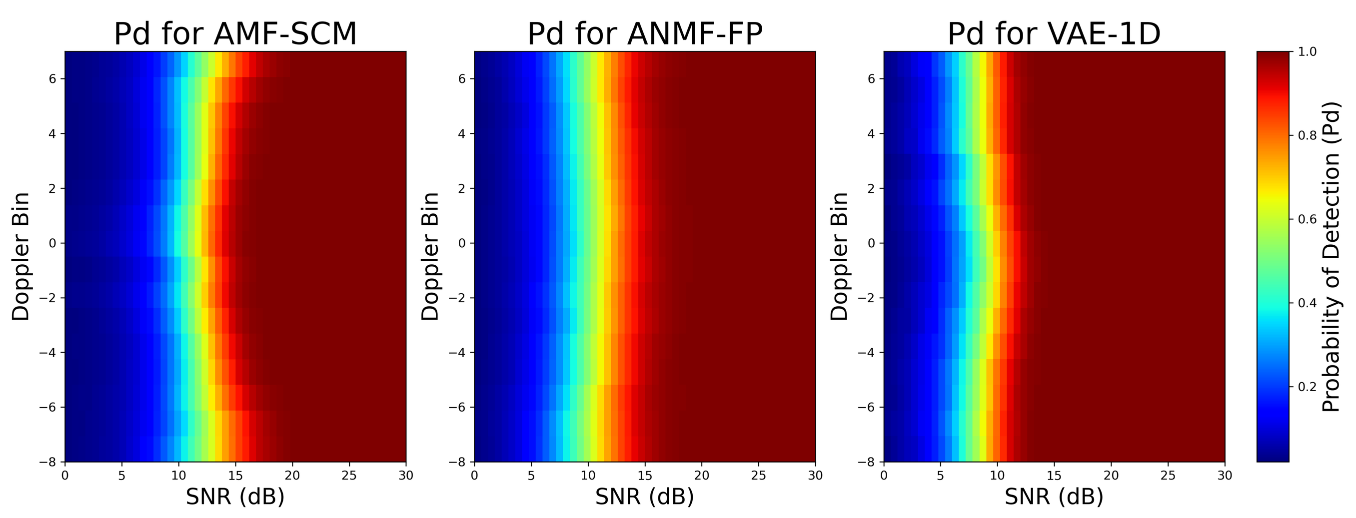
Results: Detection Performance vs SNR



(a) cGN + AWGN



(b) cCGN



(c) cCGN + AWGN

Figure 3: P_d-SNR-Doppler bin map comparing VAE to AMF and ANMF, under different noise scenarii.

Statistical Model

Hypothesis Testing and Signal Model:

The radar detection problem can be stated as the following binary hypothesis test:

$$\begin{cases} \text{Hypothesis } H_0: & \mathbf{z} = \mathbf{c} + \mathbf{n}, \\ \text{Hypothesis } H_1: & \mathbf{z} = \alpha\,\mathbf{p} + \mathbf{c} + \mathbf{n}, \end{cases}$$

where $\mathbf{z}$ is the received signal, α is target amplitude, $\mathbf{p}$ is steering vector, $\mathbf{c}$ is clutter noise, and $\mathbf{n}$ is thermal white Gaussian noise with covariance $\sigma^2\mathbf{I}$.

Clutter Models: Homogeneous clutter: $\mathbf{c} \sim \mathcal{CN}(0, \mathbf{\Sigma}_c)$; Heterogeneous clutter: compound Gaussian $\mathbf{c} = \sqrt{\tau}\mathbf{g}$, where $\mathbf{g} \sim \mathcal{CN}(0, \tau\mathbf{\Sigma}_c)$ conditioned on texture $\tau \in \mathbb{R}^+$.

SNR: The Signal-to-Noise Ratio is defined by: $\text{SNR} = |\alpha|^2\, \mathbf{p}^H\, \mathbf{\Sigma}^{-1}\, \mathbf{p}$, where $\mathbf{\Sigma} = \mathbf{\Sigma}_c + \sigma^2\mathbf{I}$.

Proposed VAE Detector

VAE Architecture:

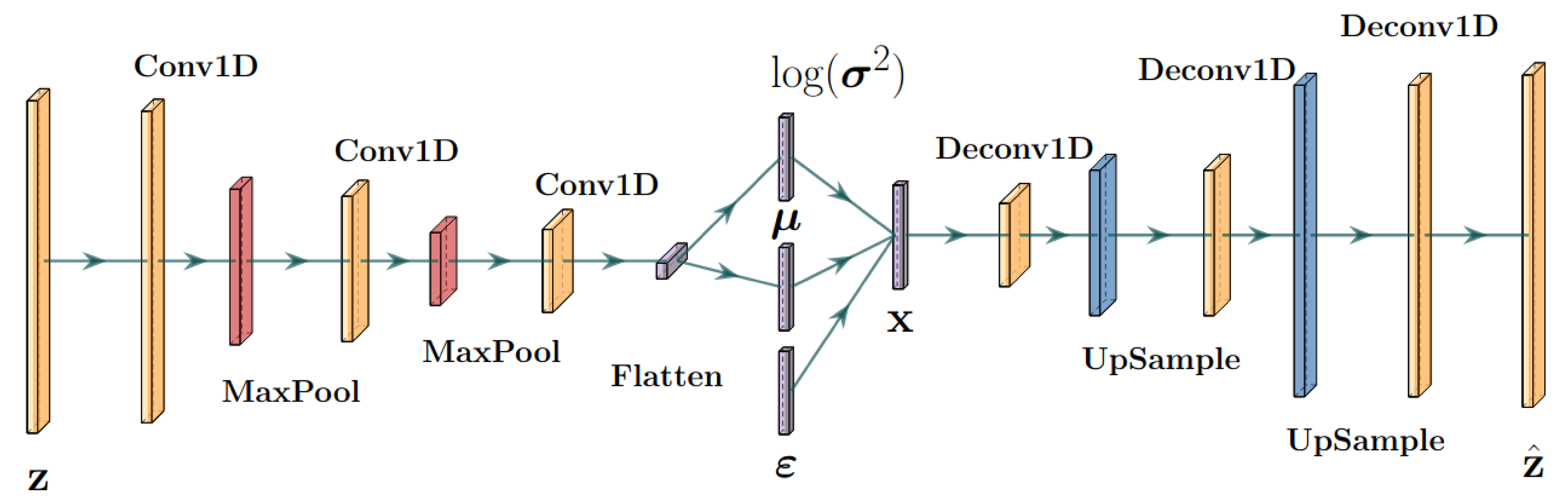


Figure 1: VAE architecture.

VAE Training:

- Trained on noise-only data $\mathcal{D}_{H_0} = \{\mathbf{z}_1, \cdots, \mathbf{z}_N \in H_0\}$
- Training loss: $\mathcal{L}_{VAE} = \mathcal{L}_{rec} + \beta\mathcal{L}_{KL}$
- Reconstruction loss: $\mathcal{L}_{rec}(\mathbf{z}, \hat{\mathbf{z}}) = \|\mathbf{z} - \hat{\mathbf{z}}\|^2$
- Kullback-Leibler divergence: $\mathcal{L}_{KL} = -\frac{1}{2}\sum_{i=1}^{q}(1 + \log(\sigma_i^2) - \mu_i^2 - \sigma_i^2)$
- Trained over 50 epochs using Adam optimizer with learning rate 10^{-3}

Detection Strategy:

- Detection test: $\mathcal{L}_{rec}(\mathbf{z}, \hat{\mathbf{z}}) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{VAE}$
- Threshold λ_{VAE} calibrated on evaluation dataset to regulate false alarm rate

MSE Distribution and Findings

MSE Distribution:

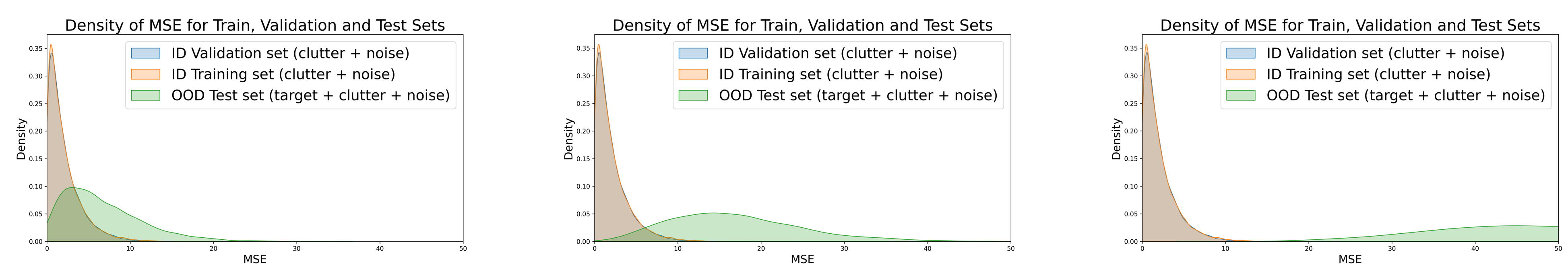


Figure 2: $\mathcal{L}_{\text{rec}}$ histogram across SNRs for cCGN + AWGN scenario.

Key Findings:

- In Gaussian + thermal noise: VAE performs on par with MF
- In compound Gaussian noise: VAE is competitive with NMF/ANMF-FP
- In compound Gaussian + thermal noise: VAE significantly outperforms adaptive detectors
- Consistent performance across all Doppler bins

Conclusion: The VAE-based detector effectively distinguishes radar targets from various noise types, consistently outperforming traditional methods in complex noise scenarios. Its superior performance stems from its ability to model intricate data distributions without assuming noise characteristics.

References

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