# Adaptive Detection Algorithms for slow moving Targets in Non-Gaussian Clutter

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Abstract— The purpose of this paper is to present a study of non-Gaussian detectors for the detection of small, slow moving targets in clutter. These detectors belong to the family of Adaptive Normalized Matched Filter. The noise-clutter covariance matrix will be computed by the classic fixed point estimator [1, 2, 3] or with an iterative estimator based on the multi-segment Burg algorithm [4]. We will also propose to add a data selection algorithm based on order statistics in order to improve the estimation of this covariance matrix when targets are in the clutter alone reference cells.

Keywords —Doppler, Radar Detection, Non-Gaussian Clutter, False Alarm rate, Adaptive processing

#### I. Introduction

The hypothesis of Gaussian clutter leads to a detection statistic provided by the adaptive matched filter after whitening, and an estimator of the covariance matrix known as the Sample Covariance Matrix (SCM). Concerning non-Gaussian signals, robust methods, such as ANMF with a Fixed Point estimator of the covariance matrix, have been recently proposed. They can tackle more complex clutter such as Complex Elliptically Symmetric (CES) [6] distribution. We will study in this article the performance in detection of these algorithms on slow moving targets. We begin by presenting the algorithms and their modifications to take into account the presence of potential targets in the reference data. The performance will be studied through Monte Carlo simulation.

## II. PROBLEM STATEMENT

# A. Adaptive Matched Filter and Doppler filtering

A classic detection function analyses raw data to decide whether a target is actually present at a specific range cell or not. To make this decision, the analysis is performed on raw complex observations data composed of  $N_{rec}$  values,  $N_{rec}$  is the number of transmitted pulses. The decision process is based on a statistical hypothesis test [7] which intends to evaluate the probability that an event would occur under the two following hypothesis:

• Null hypothesis H0:  $z = b_{bth} + b_{clutter} = c$ 

At the range cell under test, the signal z is assumed to be the linear sum of thermal noise:  $b_{th}$  and clutter:  $b_{clutter}$ 

• Alternative hypothesis H1:  $z = \alpha p + c$ 

where the signal of interest reads:  $\alpha p$ , p is the "steering vector",  $p = \frac{1}{\sqrt{N_p}} \left( e^{2i\pi k v_i/v_{amb}} \right)_{0 \le k \le N_p - 1}$ , with  $v_i$  the velocity of the target,  $v_{amb}$  the ambiguous speed that depends on the waveform parameters (Pulse Repetition Frequency, Radar frequency,...),  $N_p$  the number of pulses of the waveform and  $\alpha$  the target complex amplitude. When the noise and clutter are Gaussian with correlation matrix  $R_c = R_{noise} + R_{clutter}$  and the amplitude of the target is unknown but deterministic, the GLRT (Generalized Likelihood Ration Test) strategy leads to the following detection test:

$$\Delta(z) = \frac{\left|p^{H}R_{c}^{-1}z\right|^{2}}{p^{H}R_{c}^{-1}p} \ge Threshold$$

This test computes the Signal to Noise Ratio (SNR) on whitened data. The adaptivity is carried by the correlation matrix. If we suppose that  $R_c = \sigma^2 I$ , then  $\Delta(z) = \frac{|p^H z|^2}{\sigma^2}$ , and any knowledge on the Clutter Doppler spectrum can be taken into account by replacing the steering vector p by a Finite Impulse Response filter, for instance one with a deep null at zero Doppler frequency. The estimation of  $\sigma^2$  may be performed by cell averaging along the range axis at the output of the same Doppler hypothesis (cell averaging, order statistics, ...)

## B. Adaptive Normalized Matched filter

The Gaussian Clutter assumption is generally no longer respected when the clutter zone is not homogeneous, when the analysis cell tends to become smaller and smaller. This results in a degradation of clutter rejection performance and implicitely detection. Non Gaussian clutter can be expressed as  $c=\sqrt{\tau}g$  [1] which is the product of the square root of a positive scalar random variable  $\tau$  - which is the texture, characterized by its probability density function  $p_{\tau}(.)$ - and a complex Gaussian vector g, the *speckle*, of dimension  $N_p$ , with zero mean and covariance matrix  $M=E[gg^H]$ . Under these hypotheses, the GLRT strategy does not lead to the AMF test anymore. A new test can be fortunately derived under various hypotheses, for instance assuming that the texture is unknown deterministic, in which case the detection test becomes:

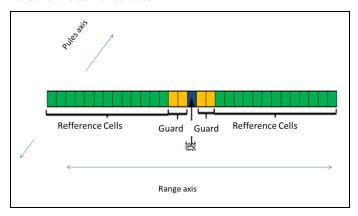
$$\Delta(z) = \frac{|p^H R_b^{-1} z|^2}{(p^H R_b^{-1} p)(z^H R_b^{-1} z)} \ge Threshold$$

This test is called ANMF which stands for Adaptive Normalized Matched Filter.

This detection test measures the squared cosine of the angle between the "steering vector" and the data, both whitened and normalized. In the first place, it requires the evaluation of the clutter correlation matrix. Estimating this matrix is a decisive step for the methods introduced in this paper. Next section presents the proposed algorithms to perform this estimation.

#### III. ESTIMATION OF THE CORRELATION MATRIX

Similarly to known estimators, evaluating the clutter distribution requires the definition of a set of reference cells around the cell under test.



The clutter covariance matrix is estimated on these Reference cells. When the clutter is Gaussian, the Sample Covariance Matrix:

$$\frac{1}{N_{ref\_cells}} \sum_{ref\_cells} z_k z_k^H$$

is well suited for the problem, otherwise one must resort to other algorithms [1].

The Tyler Fixed-Point Algorithm and Burg-Tyler algorithm are defined by the following recursive approaches.

## A. Tyler Fixed-Point Algorithm

- Initialize  $R_0 = I$
- For each step *n*

$$\circ \quad \mathbf{R}_{n+1} = \frac{N_p}{N_{ref\_cells}} \sum_{k=1}^{N_{ref\_cells}} \frac{z_k z_k^H}{z_k^H \mathbf{R}_n^{-1} z^k}$$

- If  $d(R_n, R_{n+1}) \le \epsilon$ 
  - $\circ$  Return  $R_{n+1}$

## B. Burg-Tyler Algorithm

- Initialize  $R_0^{-1} = I$
- For each step n:
  - for each sample  $z_k$ , compute de  $y_n^k = \frac{z_k}{z_k^H R_n^{-1} z_k}$

- Ocompute  $R_{n+1}^{-1}$  using Burg algorithm for multiple "snapshots":  $(y_n^k)_{1 \le n \le N_{amb}}$
- If  $d(R_{k+1}, R_k) \le \epsilon$ • Return  $R_{n+1}$

Burg algorithm for multiple segments is a method to estimate parameters of autoregressive processes in order to rebuild the correlation matrix which is theoretically Toeplitz. Additional information can be found in [4].

## IV. PERFORMANCE ON SIMULATED DATA

In order to compare the adaptive processing to a known reference, we display in this section the results of detection using a fixed Doppler bank of filters coupled with an OS-CFAR normalisation (we called it BFOS in the following) in addition to the new algorithms.

In the following, the clutter is modelled as:

$$c = \sqrt{\tau}g + b_{\text{noise}}$$
.

The texture τ follows a Weibull law:

$$p(\tau) = \frac{k}{\lambda} \Big( \frac{\tau}{\lambda} \Big)^{(k-1)} \, e^{-(\tau/\lambda)^k}$$

with parameters:  $\lambda = 0.74$ , k = 0.65

g follows a complex Gaussian law:  $N_c(0, M)$ .

The correlation matrix *M* is computed from its Doppler spectrum [5]:

$$p(v) = \frac{\beta}{2} e^{-\beta |v|} \frac{1}{1+\alpha} + \frac{\alpha}{1+\alpha} \delta(v),$$

the parameters of which are adapted to an S-band radar. The Clutter to Noise Ratio is about 60 dB.

 $b_{noise}$  follows a complex Gaussian law  $N_C(0, I)$ .

The samples along the range axis are i.i.d.

## A. False alarm management

The following figure shows the detection threshold behaviour with respect to the speed (normalized by the ambiguous speed) for a perfectly known correlation matrix (the corresponding detector is denoted as "clear-sighted ANMF"), in a situation of clutter plus thermal noise.

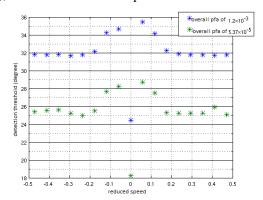


Figure 1. Detection thresholds for clear-sighted ANMF

We can notice that even when the correlation matrix is perfectly known, the ANMF is not at constant false alarm rate detector, since the threshold depends on the speed hypothesis. This is because of the addition of thermal noise in the clutter; this issue is recurrent on every detection test derived from the ANMF.

In the simulations, the thresholds were set by Monte-Carlo simulations in order to get a false alarm probability of  $5.37 \ 10^{-5}$  on all filters.

The following figures illustrate the probability of detection regarding the target speed for various SNR (from 15 dB to 28 dB, SW1 fluctuation).

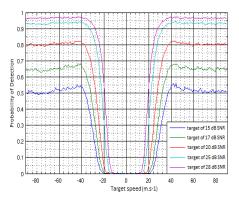


Figure 2. BFOS Algorithm

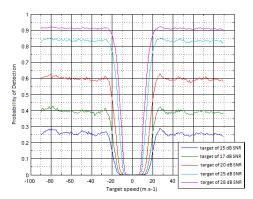


Figure 3. ANMF-FP Algorithm

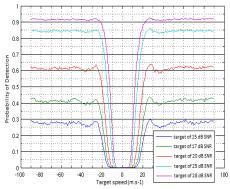


Figure 4. Burg-Tyler Algorithm

All the algorithms behave the same way (flat Pd) when the target speed is far enough from the central speed of the clutter (beyond  $\pm 40 \, m/s$ ).

For speed ranges above 40 m/s, the BFOS always outperforms the adaptive algorithms. Normalization and estimation of correlation matrix lead to bigger losses than the Doppler filter bank weightings in those areas. However, the adaptive detection techniques do allow a better detection capability for low-speed targets.

#### V. ROBUSTNESS TO TARGETS IN REFERENCE CELLS

Detection is likely to be degraded by the presence of targets in the estimation window. For the adaptive detection algorithms, the presence of same speed targets as the target in the cell under test leads to interpret this very target as an interference and make the detection function underperform. Preselecting data before the estimation of the clutter correlation matrix lowers the desensitization due to unwanted targets.

For both of the introduced algorithms, a selection procedure is included in the computation loop. It consists in the selection of the data  $z_k$  with the smallest quantity:  $z_k^H R_{n-1}^{-1} z_k$ . This is justified as follows. If the texture  $\tau$  is unknown (and deterministic) it can be estimated with the help of a maximum likelihood approach:

$$\tau = \operatorname{argmax}_{\tau}(-\log(|\tau M|) - z^{H}(\tau M)^{-1}z)$$

The likelihood is then equal to:

$$l(x; M) = -d \log(z^H R^{-1} z) + f(M)$$

One only uses the p % samples of higher likelihood l(x; M), i.e. the smallest value of:  $z_k^H R_{n-1}^{-1} z_k$ 

The following figures show the efficiency of the propose approach when one to 4 unwanted targets are present in the reference cells. Algorithm with data selection are named: PBT and ANMF-PFP

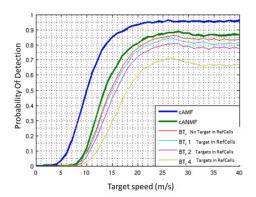


Figure 5. Robustness BT Algorithm

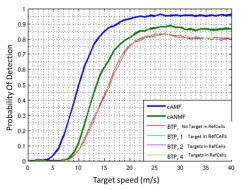


Figure 6. Robustness BTP Algorithm

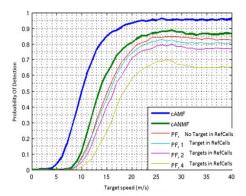


Figure 7. Robustness ANMF-FP Algorithm

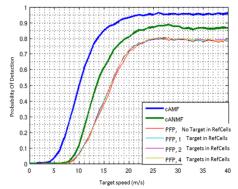


Figure 8. Robustness ANMF-PFP Algorithm

#### VI. STUDIES ON REAL DATA

Earlier we displayed the non-CFAR behavior of ANMF. It implies setting up a specific method for detection threshold definition in order to use the adaptive treatment on clutter recorded data. The approach we used is based on statistical learning. It is called Adaptation To Environment (ATE). It relies on actual environment data to split the area surrounding the radar. This division is made by land type (forests, lakes, etc.) The idea is to have a satisfyingly homogenous clutter for each subpart of the division; the purpose is to define one threshold per clutter type present in the radar surroundings. Thus, for each area, and with segmentation along the speed axis, we set a factor c which is updated through the learning. Whichever the covariance matrix estimator chosen, noting  $s(v_i)$  the Doppler spectrum evaluated for reduced speed  $v_i$ , the detection test becomes:

$$ANMF(x|\hat{R}, v_i) \ge \lambda_{PFA} * \left(\frac{s(v_i)}{\sigma_{hth}}\right)^c$$

where  $\sigma_{bth}$  is the thermal noise level, which is known or evaluated and  $\lambda_{PFA}$  is the thermal noise threshold. This method allows to set a threshold taking the clutter into account on two scales: by clutter type since the factor c is the same for a homogeneous area, but also locally with  $s(v_i)$  which is the local estimated spectrum at the range cell.

## VII. CONCLUSION

The adaptive processing proposed in this article are « invariant » to the target power. Indeed these detection methods measure an angle between observation and speed hypothesis; therefore, beyond some level of SNR the probability of detection only depends on the angle between signal of use and clutter. Hence they are not optimal to treat speeds distant from the clutter speed in comparison to a more usual technique. The purpose is not to use them to manage any situation. They should rather be paired with other detectors which would handle cases where the evaluated correlation matrix is not close to the clutter (like with BFOS).

#### ACKNOWLEDGMENT

The authors thank the French Ministry of Defense for its support.

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