# Robust target detection for Hyperspectral Imaging

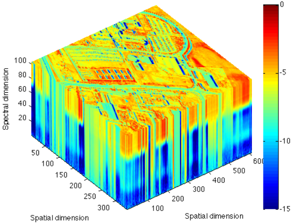
Joana Frontera-Pons
SONDRA

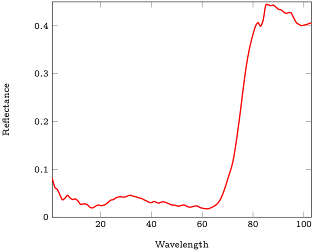
PhD defense
Under the supervision of Frédéric Pascal & Jean-Philippe Ovarlez (PhD director)

December 10, 2013

## Hyperspectral Imaging (HSI)

- **ANOMALY DETECTION IN HYPERSPECTRAL IMAGES**
  To detect all that is "different " from the background (Mahalanobis distance) - No information about the targets of interest available.
- **DETECTION OF TARGETS IN HYPERSPECTRAL IMAGES**
  To detect targets characterized by a given spectral signature **p** - Regulation of False Alarm.



- Many methodologies for detection and classification in hyperspectral images can be found in radar detection community. We can retrieve all the detectors family commonly used in radar detection (AMF (intensity detector), ACE (angle detector), sub-spaces detectors, ...).
- **Almost all** the conventional techniques for anomaly detection and targets detection are based on Gaussian assumption and on spatial homogeneity in hyperspectral images.

**All these techniques need to estimate the data covariance matrix $\Sigma$ (whitening process) and the mean vector $\mu$.**

# Outline

# Outline

## Problem Statement

- In a $m$-dimensional observation vector $\mathbf{x}$, the problem of detecting a complex known signal $\mathbf{s} = \alpha\,\mathbf{p}$ ($\mathbf{p}$ is the steering vector and $\alpha$ the target amplitude), corrupted by an additive noise $\mathbf{b}$, can be stated as the following binary hypothesis test :

$$\begin{cases} \text{Hypothesis } \mathcal{H}_0: & \mathbf{x} = \mathbf{b} & \mathbf{x}_i = \mathbf{b}_i & i = 1, \dots, N \\ \text{Hypothesis } \mathcal{H}_1: & \mathbf{x} = \mathbf{s} + \mathbf{b} & \mathbf{x}_i = \mathbf{b}_i & i = 1, \dots, N \end{cases}$$

where the $\mathbf{x}_i$'s are $N$ "signal-free" independent observations (secondary data) used to estimate the background parameters .

$\Rightarrow$ Neyman-Pearson criterion : Mazimize the probability of detection for a fixed probability of false alarm.

## Problem Statement

- Detection test: comparison between the Likelihood Ratio (LR) $\Lambda(\mathbf{x})$ and a detection threshold $\lambda$:

$$\Lambda(\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{H}_1)}{p(\mathbf{x}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta \, .$$

$\lambda$ is determined for a fixed value of $PFA$ (set by the user):

- Probability of False Alarm (type-I error):

$$PFA = \mathbb{P}(\Lambda(\mathbf{x}; \mathcal{H}_0) > \lambda)$$

- Probability of Detection (to evaluate the performance):

$$PD = \mathbb{P}(\Lambda(\mathbf{x}; \mathcal{H}_1) > \lambda)$$

for different Signal-to-Noise Ration (SNR).

## Gaussian distribution

A $m$-dimensional vector $\mathbf{x}$ has a complex Gaussian distribution denoted $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. If the probability density function exists, it is of the form:

$$f_{\mathbf{x}}(\mathbf{x}) = \pi^{-m} |\boldsymbol{\Sigma}|^{-1} \exp\{-(\mathbf{x}-\boldsymbol{\mu})^H \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})\}.$$

### Maximum Likelihood Estimators:

Let $\mathbf{x}_1, \ldots, \mathbf{x}_N$ be an IID $N$-sample, where $\mathbf{x}_i \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Thus, the SMV and the SCM can be written as:

$$\hat{\boldsymbol{\mu}}_{SMV} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_i, \quad \hat{\boldsymbol{\Sigma}}_{SCM} = \frac{1}{N} \sum_{i=1}^{N} (\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})^H .$$

- Simplicity of analysis and well-known statistical properties: consistent, unbiased and efficient,
- $\hat{\boldsymbol{\Sigma}}_{SCM}$ is Wishart distributed.

# Outline

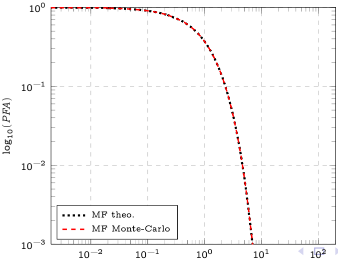
# Matched Filter

The **Matched Filter** is the optimal filter for maximizing the SNR under Gaussian background assumption:

$$\Lambda_{MF} = \frac{\left|\mathbf{p}^H \, \boldsymbol{\Sigma}^{-1} \left(\mathbf{x} - \boldsymbol{\mu}\right)\right|^2}{\left(\mathbf{p}^H \, \boldsymbol{\Sigma}^{-1} \, \mathbf{p}\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{MF} = \exp\left(-\lambda\right)$$



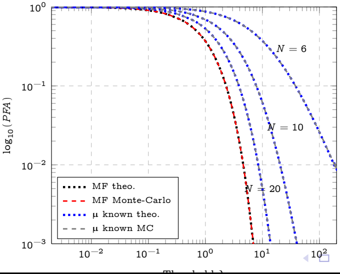
# Adaptive Matched Filter

Unknown Covariance matrix:

$$\Lambda^{(N)}_{AMF\,\hat{\boldsymbol{\Sigma}}} = \frac{\left|\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right|^2}{\left(\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,\mathbf{p}\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{AMF\,\hat{\boldsymbol{\Sigma}}} = {}_2F_1\left(N-m+1,\; N-m+2;\; N+1;\; -\frac{\lambda}{N}\right)$$



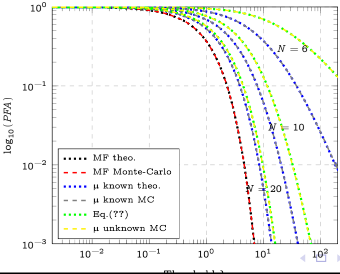
# Adaptive Matched Filter

Unknown Covariance matrix and Mean Vector:

$$\Lambda^{(N)}_{AMF\,\hat{\mathbf{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{|\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,(\mathbf{x}-\hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,\mathbf{p})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{AMF\,\hat{\mathbf{\Sigma}},\hat{\boldsymbol{\mu}}} = {}_2F_1\left(N-m,\ N-m+1;\ N;\ -\frac{\lambda'}{N-1}\right)$$



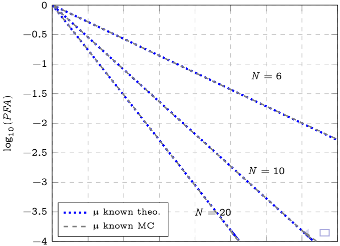
# Kelly detection test

The **Kelly detector** is based on the Generalized Likelihood Ratio Test assuming Gaussian distribution and unknown covariance matrix $\boldsymbol{\Sigma}$:

$$\Lambda^{(N)}_{Kelly\,\hat{\boldsymbol{\Sigma}}} = \frac{\left|\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right|^2}{\left(\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\mathbf{p}\right)\left(N+(\mathbf{x}-\boldsymbol{\mu})^H\,\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{Kelly} = (1-\lambda)^{N-m+1}$$



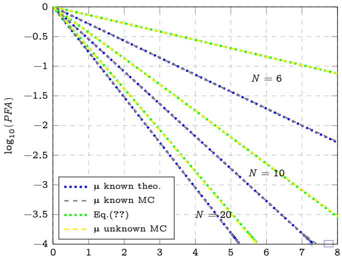
# Kelly "Plug-in" detection test

Unknown Covariance matrix and Mean Vector:

$$\Lambda^{(N)}_{Kelly\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{\left|\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\,(\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV})\right|^2}{\left(\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\mathbf{p}\right)\,\left(N+(\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV})^H\,\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\,(\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV})\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{Kelly\,\hat{\boldsymbol{\Sigma}},\,\hat{\boldsymbol{\mu}}} = \frac{\Gamma(N)}{\Gamma(N-m+1)\,\Gamma(m-1)}\int_0^1 \left[1+\frac{\lambda}{1-\lambda}\left(1-\frac{u}{N+1}\right)\right]^{m-N} u^{N-m}\,(1-u)^{m-2}\,du$$



# New Kelly detection test

Unknown Covariance matrix and Mean Vector:

## Generalized Kelly detector

$$\Lambda = \frac{\beta(N)\,\left|\mathbf{p}^H \widehat{\mathbf{S}}_0^{-1}(\mathbf{x}-\hat{\mu}_0)\right|^2}{\left(\mathbf{p}^H \widehat{\mathbf{S}}_0^{-1}\mathbf{p}\right)\left(1+(\mathbf{x}-\hat{\mu}_0)^H\,\widehat{\mathbf{S}}_0^{-1}\,(\mathbf{x}-\hat{\mu}_0)\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

where $\widehat{\mathbf{S}}_0 = \sum_{i=1}^{N}(\mathbf{x}_i-\hat{\mu}_0)(\mathbf{x}_i-\hat{\mu}_0)^H$, and $\hat{\mu}_0 = \frac{1}{N+1}\left(\mathbf{x}+\sum_{i=1}^{N}\mathbf{x}_i\right)$.

- New detector derived when both the mean vector and the covariance matrix are unknown, Generalized Likelihood Ratio Test,
- The covariance matrix $\widehat{\mathbf{S}}_0$ and the mean vector $\hat{\mu}_0$ estimates depend on the vector under test $\mathbf{x}$,
- $\widehat{\mathbf{S}}_0$ and $\mathbf{x}-\hat{\mu}_0$ are not independent and $\widehat{\mathbf{S}}_0$ is NOT Wishart distributed,
- The distribution of the detector is unknown.

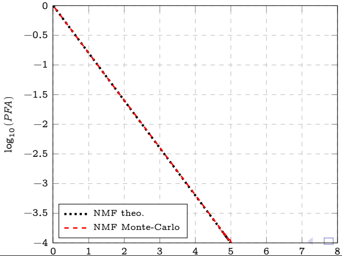
# Normalized Matched Filter

The **Normalized Matched Filter** is obtained when considering that the background and the target have the same covariance structure but different variance.

$$\Lambda_{NMF} = \frac{\left|\mathbf{p}^H\, \mathbf{\Sigma}^{-1}\, (\mathbf{x}-\boldsymbol{\mu})\right|^2}{\left(\mathbf{p}^H\, \mathbf{\Sigma}^{-1} \mathbf{p}\right)\, \left((\mathbf{x}-\boldsymbol{\mu})^H\, \mathbf{\Sigma}^{-1}\, (\mathbf{x}-\boldsymbol{\mu})\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{NMF} = (1-\lambda)^{m-1}$$



# Adaptive Normalized Matched Filter

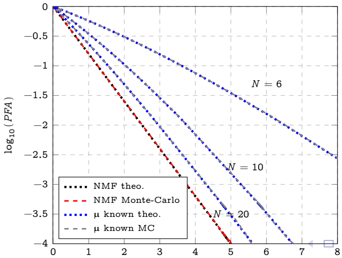
Unknown Covariance matrix:

$$\Lambda^{(N)}_{ANMF\,\hat{\boldsymbol{\Sigma}}} = \frac{\left|\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right|^2}{\left(\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p}\right)\,\left((\mathbf{x}-\boldsymbol{\mu})^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

## PFA-threshold relationship

$$PFA_{ANMF\,\hat{\boldsymbol{\Sigma}}} = (1-\lambda)^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda)\,,$$

where $a = N - m + 2$ and $b = N + 2$.



# Adaptive Normalized Matched Filter

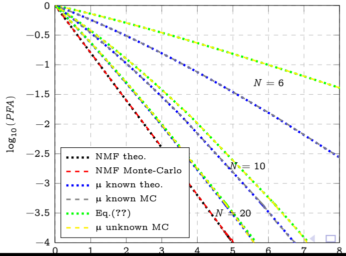
Unknown Covariance matrix and Mean Vector:

$$\Lambda_{ANMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{|\mathbf{p}^H \hat{\boldsymbol{\Sigma}}^{-1} (\mathbf{x} - \hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H \hat{\boldsymbol{\Sigma}}^{-1} \mathbf{p}) \left( (\mathbf{x} - \hat{\boldsymbol{\mu}})^H \hat{\boldsymbol{\Sigma}}^{-1} (\mathbf{x} - \hat{\boldsymbol{\mu}}) \right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

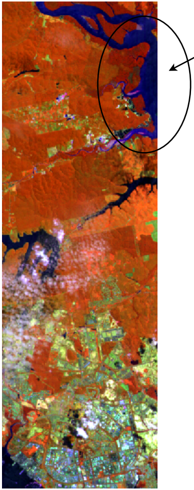
## PFA-threshold relationship

$$PFA_{ANMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = (1-\lambda)^{a-1} {}_2F_1 \left(a, a-1; b-1; \lambda\right) ,$$

where $a = (N-1) - m + 2$ and $b = (N-1) + 2$.



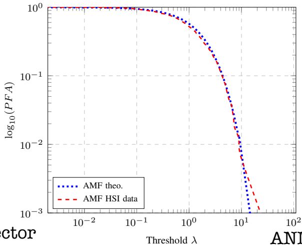
# Adaptive detection on real Hyperspectral Image



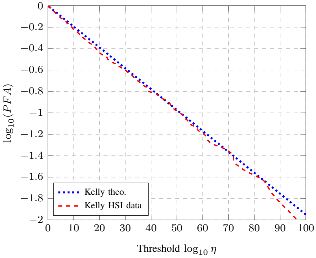
Gaussian region

## FALSE ALARM REGULATION FOR GAUSSIAN-BASED DETECTORS

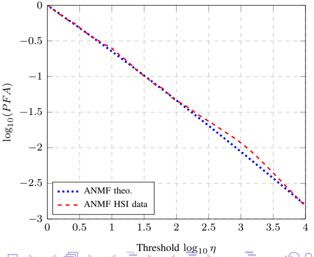
AMF




Kelly Detector

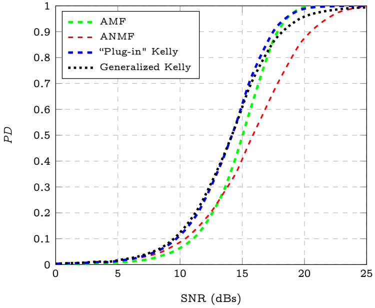



ANMF

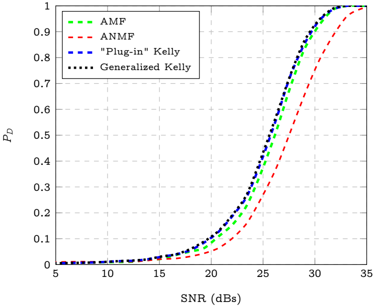


# Performance evaluation

Synthetic target with known spectral signature $\mathbf{p}$ embedded in the background.




with simulated background

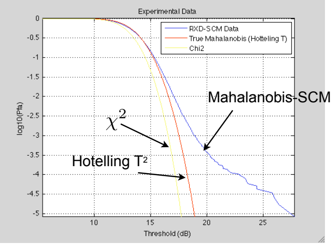



with Hyperion image

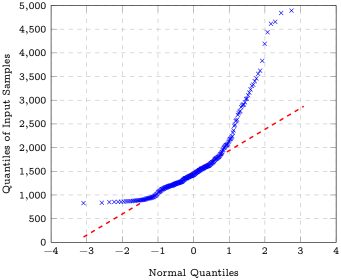
The performance results are obtained for a fixed $PFA = 10^{-3}$.

# First comments and adequacy with some results found in the literature

- Hyperspectral data are generally spatially heterogeneous in intensity and they cannot be only characterized by Gaussian distribution:




*Mahalanobis on DSO Experimental data*




- Elliptical distribution models have started to be studied in the hyperspectral scientific community but one generally uses .... Gaussian estimates !

# Outline

# Elliptical distributions for Hyperspectral background modeling

## Complex Elliptically Contoured Distributions

Let $\mathbf{z}$ be a complex circular random vector of dimension $m$. $\mathbf{z}$ has a complex elliptically (CE) distribution ($\mathcal{CE}(\boldsymbol{\mu}, \boldsymbol{\Sigma}, h_m)$) if its PDF is of the form:

$$f_{\mathbf{z}}(\mathbf{z}) = |\boldsymbol{\Sigma}|^{-1} h_m\left((\mathbf{z}-\boldsymbol{\mu})^H \boldsymbol{\Sigma}^{-1} (\mathbf{z}-\boldsymbol{\mu})\right) \tag{1}$$

where $h_m : [0, \infty) \to [0, \infty)$ is the density generator and is such as (1) defines a PDF.

- $\boldsymbol{\mu}$ is the mean vector,
- $\boldsymbol{\Sigma}$ is the scatter matrix.

In general, $\boldsymbol{\Sigma}$ equals to the covariance matrix up to a scalar factor. It characterizes the correlation structure existing within the spectral bands

Powerful statistical model that allows:

- to extend the Gaussian model (K, Weibull, Fisher, Cauchy, Alpha-Stable, Generalized Gaussian, etc.),
- to encompass the Gaussian model,
- to take into account the heterogeneity of the background power with the texture,
- to take into account possible correlation existing within the $m$-channels of observation.

# Robust $M$-estimators

## $M$-estimators

The complex $M$-estimators of location and scatter are defined as the joint solutions of:

$$\hat{\boldsymbol{\mu}}_N = \frac{\sum_{i=1}^{N} u_1(t_i)\, \mathbf{z}_i}{\sum_{i=1}^{N} u_1(t_i)}, \quad \hat{\boldsymbol{\Sigma}}_N = \frac{1}{N}\sum_{i=1}^{N} u_2\left(t_i^2\right) (\mathbf{z}_i - \hat{\boldsymbol{\mu}})(\mathbf{z}_i - \hat{\boldsymbol{\mu}})^H,$$

where $t_i = \left( (\mathbf{z}_i - \hat{\boldsymbol{\mu}})^H \hat{\boldsymbol{\Sigma}}^{-1} (\mathbf{z}_i - \hat{\boldsymbol{\mu}}) \right)^{1/2}$.

- $u_1(\cdot)$, $u_2(\cdot)$ are two weighting functions acting on the quadratic form, i.e. Mahalanobis distance,
- The choice of $u_1(\cdot)$, $u_2(\cdot)$ results in different estimates for the covariance matrix and the mean vector,
- Existence and uniqueness of the solution have been proven provided $u_1(\cdot)$, $u_2(\cdot)$ satisfy given conditions [Maronna 1976],

# The Fixed Point Estimators

The Fixed Point Estimators (FPE) firstly introduced in [Tyler 1987], satisfy the following implicit equations:

$$\hat{\boldsymbol{\mu}}_{FP} = \frac{\displaystyle\sum_{i=1}^{N} \frac{\mathbf{z}_i}{\sqrt{\left((\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})^H \widehat{\mathbf{M}}_{FP}^{-1} (\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})\right)}}}{\displaystyle\sum_{i=1}^{N} \frac{1}{\sqrt{\left((\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})^H \widehat{\mathbf{M}}_{FP}^{-1} (\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})\right)}}}, \quad \widehat{\mathbf{M}}_{FP} = \frac{m}{N} \sum_{i=1}^{N} \frac{(\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})(\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})^H}{\left((\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})^H \widehat{\mathbf{M}}_{FP}^{-1} (\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{FP})\right)}$$

These two quantities can be jointly reached by an iterative algorithm

- This estimator does not depend on the elliptical distribution density generator,
- Robust to outliers, strong targets or scatterers in the reference cells,
- FPE matrix estimator is consistent, unbiased, asymptotically Gaussian and is, for a fixed number $N$ of secondary data, Wishart distributed with $\frac{m}{m+1}N$ degrees of freedom.

## The Huber's $M$-estimators

The Huber´s $M$-Estimators are defined when taking the following weighting functions $u_1(\cdot)$ and $u_2(\cdot)$:

$$u_1(t) = \min(t, k), \; u_2(t^2) = \frac{1}{\beta} \min(t^2, k^2)$$

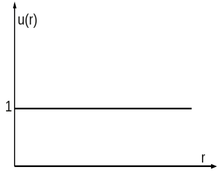
where $q = F_{\chi^2_{2m}}(2\,k^2)$ and $\beta = F_{\chi^2_{2m}}(2\,k^2) + k^2\,\frac{1-q}{m}$.

- Extreme values of $t_i$ outside the interval $[0, k^2]$ are attenuated (Fixed Point behavior),
- Normal values below $k^2$ are uniformly kept (SCM behavior),
- The parameter $k$ can be adjusted to choose the percentage of data treated as Gaussian,
- The Huber estimate is consistent, unbiased, asymptotically Gaussian and is, for a fixed number $N$ Whishart distributed with $\nu_1 N$ degrees of freedom ($\nu_1$ very close to 1).

# Examples of $M$-estimators

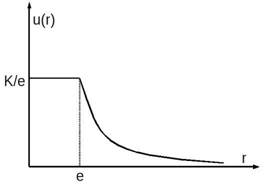
**SCM:**

$$u(t) = 1$$



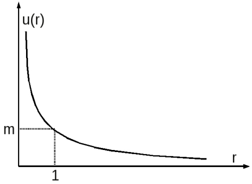

**Huber's $M$-estimator:**

$$u(t) = \begin{cases} 1/k^2 \text{ if } t <= k^2 \\ 1/t \text{ if } t > k^2 \end{cases}$$




**FPE (Tyler):**

$$u(t) = \frac{m}{t}$$



# Asymptotic distribution of complex $M$-estimators

## Asymptotic distribution of $\hat{\mathbf{\Sigma}}_N$

$$\sqrt{N}(\hat{\mathbf{\Sigma}}_N - \mathbf{\Sigma}) \xrightarrow{d} \mathcal{CN}\left(\mathbf{0}, \nu_1 \left(\mathbf{\Sigma}^T \otimes \mathbf{\Sigma}\right) + \nu_2 \,\mathrm{vec}(\mathbf{\Sigma})\mathrm{vec}(\mathbf{\Sigma})^H\right) ,$$

where $\nu_1$ and $\nu_2$ are completely defined.

- Let $H(\mathbf{V})$ be a a function on the set of complex positive definite Hermitian $m \times m$ matrices that satisfies $H(\mathbf{V}) = H(c\,\mathbf{V})$ for any positive scalar $c$ and let us assume that all the partial derivates are continuous, e.g. the ANMF statistic, the MUSIC statistic.

# An important property of complex $M$-estimators

## Asymptotic distribution of $H(\Sigma)$

$$\sqrt{N}\left(H(\hat{\boldsymbol{\Sigma}}) - H(\hat{\boldsymbol{\Sigma}})\right) \xrightarrow{d} \mathcal{CN}\left(\mathbf{0}, \vartheta_1\, H'(\boldsymbol{\Sigma})(\boldsymbol{\Sigma}^T \otimes \boldsymbol{\Sigma})H'(\boldsymbol{\Sigma})^H\right),$$

where $H'(\boldsymbol{\Sigma}) = \dfrac{\partial H(\boldsymbol{\Sigma})}{\partial \mathrm{vec}(\boldsymbol{\Sigma})}$.

$H(SCM)$ and $H(M\text{-estimators})$ share the same asymptotic distribution (differs from $\vartheta_1$).

# Adaptive Detection in Elliptical Background

ANMF built with $M$-estimators

$$\Lambda_{ANMF\,\hat{\Sigma},\hat{\mu}} = \frac{\left|\mathbf{p}^H\,\hat{\Sigma}_N^{-1}\,(\mathbf{x}-\hat{\mu}_N)\right|^2}{(\mathbf{p}^H\,\hat{\Sigma}_N^{-1}\mathbf{p})\,\left((\mathbf{x}-\hat{\mu}_N)^H\,\hat{\Sigma}_N^{-1}\,(\mathbf{x}-\hat{\mu}_N)\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

**PFA-threshold relationship**

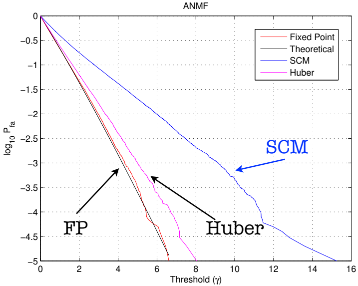
$$PFA_{ANMF\,\hat{\Sigma},\hat{\mu}} = (1-\lambda)^{a-1}\,{}_2F_1\,(a, a-1; b-1; \lambda)\ ,$$

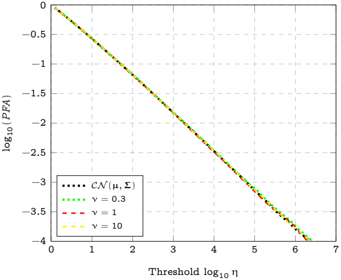
where $a = \vartheta_1(N-1) - m + 2$ and $b = \vartheta_1(N-1) + 2$.

The parameter $\vartheta_1$ is very close to 1 but depends on the $M$-estimator: Ex: for the FPE, $\vartheta_1 = m/(m+1)$.

# Adaptive Detection in Elliptical Background

- This two-step GLRT test is homogeneous of degree 0: it is independent of any particular Elliptical distribution: CFAR texture and CFAR Matrix properties,
- Under homogeneous Gaussian region, it reaches the same performance than those of the detector built with the SCM estimate.

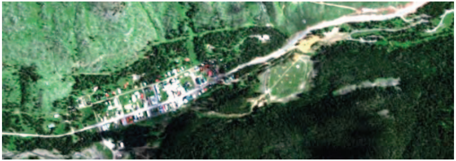


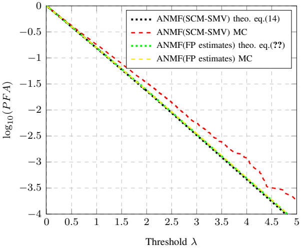
# Results in real Hyperspectral Images

**The hyperspectral data are real and positive as they represent radiance or reflectance.**

- A mean vector has to be included in the model and estimated jointly with the scatter matrix,
- The real data has been transformed into complex ones by a linear Hilbert filter and then be decimated by a factor 2 (principle of analytic signals)

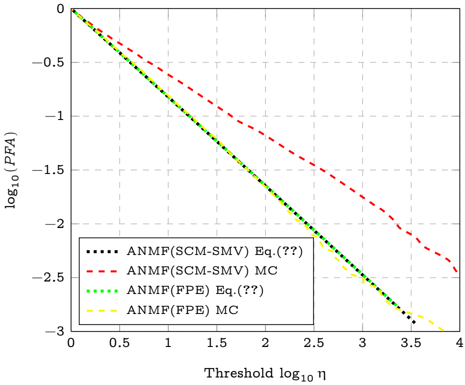


Original data set (Hymap data)



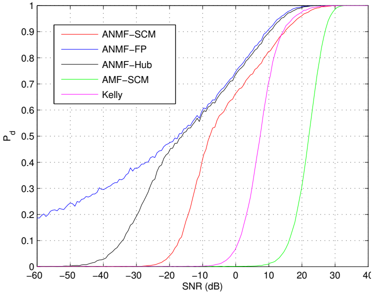

False Alarm Regulation

# Results in real Hyperspectral Images

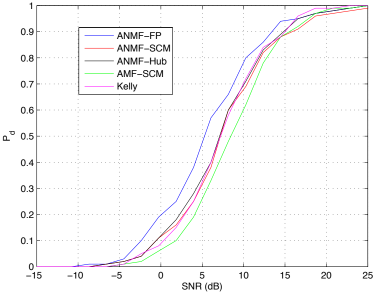


# Performance evaluation

Synthetic target with known spectral signature $\mathbf{p}$ embedded in the background.




**with simulated data**




**with Hymap data**

The performance results are obtained for a fixed $PFA = 10^{-3}$.

## Shrinkage Covariance Matrix Estimators

Small number of observations or under-sampling $N < m$: matrix is not invertible $\Rightarrow$ Problem when using $M$-estimators or FPE!.

**Regularized SCM:**

$$\hat{\mathbf{M}}_{SCM-DL}(\beta) = \frac{1-\beta}{N} \sum_{i=1}^{N} (\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{SMV}) (\mathbf{z}_i - \hat{\boldsymbol{\mu}}_{SMV})^H + \beta\, \mathbf{I}_m$$

- Not appropriate for non-Gaussian, impulsive background.

# Shrinkage Fixed Point Estimator

## Shrinkage FPE

The shrinkage FPE introduced in [Pascal2013] is defined as the solution of the following fixed point equation:

$$\hat{\mathbf{M}}_{FP}(\beta) = (1-\beta)\,\frac{m}{N}\sum_{i=1}^{N}\frac{(\mathbf{z}_i-\hat{\mu}_{FP})(\mathbf{z}_i-\hat{\mu}_{FP})^H}{((\mathbf{z}_i-\hat{\mu}_{FP})^H\,\hat{\mathbf{M}}_{FP}^{-1}(\beta)\,(\mathbf{z}_i-\hat{\mu}_{FP}))}+\beta\,\mathbf{I},$$

subject to the no trace constraint but for $\beta \in (\bar{\beta}, 1]$, where $\bar{\beta} := \max(0, 1 - N/m)$.

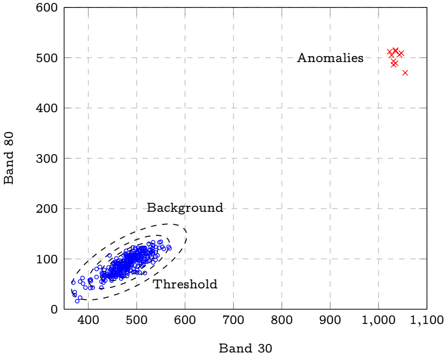
- $\hat{\mathbf{M}}_{FP}(\beta)$ verifies $\mathrm{Tr}\left(()\,\hat{\mathbf{M}}_{FP}(\beta)^{-1}\right) = m$ for all $\beta \in (0, 1]$.

**The main challenge is to find the optimal $\beta$!**

# Outline

# Anomaly Detection

- To detect all that is "different " from the background - No information about the targets of interest available.
- Anomaly Detectors cannot distinguish between true targets and detections of bright pixels of the background or targets that are not of interest.



## Reed-Xiaoli Detector

The RXD [Reed1990] is commonly considered as the benchmark anomaly detector for hyperspectral data:

$$\Lambda(\mathbf{X}) = \frac{(\mathbf{X}\boldsymbol{\alpha}^T)^T(\mathbf{X}\mathbf{X}^T)^{-1}(\mathbf{X}\boldsymbol{\alpha}^T)}{\boldsymbol{\alpha}\boldsymbol{\alpha}^T}$$

The sampled version when assuming non-zero mean Gaussian background yields:

$$\Lambda_{ARXD} = (\mathbf{x}_i - \hat{\boldsymbol{\Sigma}}_{SMV})^T \hat{\boldsymbol{\Sigma}}_{SCM}^{-1} (\mathbf{x}_i - \hat{\boldsymbol{\mu}}_{SMV}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

- $\mathbf{x}_i$ is present in the covariance estimation,
- $N$ secondary data are NOT signal-free,
- Global strategy.

## Kelly Anomaly Detector

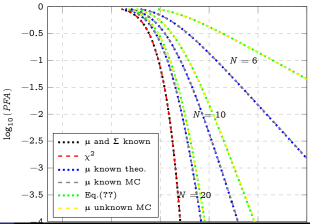
Obtained when deriving the Kelly's LR w.r.t. the steering vector $\mathbf{p}$.

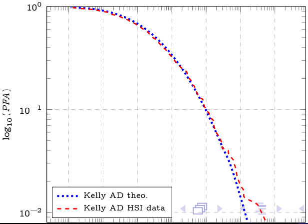
$$\Lambda^{(N)}_{KellyAD\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = \left(\mathbf{x} - \hat{\boldsymbol{\mu}}_{SMV}\right)^T \hat{\boldsymbol{\mu}}_{SCM}^{-1} \left(\mathbf{x} - \hat{\boldsymbol{\mu}}_{SMV}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda\boldsymbol{\Sigma}$$

### Detector distribution under Gaussian hypothesis

$$\frac{N-m}{m\,(N+1)}\Lambda^{(N)}_{KellyAD\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\Sigma}}} \sim F_{m,N-m}\ ,$$

with $F_{m,N-m}$ is the non-central $F$-distribution with $m$ and $N-m$ degrees of freedom.



## Other Anomaly Detectors

- **Normalized-RXD**

$$\Lambda_{N-RXD} = \frac{(\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV})^T}{\|\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV}\|}\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}\frac{(\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV})^T}{\|\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV}\|} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

- **Uniform Target Detector**

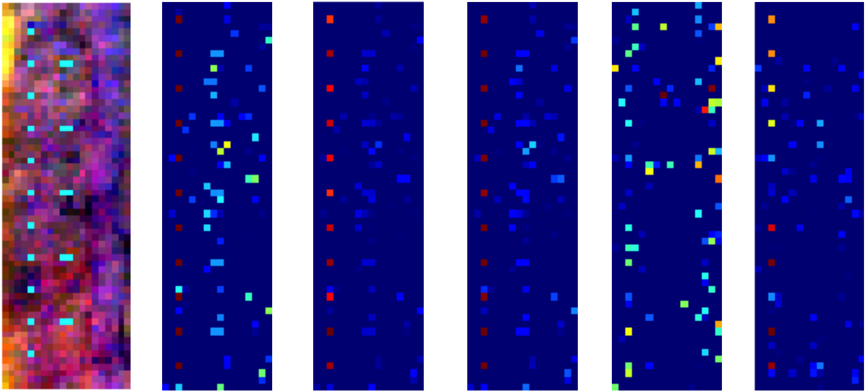
$$\Lambda_{UTD} = (\mathbf{1}-\hat{\boldsymbol{\mu}}_{SMV})^T\hat{\boldsymbol{\Sigma}}_{SCM}^{-1}(\mathbf{x}-\hat{\boldsymbol{\mu}}_{SMV}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda\,.$$

### Generalized Kelly Anomaly Detector

$$\Lambda_{G-KellyAD} = (\mathbf{x}-\hat{\boldsymbol{\mu}}_0)^H\,\hat{\mathbf{S}}_0^{-1}\,(\mathbf{x}-\hat{\boldsymbol{\mu}}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda$$

where $\hat{\mathbf{S}}_0 = \sum_{i=1}^{N}(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_0)(\mathbf{x}_i-\hat{\boldsymbol{\mu}}_0)^H$, and $\hat{\boldsymbol{\mu}}_0 = \dfrac{1}{N+1}\left(\mathbf{x}+\sum_{i=1}^{N}\mathbf{x}_i\right)$.

# Results on Hyperion image



(a) Original (b) RXD (c) Kelly AD (d) G-Kelly (e) N-RXD (f) UTD

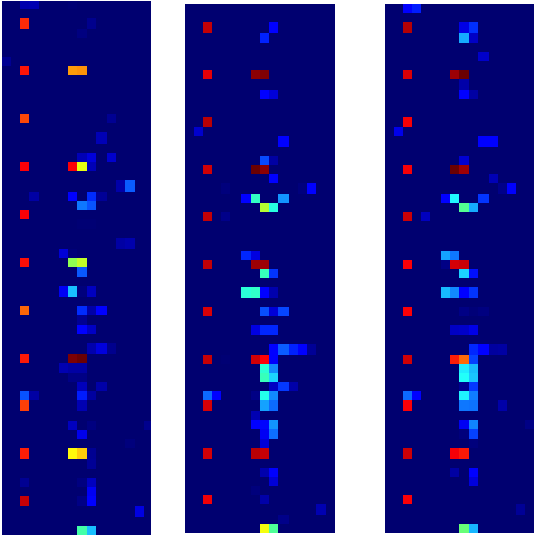
# Anomaly Detection in non-Gaussian environment

Robust Kelly Anomaly detector built with $M$-estimators:

$$\Lambda_{KellyAD\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = (\mathbf{x} - \hat{\boldsymbol{\mu}}_N)^T \hat{\boldsymbol{\Sigma}}_N^{-1} (\mathbf{x} - \hat{\boldsymbol{\mu}}_N) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda,$$

- Replace the unknown parameters by robust estimators ($M$-estimators or Shrinkage estimators),
- The detector's distribution depends on the underlying non-Gaussian distribution.

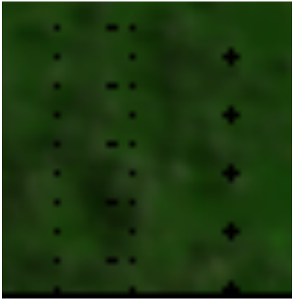
# Results on Hyperion image



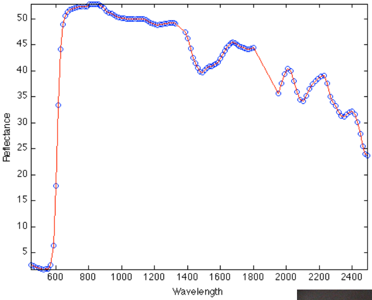
(a) FP (b) SCM-DL (c) Shrinkage FPE

# Results on Hymap image

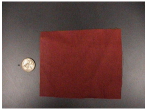
**Results obtained with artificial targets**



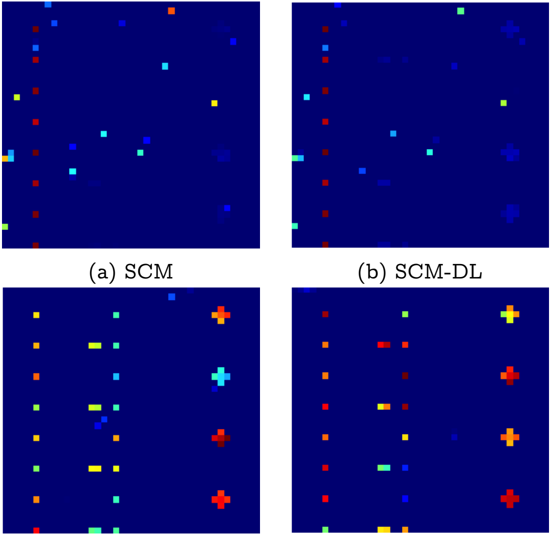
**Original image (Forest Region)**




**Target Spectrum**



# Results on Hymap image



(a) SCM (b) SCM-DL

(c) FPE (d) Shrinkage FPE

# Outline

## Conclusions

- Extension of classical Target detection and Anomaly detection techniques for non-zero mean case under Gaussian assumption,
- Hyperspectral images like radar or SAR images can suffer from non-Gaussianity or heterogeneity that can reduce the performance of anomaly detectors (RXD) and target detectors (AMF, ANMF),
- Elliptical Distributions modeling is a very useful theoretical tool for the hyperspectral context that can match and overcome the heterogeneity and non-Gaussianity of the images,
- Jointly used with robust estimates, the proposed hyperspectral detectors may provide better performances and a more accurate false alarm regulation. And they keep the same performance than the conventional Gaussian detectors for homogeneous and Gaussian data.

## Perspectives

- Subspace Projectors
- Random Matrix Theory
- Change Detection problems,

Thank you for your attention