A combined TSVM model and GLRT detector for a roll invariant target detection.

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Abstract

In this paper, a novel method is proposed for detecting Polarimetric Synthetic Aperture Radar (PolSAR) targets. The proposed method is a combination of the Target Scattering Vector Model (TSVM) and the Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) detector. The TSVM provides an unique and roll-invariant decomposition of the observed target vector by means of four independent parameters. The combination of those two methods will allow the detection of any oriented targets.

1 Introduction

The new generation of recently launched Synthetic Aperture Radar (SAR) sensors are able to produce high quality images of the Earth's surface with meter resolution. The number of scatterers present in each resolution cell decreases considerably and homogeneous hypothesis of the Polarimetric SAR (PolSAR) clutter can be reconsidered. Heterogeneous clutter models have therefore recently been studied.
In 1973, Kung Yao has first introduced the use of Spherically Invariant Random Vectors (SIRV) and their applications to estimation and detection in communication. From a PolSAR point of view, the observed target vector $\mathbf{k}$ is defined as the product of a square root of a positive random variable τ (representing the texture) with an independent complex Gaussian vector $\mathbf{z}$ with zero mean. SIRVs describe a whole class of stochastic processes. This class includes the conventional clutter models having Gaussian, $\mathcal{K}$, $\mathcal{G}^0$ and KummerU PDFs which correspond respectively to dirac, Gamma, Inverse Gamma and Fisher distributed texture.
Once the SIRV parameters are estimated (covariance matrix and texture parameters), optimal SAR detectors can be applied to detect particular targets. Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) detectors have been successfully applied to detect trihedral scattering. In 2007, Ridha Touzi has proposed a new target scattering vector model (TSVM) to extract physical parameters [1]. Based on the Kennaugh-Huynen decomposition, the TSVM allows to extract four roll-invariant parameters (independent of the incidence angle). Those parameters are necessary for an unambiguous characterization of target scattering. The proposed method consists in applying the TSVM prior to the GLRT-LQ for target detection (trihedral, dihedral, dipole, helix, ...).

2 Roll-invariant decomposition

2.1 Problem formulation

Let $\mathbf{k}_{dip}$ and $\mathbf{k}_{dih}$ be respectively the steering vectors in the Pauli basis of dipole and dihedral targets. They are expressed as follows :

$$\mathbf{k}_{dip} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \cos(2\psi) \\ \sin(2\psi) \end{bmatrix} \text{ and } \mathbf{k}_{dih} = \begin{bmatrix} 0 \\ \cos(2\psi) \\ \sin(2\psi) \end{bmatrix} \quad (1)$$

where ψ is the orientation of the maximum polarization with respect to the horizontal polarization [1].
Consequently, for a roll invariant-target dipole or dihedral detection, the tilt angle ψ should be removed. In 1993, Krogager has proposed an algorithm to derive ψ which uses the phase difference between right-right (S_{RR}) and left-left (S_{LL}) circular polarizations [2]. S_{RR} and S_{LL} are respectively defined by :

$$\begin{aligned} S_{RR} &= (S_{HH} - S_{VV} + 2jS_{HV})/2 \\ S_{LL} &= (S_{VV} - S_{HH} + 2jS_{HV})/2 \end{aligned} \quad (2)$$

The orientation angle $\psi_{Krogager}$ estimated by Krogager is given by :

$$\psi_{Krogager} = \left[\text{Arg}(S_{RR}S_{LL}^{*} + \pi)\right]/4 \quad (3)$$

This estimated orientation angle (Eq. 3) is valid under certain condition on the target. To overcome this problem, authors propose to apply the TSVM method which provide an unique and roll-invariant decomposition of any targets.

2.2 The Kennaugh-Huynen con-diagonalization

Coherent targets are fully described by their scattering matrix $\tilde{\mathbf{S}}$. Kennaugh and Huynen have proposed to apply the characteristic decomposition on the scattering matrix to retrieve physical parameters [3] [4] [5]. Under the reciprocity assumption, it yields :

$$\tilde{\mathbf{S}} = \tilde{\mathbf{R}}(\psi)\tilde{\mathbf{T}}(\tau_m)\tilde{\mathbf{S}}_{\mathbf{d}}\tilde{\mathbf{T}}(\tau_m)\tilde{\mathbf{R}}(-\psi) \tag{4}$$

where $\tilde{\mathbf{R}}(\boldsymbol{\psi})$ and $\tilde{\mathbf{T}}(\tau_m)$ are defined by :

$$\tilde{\mathbf{R}}(\boldsymbol{\psi}) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \tag{5}$$

and :

$$\tilde{\mathbf{T}}(\tau_m) = \begin{bmatrix} \cos\tau_m & -j\sin\tau_m \\ -j\sin\tau_m & \cos\tau_m \end{bmatrix} \tag{6}$$

$\tilde{\mathbf{S}}_{\mathbf{d}}$ is a diagonal matrix which contains the coneigenvalues μ_1 and μ_2 of $\tilde{\mathbf{S}}$ as :

$$\tilde{\mathbf{S}}_{\mathbf{d}} = \begin{bmatrix} me^{2j(\nu+\rho)} & 0 \\ 0 & m\tan^2\gamma\, e^{-2j(\nu-\rho)} \end{bmatrix} = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \tag{7}$$

The Kennaugh-Huynen con-diagonalization allows to characterize any targets by means of six independent parameters : ψ, τ_m, m, γ, ν and ρ. ψ is the rotation angle (see Eq. 1). This parameter is used for the subtraction of the target orientation from the target vector, which leads to a roll-invariant decomposition. This step is named desying. τ_m is the target helicity, it characterizes the symmetry of the target. m is the maximum amplitude return. γ and ν are respectively the characteristic and skip angles. ρ is the absolute phase of the target. This term is not observable except for interferometric applications.

2.3 The Target Scattering Vector Model

The TSVM, proposed by Touzi in 2007, consists in the projection in the Pauli basis of the scattering matrix con-diagonalized by the Takagi method [1]. It leads :

$$\begin{aligned} \overrightarrow{e_T}^{\mathbf{SV}} &= m|\overrightarrow{e_T}|_m e^{j\Phi_s} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\psi) & -\sin(2\psi) \\ 0 & \sin(2\psi) & \cos(2\psi) \end{bmatrix} \\ &\times \begin{bmatrix} \cos\alpha_s\cos(2\tau_m) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}} \\ -j\cos\alpha_s\sin(2\tau_m) \end{bmatrix} \end{aligned} \tag{8}$$

where α_s and Φ_{α_s} are derived from the coneigenvalues μ_1 and μ_2 by :

$$\tan(\alpha_s)\, e^{j\Phi_{\alpha_s}} = \frac{\mu_1-\mu_2}{\mu_1+\mu_2} \tag{9}$$

Because of the coneigenvalue phase ambiguity, Huynen's orientation angle ψ should be reevaluated. To remove this ambiguity, the following relation is applied to restrict the interval of ψ to $[-\pi/4, \pi/4]$:

$$\begin{aligned} &\overrightarrow{e_T}^{\mathbf{SV}}(\Phi_s, \psi, \tau_m, m, \alpha_s, \Phi_{\alpha_s}) \\ &\quad = \overrightarrow{e_T}^{\mathbf{SV}}(\Phi_s, \psi \pm \frac{\pi}{2}, -\tau_m, m, -\alpha_s, \Phi_{\alpha_s}) \end{aligned} \tag{10}$$

As the last term of Eq. 8 is independent of the target orientation angle, it yields that the four parameters m, α_s, Φ_{α_s} and τ_m allow an unique and unambiguous description of any target. In the following, the TSVM method is first applied on the original PolSAR data-set to provide a roll-invariant target vector.

2.4 Comparison between ψ and $\psi_{Krogager}$

According to the TSVM, one can easily prove the following relation between the orientation angle ψ estimated by the TSVM method and $\psi_{Krogager}$ estimated with the phase difference between right-right and left-left circular polarizations (Eq. 3) :

$$\begin{aligned} \psi &= \psi_{Krogager} - \frac{1}{4}\text{atan}\left(\frac{\tan(\alpha_s)\sin(\Phi_{\alpha_s})}{\tan(\alpha_s)\cos(\Phi_{\alpha_s})+\sin(2\tau_m)}\right) \\ &+ \frac{1}{4}\text{atan}\left(\frac{\tan(\alpha_s)\sin(\Phi_{\alpha_s})}{\tan(\alpha_s)\cos(\Phi_{\alpha_s})-\sin(2\tau_m)}\right) \end{aligned} \tag{11}$$

Figure 1 shows a comparison between the orientation angle ψ estimated by the TSVM and $\psi_{Krogager}$ as a function of the helicity τ_m for $\alpha_s = \pi/3$ and $\Phi_{\alpha_s} = \pi/3$, and as a function of the target scattering phase Φ_{α_s} for $\alpha_s = \pi/3$ and $\tau_m = \pi/8$. For $\tau_m = 0$, the target is symmetric. It leads that ψ is equal to $\psi_{Krogager}$, as observed in black in **Figure 1**. Moreover, according to Eq. 11, ψ and $\psi_{Krogager}$ are equal for $\Phi_{\alpha_s} = 0$, which correspond to a wide class of targets including trihedral, dihedral, helix, dipole, . . . For all other cases, the orientation angle ψ should be used instead of $\psi_{Krogager}$ for a roll-invariant target characterization.

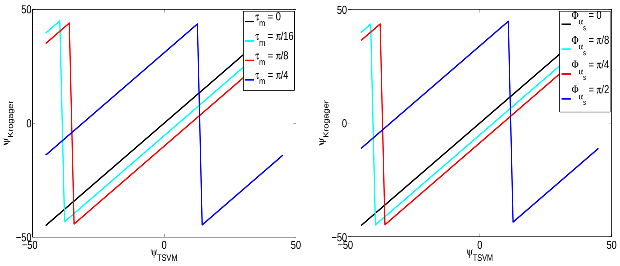

Figure 1: Comparison between ψ and $\psi_{Krogager}$ as a function of τ_m for $\alpha_s = \pi/3$ and $\Phi_{\alpha_s} = \pi/3$ and as a function of Φ_{α_s} for $\alpha_s = \pi/3$ and $\tau_m = \pi/8$.

3 Roll-invariant target detection

The general principle of the proposed roll-invariant target detection algorithm can be divided into five steps :

1. Estimation of the tilt orientation angle and extraction of the "roll-invariant" target vector.
2. Estimation of the fixed point covariance matrix estimator.
3. Computation of the similarity measure between the steering vector and the "roll-invariant" target vector.
4. Choice of the false alarm probability.
5. Thresholding of the similarity image and conclude or not on the detection.

In this paper, a Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) detector is used to detect a particular target. Let $\mathbf{p}$ be a steering vector and $\mathbf{k}$ the observed signal. The GLRT-LQ between $\mathbf{p}$ and $\mathbf{k}$ is given by :

$$\Lambda\left([M]\right) = \frac{|\mathbf{p}^H [M]^{-1}\mathbf{k}|^2}{(\mathbf{p}^H [M]^{-1}\mathbf{p})\,(\mathbf{k}^H [M]^{-1}\mathbf{k})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad (12)$$

where $[M]$ is covariance matrix of the population under the null hypothesis H_0, i.e. the observed signal is only the clutter.
In general, the covariance matrix is unknown. One solution consists in estimating the covariance matrix $[M]$ by $[\hat{M}]_{FP}$, the fixed point covariance matrix estimator. It is the maximum likelihood estimator of the normalized covariance matrix under the deterministic texture in a Spherically Invariant Random Process. Its expression is given by the solution of the following recursive equation [6] [7] :

$$[\hat{M}]_{FP} = f([\hat{M}]_{FP}) = \frac{p}{N}\sum_{i=1}^{N} \frac{\mathbf{k}_i\mathbf{k}_i^H}{\mathbf{k}_i^H [\hat{M}]_{FP}^{-1}\mathbf{k}_i}. \qquad (13)$$

Replacing $[M]$ by $[\hat{M}]_{FP}$ in Eq. 12 leads to an adaptive version of the GLRT-LQ detector.
If the covariance matrix is estimated by the fixed point estimator (Eq. 13), it has been proved, for large N, the relation between false alarm probability p_{fa} and the detection threshold λ :

$$p_{fa} = (1-\lambda)^{(a-1)}\; {}_2F_1(a, a-1; b-1; \lambda) \qquad (14)$$

with $a = N - p + 2$ and $b = N + 2$. N is the number of pixels used to estimate the covariance matrix $[M]$. ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function.

4 Detection results

4.1 On synthetic targets

To evaluate the potential of the GLRT-LQ detectors, a real data-set acquired by the RAMSES sensor at P-band on the Nezer forest, France is analyzed. **Figure 2** shows a colored composition in the Pauli basis of the target vector. In this data-set, six synthetic targets have been added (named A to F). Targets $\mathbf{A}$ and $\mathbf{B}$ are two dihedral oriented respectively at $\psi = 0$ and $\psi = \pi/5$. Target $\mathbf{C}$ have the following characteristics : $\alpha_s = \Phi_{\alpha_s} = \tau_m = \pi/3$ and $\psi = \pi/5$. Target $\mathbf{D}$ is a pure trihedral $\left(\mathbf{p}_{tri} = [1,0,0]^T\right)$. Target $\mathbf{E}$ is an oriented dipole at $\psi = \pi/11$. Target $\mathbf{F}$ is defined by $\alpha_s = \pi/4$, $\Phi_{\alpha_s} = \pi/5$ and $\tau_m = \pi/8$ with a tilt angle of $\pi/6$.

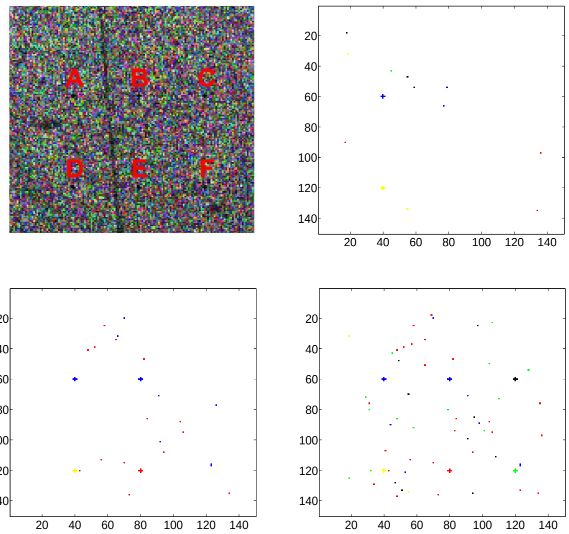

Figure 2: Nezer, RAMSES PolSAR data, P-band (150×150 pixels). Colored composition in the Pauli basis of the target vector $[k]_1$-$[k]_3$-$[k]_2$. Classification results of the Nezer data-set based on the GLRT-LQ, GLRT-LQ Krogager and GLRT-LQ TSVM detectors.

Three detectors are implemented : the GLRT-LQ applied on the original data-set, the GLRT-LQ detectors with tilt compensation estimated by the phase difference between right-right and left-left circular polarizations (GLRT-LQ Krogager) and by the TSVM (GLRT-LQ TSVM). Those detectors are implemented for the five roll-invariant studied steering vectors (pure dihedral, trihedral, dipole, ...). The false alarm probability is fixed to 10^{-4}. According to Eq. 14, it leads to a detection threshold λ of 0.99 for $N = 128$. **Figure 2** shows the detection results into four classes for the three detectors. Pixels in white are undetected objects. Pixels which belong to the blue, yellow and red classes are respectively classified to dihedral, trihedral and dipole with $p_{fa} = 10^{-4}$. Black and green pixels are respectively classified to $\mathbf{C}$ and $\mathbf{F}$ targets.
By analyzing **Figure 2**, it could be noticed that the six synthetic targets are perfectly retrieved with the roll-invariant detector GLRT-LQ TSVM. Concerning the GLRT-LQ detector, the trihedral (target $\mathbf{D}$ in yellow) and the non-oriented dihedral (target $\mathbf{A}$ in blue) are detected. This results is quite logical because a trihedral has a steering vector $\mathbf{p}_{tri}$ independent of the orientation angle ψ. For the

pure dihedral (target $\mathbf{A}$), the tilt angle doesn't need to be removed because it is null in this case. For the GLRT-LQ Krogager detector, two other synthetic targets are retrieved compared to the GLRT-LQ detector : the oriented dihedral and dipole (targets $\mathbf{B}$ and $\mathbf{E}$). As those two targets have a null target scattering phase Φ_{α_s}, the tilt angle ψ estimated with Eq. 3 is valid. For targets $\mathbf{C}$ and $\mathbf{F}$, the orientation angle estimated with the Krogager method is not valid. It leads that only the GLRT-LQ TSVM method is able to detect those targets.

4.2 On a RAMSES X-band data-set

In this section, a real data-set acquired by the RAMSES sensor at X-band is analyzed. **Figure 3** shows a colored composition in the Pauli basis of the target vector. This data-set is composed by two particular targets : a dihedral (in green) and a narrow diplane (in red).

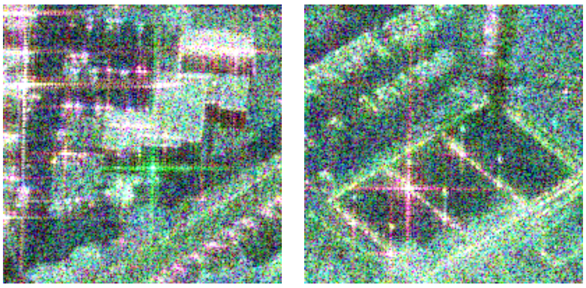

Figure 3: Toulouse, RAMSES PolSAR data, X-band (150×150 pixels). Colored composition in the Pauli basis of the target vector $[k]_1$-$[k]_3$-$[k]_2$. Images containing a dihedral (left) and a narrow diplane (right).

Both GLRT-LQ Krogager and GLRT-LQ TSVM detectors are applied on this data-set. **Table 1** and **Table 2** show respectively the criterion characteristics for the dihedral and narrow diplane. As those two targets have a null helicity τ_m, both detectors should have similar performance. Nevertheless, it can be noticed that the GLRT-LQ TSVM detector provide better results than the GLRT-LQ Krogager. Concerning the dihedral, the false alarm probability is decreased by a factor of 4 for the GLRT-LQ TSVM ($p_{fa} = 2.1 \times 10^{-3}$) compared to the GLRT-LQ Krogager ($p_{fa} = 8.2 \times 10^{-3}$).

	dihedral				
	GLRT-LQ	ψ	α_s	Φ_{α_s}	τ_m
Krogager	0.912	0.761			
TSVM	0.956	0.770	-1.453	0.450	-0.178
Pure target			1.571	∞	0

Table 1: Detector characteristics for the dihedral.

	narrow diplane				
	GLRT-LQ	ψ	α_s	Φ_{α_s}	τ_m
Krogager	0.828	-0.023			
TSVM	0.849	-0.026	1.210	-0.172	0.052
Pure target			1.249	0	0

Table 2: Detector characteristics for the narrow diplane.

5 Conclusion

In this paper, authors have proposed to combine the target scattering vector model method and the GLRT-LQ detector for a roll invariant target detection. The TSVM allows the extraction of a roll-invariant target vector by means of four independent parameters m, α_s, Φ_{α_s} and τ_m. Those parameters provide an unique and unambiguous description of any target. In the proposed method, the TSVM is first applied on the original PolSAR data-set. Then, the GLRT-LQ similarity measure is computed between the roll-invariant target vector and the steering vector.

Comparisons between the tilt angles ψ and $\psi_{Krogager}$ have been carried out. They are equal for symmetric targets ($\tau_m = 0$) and for $\Phi_{\alpha_s} = 0$, which corresponds to a wide class of targets including trihedral, dihedral, helix, dipole, ... For all other cases, the orientation angle ψ should be used instead of $\psi_{Krogager}$ to provide a roll-invariant target characterization. Detection results on both synthetic and real targets have shown that the application of the TSVM prior to the GLRT-LQ measure allow the detection of any oriented targets.

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