SPATIO-TEMPORAL ADAPTIVE DETECTOR
IN
NON-HOMOGENEOUS AND LOW-RANKCLUTTER

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PRESENTATION OUTLINE

I The studied RADAR detection problem

II The Low Rank Matched Filters (LRMF and LRNMF) tests of Rangaswami et al. (2004)

III Clutter subspace estimation from secondary data based on the Normalized Sample Covariance Matrix (NSCM)

IV Simulation results

I. The studied RADAR detection problem

► STAP framework

► Basic detection problem :

Primary data : $\begin{cases} H0 : \mathbf{y} = \mathbf{d} \\ H1 : \mathbf{y} = \mathbf{s} + \mathbf{d} \end{cases}$ Secondary data : $\mathbf{y}_k = \mathbf{d}_k \ (k = 1, \ldots, K)$

$\mathbf{y} \in \mathbb{C}^m$ = primary data (cell under test)

$\mathbf{s} = \alpha\, \mathbf{p}$ = signal to be detected of unknown amplitude $\alpha \in \mathbb{C}$

d = **c** (clutter) + **n** (noise) = disturbance

$\mathbf{d}_k$ = $\mathbf{c}_k$ (clutter) + $\mathbf{n}_k$ (noise) : i.i.d. with the same distribution as **d**

p = space-time steering vector depending on traget AOA and velocity

► Disturbance nature

Same framework as in : *Robust adaptive signal processing methods for heterogeneous radar clutter scenarios*, M. Rangaswami et al., Signal Processing (Elsevier), 2004.

$$\mathbf{d} = \mathbf{c} \text{ (SIRV clutter)} + \mathbf{n} \text{ (Gaussian noise)} = \text{disturbance}$$

- **Clutter c : SIRV**

$\mathbf{c} = \sqrt{\tau}\,\mathbf{g}$ is a SIRV noise

= product of the square root of a positive random variable τ (the texture) and an independent zero-mean complex Gaussian *m*-vector **g** (the speckle) with covariance matrix **M** normalized according to Tr(**M**) = *m*,

- **STAP context : low rank clutter c** $\Rightarrow$ rank(**M**) = $r \ll m$

- **Noise n : Zero-mean Complex Gaussian** $N_c(0, \sigma^2 \mathbf{I})$

II. The Low Rank Matched Filters (LRMF and LRNMF) tests of Rangaswami et al.

In the spirit of Low Rank approaches to Radar Detection initiated by the PCI of Kirstein and Tufts (94) and the Eigencanceler Haimovich (96)

Unknown texture τ , known speckle covariance matrix **M** of rank $r < m$

$\mathbf{M} = \sum_{i=1}^{r} \lambda_i \, \mathbf{u}_i \, \mathbf{u}_i^H$ = eigen-decomp. of rank r speckle cov. matrix **M**

$\mathbf{u}_1, \ldots, \mathbf{u}_r, \mathbf{u}_{r+1}, \ldots, \mathbf{u}_m$ = orthonormal basis of $\mathbb{C}^m$

$\mathbf{U}_c = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_r]$, $\mathbf{U}_o = [\mathbf{u}_{r+1} \ \cdots \ \mathbf{u}_m]$

$\mathbf{P} = \sum_{i=1}^{r} \mathbf{u}_i \, \mathbf{u}_i^H$ = rank r projector onto the *clutter subspace*

$\mathbf{P}^{\perp} = \sum_{i=r+1}^{m} \mathbf{u}_i \, \mathbf{u}_i^H = \mathbf{I} - \mathbf{P}$ = projector onto the *orthogonal* of the *clutter subspace*

► **Pre-process** the primary data **y** by projection onto the *orthogonal* of the *clutter subspace* → get read of the Low Rank SIRV clutter

$$\mathbf{x} = \mathbf{U}_o^H \mathbf{y} = \begin{cases} \cancel{\mathbf{U}_o^H \mathbf{c}} + \mathbf{U}_o^H \mathbf{n} & (H0) \\ \alpha\, \mathbf{U}_o^H \mathbf{p} + \cancel{\mathbf{U}_o^H \mathbf{c}} + \underbrace{\mathbf{U}_o^H \mathbf{n}}_{N_c(0,\, \sigma^2 \mathbf{I})} & (H1) \end{cases}$$

► **Treat the detection problem after projection**

- Case of known noise power σ^2 : the Low Rank Matched Filter (LRMF)

$$\Lambda_{LRMF} = \frac{\left| \mathbf{p}^H \mathbf{P}^{\perp} \mathbf{y} \right|^2}{(\mathbf{p}^H \mathbf{P}^{\perp} \mathbf{p})} \underset{H0}{\overset{H1}{\gtrless}} \eta_{LRMF}$$

- Case of unknown σ^2 : the Low Rank Normalized Matched Filter (LRNMF)

$$\Lambda_{LRNMF} = \frac{\left| \mathbf{p}^H \mathbf{P}^{\perp} \mathbf{y} \right|^2}{(\mathbf{p}^H \mathbf{P}^{\perp} \mathbf{p})\ (\mathbf{y}^H \mathbf{P}^{\perp} \mathbf{y})} \underset{H0}{\overset{H1}{\gtrless}} \eta_{LRNMF}$$

(after the Normalized Matched Filter (NMF) [Scharf 94,Conte 95,Gini 97])

III. Clutter subspace estimation from secondary data

► **Clutter subspace must be estimated** from secondary data :

► **Secondary data** : SIRV clutter, possible target contamination

$I = \{1, \ldots, K\}$ = indexes of secondary data $\mathbf{y}_k$

I_0 = indexes of non target-contaminated $\mathbf{y}_k$

I_1 = indexes of target-contaminated $\mathbf{y}_k$

$I = I_0 \cup I_1$

$$\text{for } k \in I_0 : \quad \mathbf{y}_k = \underbrace{\mathbf{c}_k}_{\text{SIRV clutter}} + \underbrace{\mathbf{n}_k}_{\text{Gaussian noise}}$$

$$\text{for } k \in I_1 : \quad \mathbf{y}_k = \underbrace{\alpha_k \mathbf{p}_k}_{\text{target contamination}} + \underbrace{\mathbf{c}_k}_{\text{SIRV clutter}} + \underbrace{\mathbf{n}_k}_{\text{Gaussian noise}}$$

► **Problem** : the secondary data SCM yields a poor estimate of the clutter subspace in case of heavily-tailed non Gaussian clutter or target-contamination

► **Proposition** : estimate the clutter subspace from the secondary data Normalized Sample Covariance Matrix (NSCM)

III.1 The Normalized Sample Covariance Matrix (NSCM)

► **NSCM** : $\mathbf{R}_{NSCM} = \frac{m}{K} \sum_{k=1}^{K} \frac{1}{\|\mathbf{y}_k\|^2} \mathbf{y}_k \mathbf{y}_k^H$ = SCM of $\frac{1}{\|\mathbf{y}_k\|} \mathbf{y}_k$

Robust estimate in the case of SIRV clutter

Introduced for adaptive Radar detection by [Gini 95, Conte 96]

Statistical properties and asymptotic distribution studied for SIRV disturbance in:

S. Bausson, F. Pascal, P. Forster, J.-P. Ovarlez and P. Larzabal,
First and Second Order Moments of the Normalized Sample Covariance Matrix of Spherically Invariant Random Vectors, *IEEE Signal Processing Letters*, 2007

► **Present work** : statistical analysis in the case of SIRV clutter plus gaussian noise

► **Paper contribution** : $\mathbf{R}_{NSCM}$ yields a consistent estimate of the clutter subspace (in the case of no target-contamination)

Robustness to target-contamination (simulations)

III.2 Our result : Consistency of the clutter subspace estimate based on the NSCM (no target-contamination case)

Assumption : no target-contamination

$$\mathbf{y}_k = \underbrace{\mathbf{c}_k}_{\text{SIRV clutter}} + \underbrace{\mathbf{n}_k}_{\text{Gaussian noise}} \quad \text{i.i.d. for } k = 1, \ldots, K$$

where :

- $\mathbf{c}_k = \sqrt{\tau_k}\,\mathbf{g}_k$, texture τ has pdf $f(\tau)$, $\mathbf{g} \sim N_c(0, \mathbf{M})$ with $\mathbf{M} = \sum_{i=1}^{r} \lambda_i \mathbf{u}_i \mathbf{u}_i^H$

- $\mathbf{n} \sim N_c(0, \sigma^2 \mathbf{I})$

The NSCM is **NOT a consistent estimate of** $E\left[\mathbf{y}_k \mathbf{y}_k^H\right] = E[\tau]\,\mathbf{M} + \sigma^2 \mathbf{I}$

… **BUT**…

► $E[\frac{\mathbf{y}_k \mathbf{y}_k^H}{\|\mathbf{y}_k\|^2}]$ shares the same ordered eigenvectors $\mathbf{u}_i$ as the clutter cov. **M**

More precisely :

$$E[\frac{\mathbf{y}_k \mathbf{y}_k^H}{\|\mathbf{y}_k\|^2}] = \sum_{i=1}^{r} \mu_i \mathbf{u}_i \mathbf{u}_i^H + \mu \sum_{i=r+1}^{m} \mathbf{u}_i \mathbf{u}_i^H \quad \text{with } \mu_1 \geq \cdots \geq \mu_m \geq \mu \quad \text{where :}$$

- $\mu_i = E[\frac{(\tau \lambda_i + \sigma^2)\ \chi_i(2)}{\sum_{i=1}^{r}(\tau \lambda_i + \sigma^2)\chi_i(2) + \sigma^2 \chi_{r+1}(2(m-r))}]$

- $\mu = E[\frac{\sigma^2\ \chi_{r+2}(2)}{\sum_{i=1}^{r}(\tau \lambda_i + \sigma^2)\chi_i(2) + \sigma^2\ \chi_{r+2}(2) + \sigma^2 \chi_{r+3}(2(m-r-1))}]$

- $\chi_m(n)$ is a Chi² r.v. with n degrees of freedom, all r.v. are independent

► Consistency of the clutter subspace estimate based on the NSCM

Let $\mathbf{R}_{NSCM} = \sum_{i=1}^{m} \hat{\mu}_i \, \hat{\mathbf{u}}_i \, \hat{\mathbf{u}}_i^H$ be the eigen-decomposition of $\mathbf{R}_{NSCM}$ ($\hat{\mu}_1 \geq \cdots \geq \hat{\mu}_m$)

Thus, by the law of large numbers :

$$\mathbf{R}_{NSCM} \xrightarrow{P} m\,\mathrm{E}\left[\frac{\mathbf{y}_k \, \mathbf{y}_k^H}{\left\| \mathbf{y}_k \right\|^2} \right] = m \sum_{i=1}^{r} \mu_i \, \mathbf{u}_i \, \mathbf{u}_i^H + m \, \mu \sum_{i=r+1}^{m} \mathbf{u}_i \, \mathbf{u}_i^H$$

Therefore,

$\hat{\mathbf{P}} = \sum_{i=1}^{r} \hat{\mathbf{u}}_i \, \hat{\mathbf{u}}_i^H$ is a **consistent estimate** proj. onto clutter subspace $\mathbf{P} = \sum_{i=1}^{r} \mathbf{u}_i \, \mathbf{u}_i^H$

III.3 Robustness of the NSCM to target-contamination

Assume secondary data $\mathbf{y}_K$ is target-contaminated with : $\|\mathbf{y}_K\| >> \|\mathbf{y}_1\|,\ldots,\|\mathbf{y}_{K-1}\|$

► **SCM** : $\mathbf{R}_{SCM} = \frac{1}{K}\left(\mathbf{y}_1 \mathbf{y}_1^H + \cdots + \mathbf{y}_{K-1} \mathbf{y}_{K-1}^H + \mathbf{y}_K \mathbf{y}_K^H \right)$

→ very sensitive to large norm secondary data $\mathbf{y}_K$

► **NSCM** : $\mathbf{R}_{NSCM} = \frac{m}{K}\left(\frac{\mathbf{y}_1 \mathbf{y}_1^H}{\|\mathbf{y}_1\|^2} + \cdots + \frac{\mathbf{y}_{K-1} \mathbf{y}_{K-1}^H}{\|\mathbf{y}_{K-1}\|^2} + \frac{\mathbf{y}_K \mathbf{y}_K^H}{\|\mathbf{y}_K\|^2} \right)$

→ all terms $\frac{\mathbf{y}_k \mathbf{y}_k^H}{\|\mathbf{y}_k\|^2}$ have the same Frobenius norm (i.e. 1)

→ NSCM is much less sensitive to large norm secondary data $\mathbf{y}_K$

IV. SIMULATIONS

► STAP data provided by the french DGA/CELAR's simulator (based on real data)

center frequency f0 = 10 GHz , bandwidth B = 5 MHz , PRF = 1 kHz

N = 4 sensors , inter-element spacing is d = 0.3 m

M = 64 coherent pulses , radar velocity v = 100 m/s

$m = NM = 256$ (space-time data vectors dimension)

$N_s \approx 400$ secondary data

Brennan's rule : clutter rank $r = N + \dfrac{2vT}{d}(M-1) = 46$

► Preliminary remark :

Reed-Mallet-Brennan rule not satisfied since $N_s < 2m$

Kirstein-Tufts-Haimovich rule is satisfied since $N_s > 2r$

► Comparisons of the outputs of the :

- Adaptive Matched Filter based on **SCM** [Robey et al. 92] :

$$\Lambda_{AMF}(\theta,v) = \frac{\left| \mathbf{p}(\theta,v)^H \; \mathbf{R}_{SCM}^{-1} \; \mathbf{y} \right|^2}{\left(\mathbf{p}(\theta,v)^H \; \mathbf{R}_{SCM}^{-1} \; \mathbf{p}(\theta,v) \right)}$$

- Low Rank Adaptive Matched Filter based on **SCM** [Kirstein-Tufts 94, Haimovich 96]

$$\Lambda_{LR\text{-}SCM}(\theta,v) = \frac{\left| \mathbf{p}(\theta,v)^H \; \mathbf{P}_{\mathrm{SCM}}^{\perp} \; \mathbf{y} \right|^2}{\left(\mathbf{p}(\theta,v)^H \, \mathbf{P}_{\mathrm{SCM}}^{\perp} \, \mathbf{p}(\theta,v) \right)}$$

- Low Rank Adaptive Matched Filter based on **NSCM** (paper proposition)

$$\Lambda_{LR\text{-}NSCM}(\theta,v) = \frac{\left| \mathbf{p}(\theta,v)^H \; \mathbf{P}_{\mathrm{NSCM}}^{\perp} \; \mathbf{y} \right|^2}{\left(\mathbf{p}(\theta,v)^H \, \mathbf{P}_{\mathrm{NSCM}}^{\perp} \, \mathbf{p}(\theta,v) \right)}$$

► Common Scenario :

Three targets :

- Target 1 : 4 m/s, 0 deg, bin 216
- Target 2 : 4 m/s, 0 deg, bin 256   Cell Under Test (CUT)
- Target 3 : −4 m/s, 0 deg, bin 296

► Sub-scenario 1 : no target-contamination

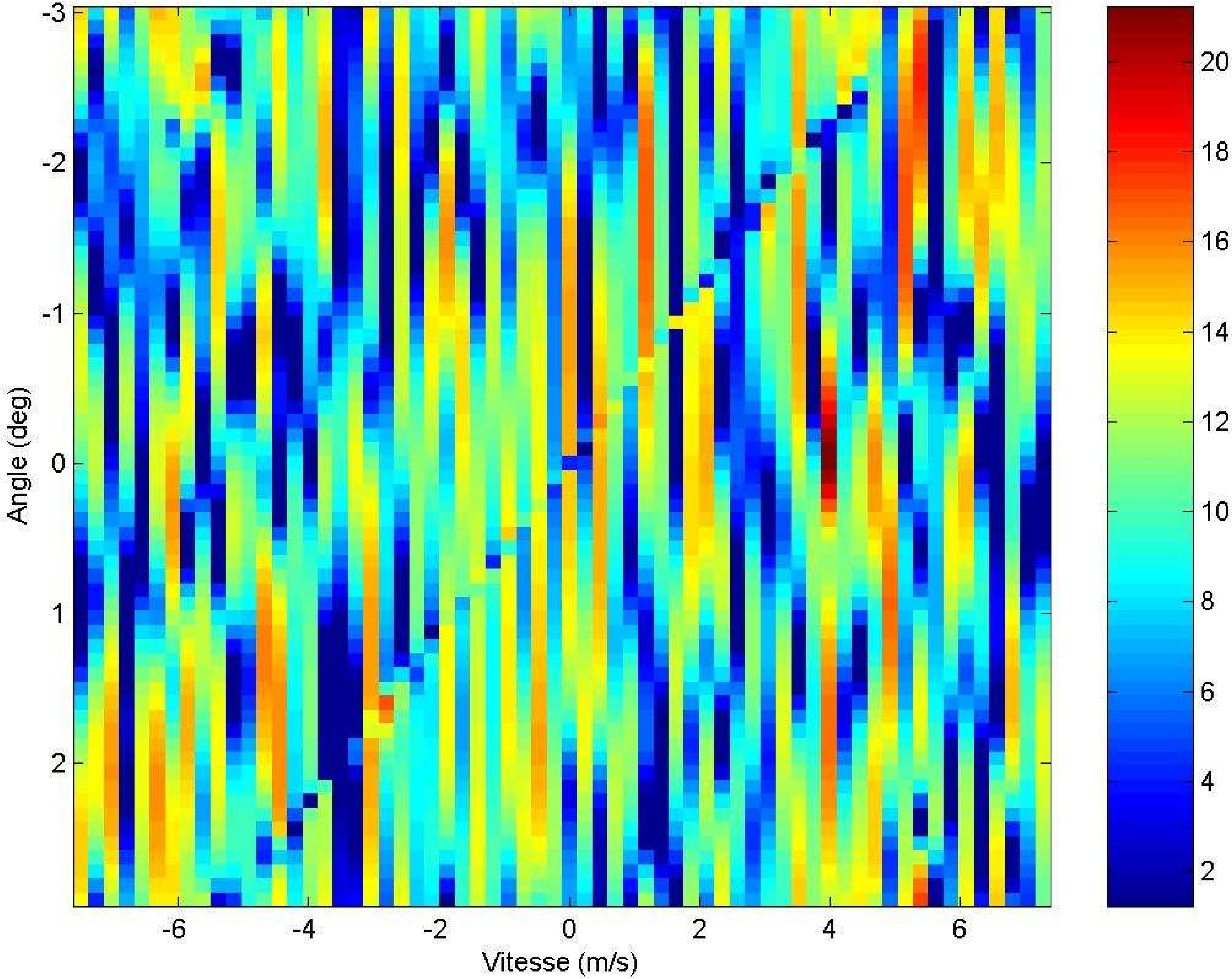
N_s = 408 secondary data composed of all range bins except: 216, 296, four guard bins around CUT 256

► Sub-scenario 2: target-contamination

N_s = 410 secondary data composed of all range bins except: four guard bins around CUT 256

No target-contamination, Target at 4 m/s, 0 deg

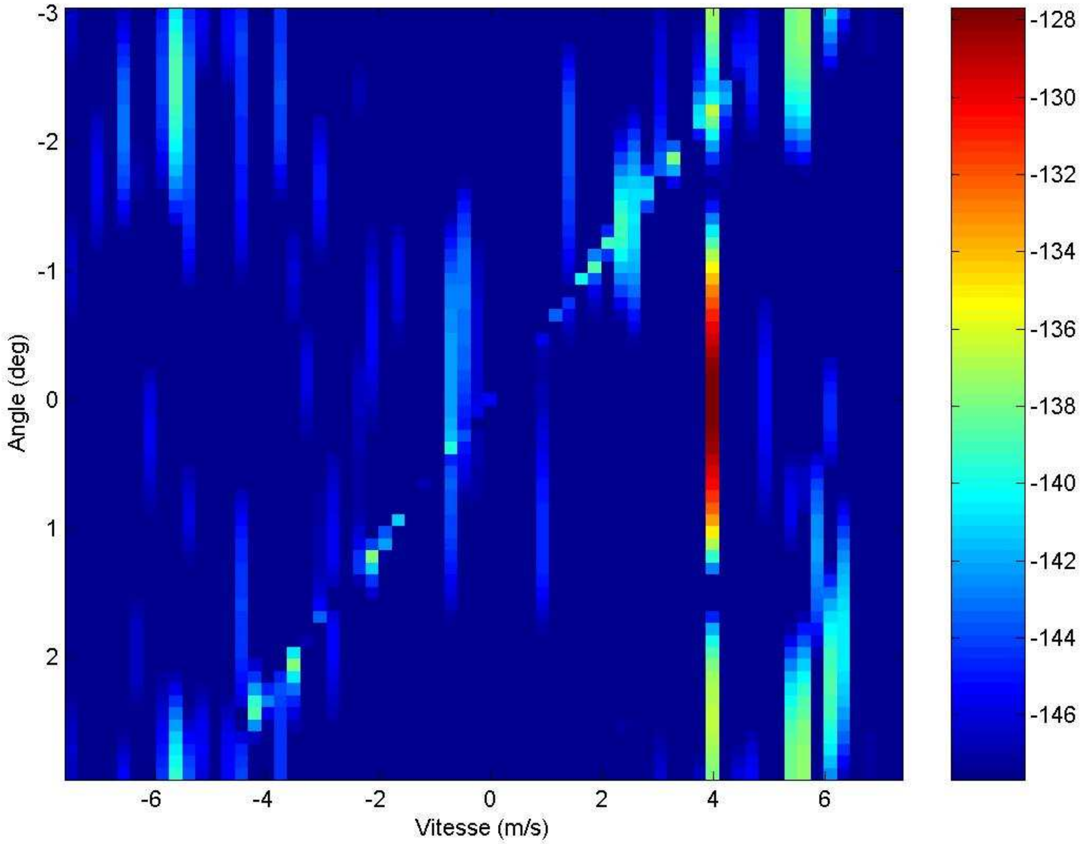
AMF based based on the SCM



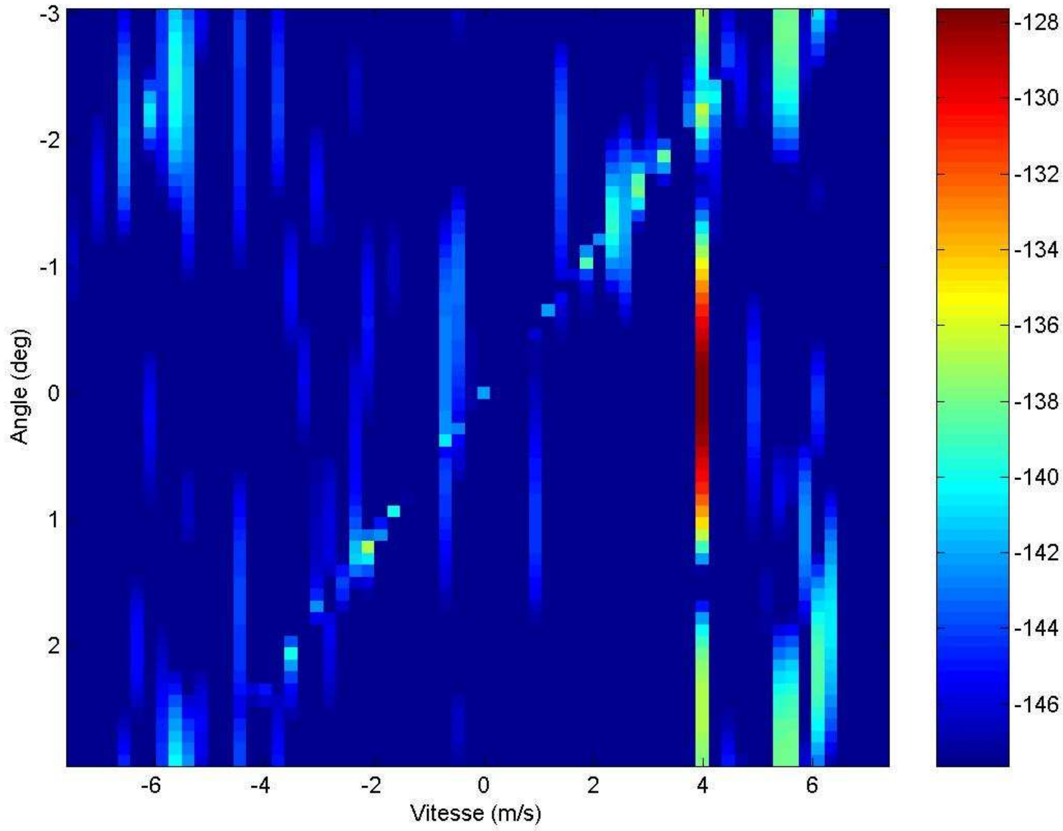
No target-contamination, Target at 4 m/s, 0 deg

Rank 45

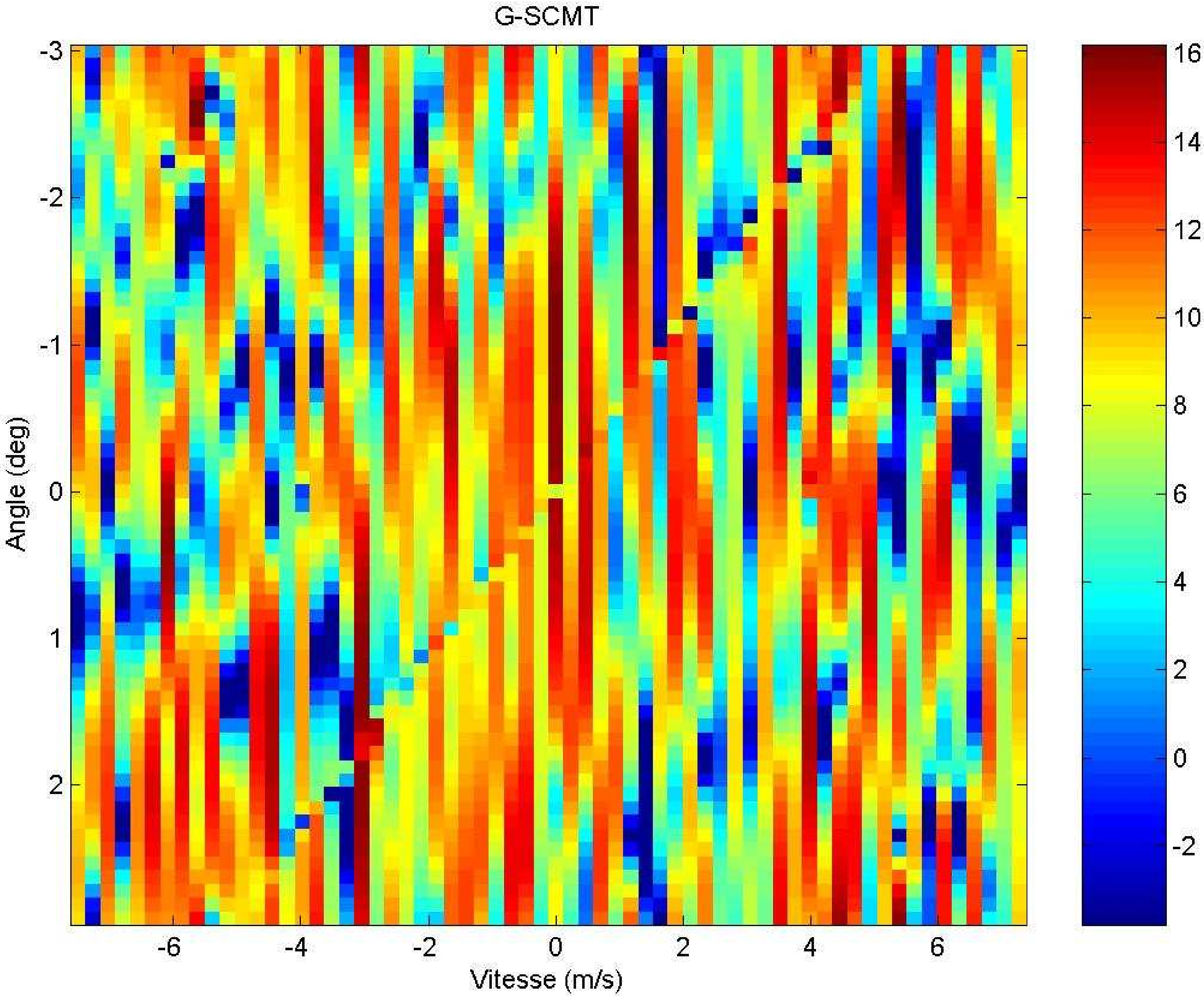
LR-AMF based on the SCM



LR-AMF based on the NSCM



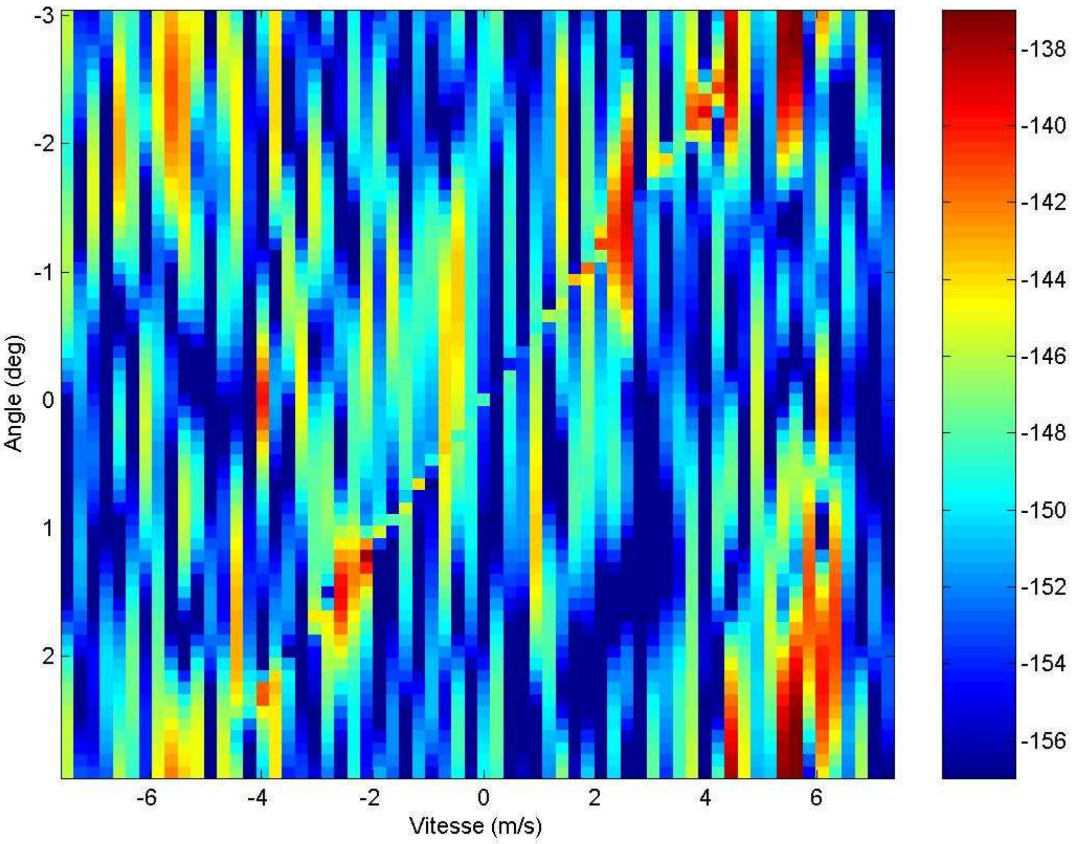
Target-contamination, Target at 4 m/s, 0 deg



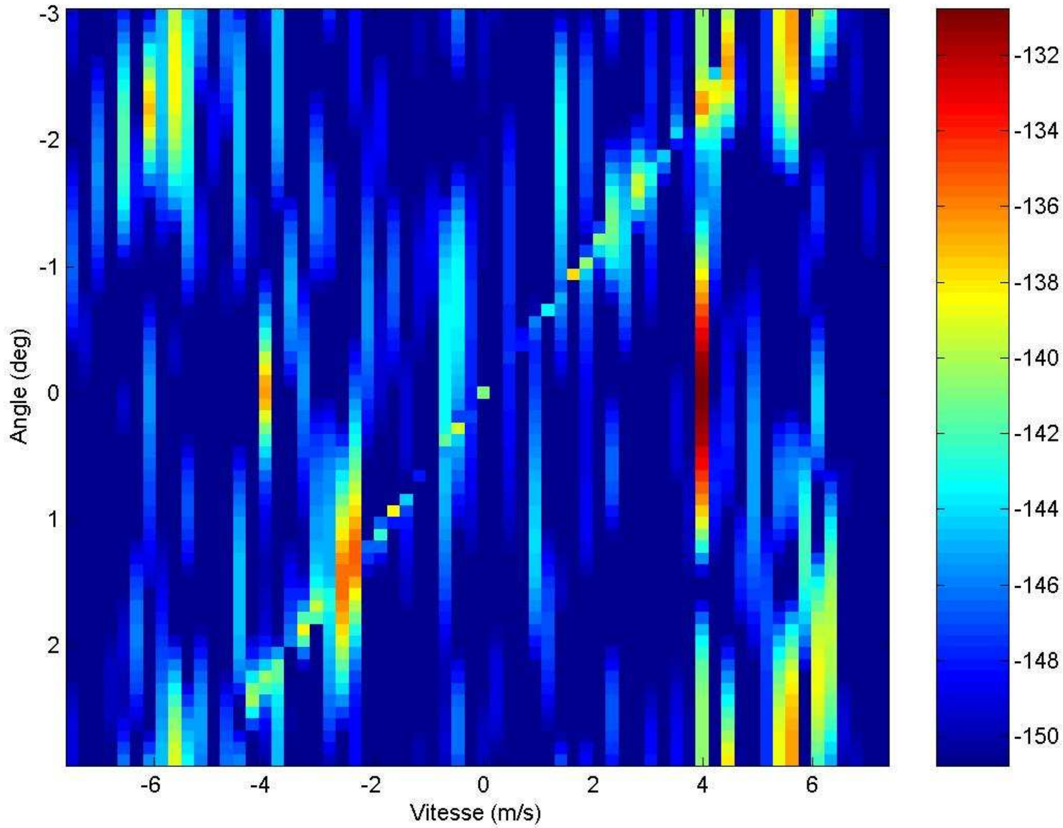
Target-contamination, Target at 4 m/s, 0 deg

Rank 45

LR-AMF based on the SCM



LR-AMF based on the NSCM



CONCLUSION

STAP context of Low Rank SIRV clutter plus white noise

► The **Normalized Sample Covariance Matrix** provides a **consistent estimate of the clutter subpace**

► The **Normalized Sample Covariance Matrix** is **robust** to target-contamination

► The **Normalized Sample Covariance Matrix** is a **good candidate for adaptive** version of Rangaswami's **Low Rank Matched Filter and Low Rank Normalized Matched Filter**