ON A SIRV-CFAR DETECTOR WITH RADAR EXPERIMENTATIONS IN IMPULSIVE CLUTTER

Frédéric Pascal[123], *Jean-Philippe Ovarlez*[1], *Philippe Forster*[2], *and Pascal Larzabal*[3]

[1] ONERA DEMR/TSI
Chemin de la Hunière , F-91761, Palaiseau Cedex, France
[2] Université Paris X - GEA
1 Chemin Desvallières, F-92410 Ville d'Avray, France
[3] ENS Cachan - SATIE
61 Avenue du Président Wilson, F-94235 Cachan Cedex, France

Email : pascal@onera.fr, ovarlez@onera.fr, philippe.forster@cva.u-paris10.fr, pascal.larzabal@satie.ens-cachan.fr

ABSTRACT

This paper deals with radar detection in impulsive clutter. Its aim is twofold. Firstly, assuming a Spherically Invariant Random Vectors (SIRV) model for the clutter, the corresponding unknown covariance matrix is estimated by a recently introduced algorithm [1, 2]. A statistical analysis (bias, consistency, asymptotic distribution) of this estimate will be summarized allowing us to give the GLRT properties: the SIRV-CFAR (Constant False Alarm Rate) property, i.e. texture-CFAR and Covariance Matrix-CFAR, and the relationship between the Probability of False Alarm (PFA) and the detection threshold.
Secondly, one of the main contributions of this paper is to give some results obtained with real non-Gaussian data. These results demonstrate the interest of the proposed detection scheme, and show an excellent correspondence between experimental and theoretical false alarm rates.

1. PROBLEM STATEMENT AND BACKGROUND

Non-Gaussian noise characterization has gained many interests since experimental radar clutter measurements, made by organizations like MIT [3], showed that these data are correctly described by non-Gaussian statistical models. One of the most tractable and elegant non-Gaussian model comes from the so-called *Spherically Invariant Random Vector* (SIRV) theory. A SIRV is the product of a Gaussian random process - called *speckle* - with the square root of a non-negative random variable - called *texture*. This model leads to many results [4, 5].

The basic problem of detecting a complex signal corrupted by an additive SIRV clutter $\mathbf{c}$ in a m-dimensional complex vector $\mathbf{y}$ can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \dots, N \\ H_1 : \mathbf{y} = \mathbf{s} + \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \dots, N \end{cases} \tag{1}$$

where the $\mathbf{c}_i$'s are N signal-free independent measurements, traditionally called the secondary data, used to estimate the clutter covariance matrix.

Under hypothesis H_1, it is assumed that the observed data consists in the sum of a signal $\mathbf{s} = \alpha\,\mathbf{p}$ and clutter $\mathbf{c}$, where $\mathbf{p}$ is a perfectly known complex steering vector and α is the signal complex amplitude.

Let us now recap some SIRV theory results. A noise modelled as a SIRV is a non-homogeneous Gaussian process with random power. More precisely, a SIRV [6] is the product of the square root of a positive random variable τ (*texture*) and a m-dimensional independent complex Gaussian vector $\mathbf{x}$ (*speckle*) with zero mean covariance matrix $\mathbf{M} = \mathbb{E}(\mathbf{x}\mathbf{x}^H)$ with normalization $\mathrm{Tr}(\mathbf{M}) = m$, where H denotes the conjugate transpose operator

$$\mathbf{c} = \sqrt{\tau}\,\mathbf{x}. \tag{2}$$

The SIRV PDF expression is

$$p_m(\mathbf{c}) = \int_0^{+\infty} g_m(\mathbf{c}, \tau)\, p(\tau)\, d\tau\,, \tag{3}$$

where

$$g_m(\mathbf{c}, \tau) = \frac{1}{(\pi\,\tau)^m\,|\mathbf{M}|} \exp\left(-\frac{\mathbf{c}^H\,\mathbf{M}^{-1}\,\mathbf{c}}{\tau}\right). \tag{4}$$

This model allowed to build several Generalized Likelihood Ratio Tests like the GLRT-Linear Quadratic (GLRT-LQ) in [4, 5] defined by

$$\Lambda(\mathbf{M}) = \frac{|\mathbf{p}^H\mathbf{M}^{-1}\mathbf{y}|^2}{(\mathbf{p}^H\mathbf{M}^{-1}\mathbf{p})(\mathbf{y}^H\mathbf{M}^{-1}\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,, \tag{5}$$

where $\mathbf{p}$ is the steering vector, $\mathbf{y}$ the observed vector and λ the detection threshold associated to this detector.

In many problems, non-Gaussian noise can be characterized by SIRVs but the covariance matrix $\mathbf{M}$ is generally not known and an estimate $\widehat{\mathbf{M}}$ is required. Obviously, it has to satisfy the $\mathbf{M}$-normalization: $\mathrm{Tr}(\widehat{\mathbf{M}}) = m$.

The next section is devoted to an adaptive GLRT built from an Approximate Maximum Likelihood (AML) estimate of the SIRV covariance matrix. Then, Section 3 presents an application of this detector to real data: experimental results perfectly match theoretical analysis.

2. THE FIXED POINT ESTIMATE $\widehat{\mathbf{M}}_{FP}$ AND THE CORRESPONDING ADAPTIVE GLRT

2.1 The AML estimate

Conte and Gini in [1, 2] have shown that an Approximate Maximum Likelihood (AML) estimate $\widehat{\mathbf{M}}$ of $\mathbf{M}$ is a solution of the following equation

$$\widehat{\mathbf{M}} = \frac{m}{N}\sum_{i=1}^{N}\left(\frac{\mathbf{c}_i\mathbf{c}_i^H}{\mathbf{c}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{c}_i}\right). \quad (6)$$

Notice that the ML estimate has been studied in [7]. Existence and uniqueness of the above equation solution, denoted $\widehat{\mathbf{M}}_{FP}$ have already been investigated in [8].

Let the function f be defined as

$$f(\widehat{\mathbf{M}}) = \frac{m}{N}\sum_{i=1}^{N}\left(\frac{\mathbf{c}_i\mathbf{c}_i^H}{\mathbf{c}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{c}_i}\right) = \frac{m}{N}\sum_{i=1}^{N}\left(\frac{\mathbf{x}_i\mathbf{x}_i^H}{\mathbf{x}_i^H\widehat{\mathbf{M}}^{-1}\mathbf{x}_i}\right), \quad (7)$$

where the right hand side of (7) is rewritten in terms of the $\mathbf{x}_i$'s and the τ_i's.

Eqn. (7) obviously implies that $\widehat{\mathbf{M}}_{FP}$ is independent of the τ_i's.

The statistical properties of $\widehat{\mathbf{M}}_{FP}$ have been investigated and published in [11], the main results are recaped here below:

Proposition 2.1

(1) $\widehat{\mathbf{M}}_{FP}$ is a consistent estimate of $\mathbf{M}$;

(2) $\widehat{\mathbf{M}}_{FP}$ is an unbiased estimate of $\mathbf{M}$;

(3) the asymptotic distribution of $\widehat{\mathbf{M}}_{FP}$ is Gaussian and its covariance matrix is fully characterized in [11];

(4) this distribution is the same as the asymptotic distribution of a Wishart matrix with $\left(\frac{m}{m+1}\right)$ N degrees of freedom.

2.2 The studied adaptive GLRT

Let us now present the adaptive GLRT [9, 10], used for detection

$$\hat{\Lambda}(\widehat{\mathbf{M}}) = \frac{|\mathbf{p}^H\widehat{\mathbf{M}}^{-1}\mathbf{y}|^2}{(\mathbf{p}^H\widehat{\mathbf{M}}^{-1}\mathbf{p})(\mathbf{y}^H\widehat{\mathbf{M}}^{-1}\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,. \quad (8)$$

In the next section dealing with applications to real data, we will use $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ as detector. This detector is obviously texture-CFAR (independent of the distribution of τ) and, an original result of this paper is to show the independence of the distribution of $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ with the covariance matrix $\mathbf{M}$: we will say that $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ is Matrix-CFAR (M-CFAR).

Definition 2.1

An adaptive detector $\hat{\Lambda}(\widehat{\mathbf{M}})$ verifies the M-CFAR property if its statistical distribution is independent of the covariance matrix $\mathbf{M}$ estimated by $\widehat{\mathbf{M}}$.

This property is of most interest in a practical work to detect targets when the covariance matrix is unknown.

Theorem 2.1

$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ is M-CFAR.

Theorem 2.1 establishes the M-CFAR property of the adaptive GLRT built with the FP estimate.

Proof 2.1

Let $\mathbf{M}$ be a covariance matrix. Let $\widehat{\mathbf{M}}_{FP}$ be the FP estimate of $\mathbf{M}$. Then, under hypothesis H_0 (no target signal), we will show that

$$\mathscr{L}\left(\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})\right) = \mathscr{L}\left(\hat{\Lambda}(\widehat{\mathbf{M}}_{FP,\mathbf{I}})\right) \quad (9)$$

where $\mathscr{L}(x)$ stands for the statistical distribution of a random variable x and $\widehat{\mathbf{M}}_{FP,\mathbf{I}}$ is the FP estimate of the identity matrix $\mathbf{I}$.

Since the statistics of $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ is independent of the texture τ, we choose $\tau = 1$: secondary data $\mathbf{x}_1,\ldots,\mathbf{x}_N$ are thus Gaussian with covariance matrix $\mathbf{M}$,

$$\mathbf{x}_i \sim \mathscr{N}(\mathbf{0},\mathbf{M})\,.$$

The FP estimate of $\mathbf{M}$ is defined as the unique solution (up to a scalar factor) of

$$\widehat{\mathbf{M}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{x}_i\,\mathbf{x}_i^H}{\mathbf{x}_i^H\,\widehat{\mathbf{M}}_{FP}^{-1}\,\mathbf{x}_i}, \quad (10)$$

and the adaptive GLRT detector is

$$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP}) = \frac{|\mathbf{p}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{x}|^2}{(\mathbf{p}^H\widehat{\mathbf{M}}_{FP}^{-1}\mathbf{p})(\mathbf{x}^H\,\widehat{\mathbf{M}}_{FP}^{-1}\,\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda,$$

where $\mathbf{x}$ is the observation vector (under hypothesis H_0) such that $\mathbf{x} \sim \mathscr{N}(\mathbf{0},\mathbf{M})$.

The first part of the proof is the whitening of the data. By applying the following change of variable, $\mathbf{y} = \mathbf{M}^{-1/2}\,\mathbf{x}$ to Eqn. (10), one has

$$\widehat{\mathbf{M}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{M}^{1/2}\,\mathbf{y}_i\,\mathbf{y}_i^H\,\mathbf{M}^{1/2}}{\mathbf{y}_i^H\,\widehat{\mathbf{T}}^{-1}\,\mathbf{y}_i}, \quad (11)$$

where

$$\widehat{\mathbf{T}} = \mathbf{M}^{-1/2}\widehat{\mathbf{M}}_{FP}\mathbf{M}^{-1/2}\,.$$

Therefore,

$$\widehat{\mathbf{T}} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{y}_i\,\mathbf{y}_i^H}{\mathbf{y}_i^H\,\widehat{\mathbf{T}}^{-1}\,\mathbf{y}_i}. \quad (12)$$

$\widehat{\mathbf{T}}$ is thus the unique FP estimate (up to a scalar factor) of the identity matrix. Its statistics is clearly independent of $\mathbf{M}$ since the $\mathbf{y}_i$'s are $\mathscr{N}(\mathbf{0},\mathbf{I})$.

Moreover, for any unitary matrix $\mathbf{U}$, one has

$$\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H = \frac{m}{N}\sum_{i=1}^{N} \frac{\mathbf{z}_i\,\mathbf{z}_i^H}{\mathbf{z}_i^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{z}_i}, \tag{13}$$

where $\mathbf{z}_i = \mathbf{U}\,\mathbf{y}_i$ *is also* $\mathcal{N}(\mathbf{0},\mathbf{I})$ *. Therefore,* $\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H$ *has the same distribution as* $\widehat{\mathbf{T}}$ *.*

In terms of the adaptive detector, one has

$$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP}) = \frac{|\mathbf{p}_1^H\widehat{\mathbf{T}}^{-1}\mathbf{y}|^2}{(\mathbf{p}_1^H\widehat{\mathbf{T}}^{-1}\mathbf{p}_1)(\mathbf{y}^H\widehat{\mathbf{T}}^{-1}\mathbf{y})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,,$$

where $\mathbf{p}_1 = \mathbf{M}^{-1/2}\,\mathbf{p}$ *and* $\mathbf{y} = \mathbf{M}^{-1/2}\,\mathbf{x}$ *is* $\mathcal{N}(\mathbf{0},\mathbf{I})$ *.*

Now let $\mathbf{U}$ *be a unitary matrix such that*

$$\mathbf{U}\mathbf{p}_1 = \|\mathbf{p}_1\|\,\mathbf{e} \tag{14}$$

where $\mathbf{e} = (1\,0\ldots0)^\top$, $\top$ *denotes the transpose operator and* $\|\mathbf{p}_1\|$ *denotes the 2-norm of vector* $\mathbf{p}_1$*.*

Thanks to Eqn. (14), one has

$$\hat{\Lambda}(\widehat{\mathbf{M}}_{FP}) = \frac{|\mathbf{e}^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{z}|^2}{(\mathbf{e}^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{e})(\mathbf{z}^H\left(\mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H\right)^{-1}\mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,,$$

where $\mathbf{z} = \mathbf{U}\,\mathbf{y}$ *is also* $\mathcal{N}(\mathbf{0},\mathbf{I})$ *.*

By setting $\widehat{\mathbf{M}}_{FP,\mathbf{I}} = \mathbf{U}\widehat{\mathbf{T}}\mathbf{U}^H$ *, we see that the distribution of* $\hat{\Lambda}(\widehat{\mathbf{M}}_F P)$ *does not depend on* $\mathbf{M}$*, which completes the proof of Theorem 2.1. Note also that the distribution of* $\hat{\Lambda}(\widehat{\mathbf{M}}_F P)$ *does not depend on the steering vector* p*.*

In this section, the statistical performance of the FP estimate has been investigated as well as the SIRV-CFAR (texture-CFAR and M-CFAR) property of the adaptive GLRT, built with $\widehat{\mathbf{M}}_{FP}$. One of the first deduction of previous results is that whatever the SIRV used, for different distributions of the texture and for different covariance matrices, the resulting GLRT $\hat{\Lambda}(\widehat{\mathbf{M}})$ follows the same distribution. This is of a major interest in areas of clutter transition like for example, in coastal areas (ground and sea) or at the edge of forests (fields and trees) because the detector should be insensitive to the different clutter areas. This is the object of the next section.

3. RADAR APPLICATIONS

This section is devoted to the analysis of different radar measurements in which the clutter is strongly impulsive. In a first time, let us give some generalities.

In radar detection, the analysis falls into two independent stages:

- The calculation of the detection threshold λ to ensure a false alarm rate, given by the operator. This part is performed by a learning of the clutter.
- The comparison of the adaptive GLRT $\hat{\Lambda}(\widehat{\mathbf{M}})$ with the detection threshold.

Let us define some notations:

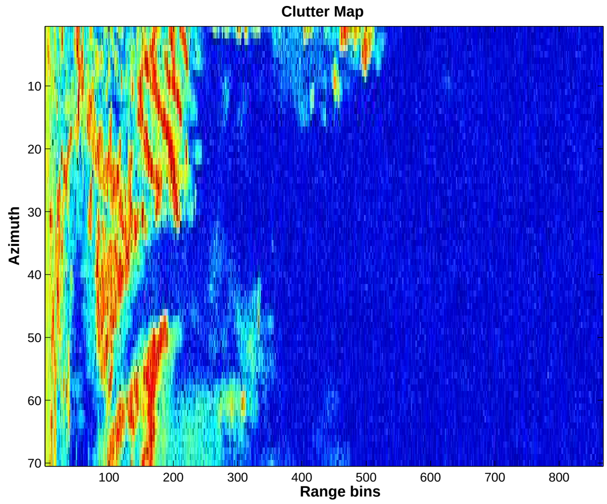

Figure 1: Ground clutter data level (in dB) corresponding to the first pulse echo. Y-coordinates represent 70 azimuth angles and X-coordinates represent $N = 868$ range bins.

- the Probability of False Alarm

$$P_{fa} = \mathbb{P}(\Lambda > \lambda | H_0)\,, \tag{15}$$

- the Probability of Detection (PD)

$$P_d = \mathbb{P}(\Lambda > \lambda | H_1)\,. \tag{16}$$

In [12], a theoretical relationship between the detection threshold λ and the PFA has been established when the covariance matrix $\mathbf{M}$ is estimated by the well known Sample Covariance Matrix (SCM) estimate defined by

$$\widehat{\mathbf{M}}_{SCM} = \frac{m}{N}\sum_{i=1}^{N}\mathbf{c}_i\,\mathbf{c}_i^H\,. \tag{17}$$

Now, the expression of PFA-threshold relationship in this specific case (SCM estimate) is

$$P_{fa} = (1-\lambda)^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda)\,, \tag{18}$$

where $a = N - m + 2$, $b = N + 2$ and ${}_2F_1$ is the hypergeometric function [13] defined as

$${}_2F_1(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{k=0}^{\infty}\frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(c+k)}\frac{x^k}{k!}. \tag{19}$$

Moreover, thanks to point (4) of proposition 2.1 and since the SCM (17) is Wishart distributed [14], expression 18 still holds for large N, when the covariance matrix $\mathbf{M}$ is estimated by the FP estimate:

$$P_{fa} = (1-\lambda)^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda)\,, \tag{20}$$

where $a = \dfrac{m}{m+1}N - m + 2$ and $b = \dfrac{m}{m+1}N + 2$. It means that it is the same relationship but with less secondary data ($\dfrac{m}{m+1}N$ data instead of N in the Wishart case).

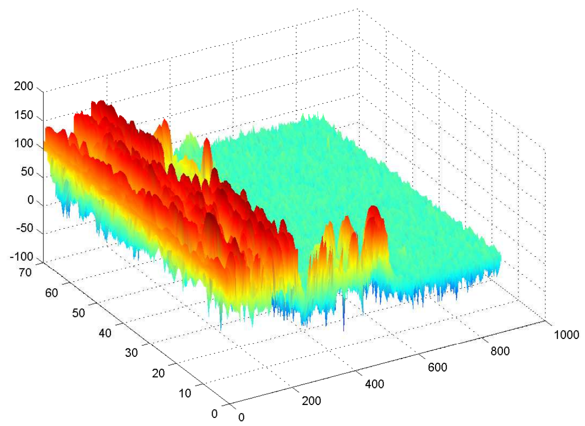

Figure 2: Ground clutter data level (in dB) corresponding to the first pulse echo. Y-coordinates represent 70 azimuth angles and X-coordinates represent $N = 868$ range bins.

This result has never been validated on real measurements: this is one of the purposes of this section.

The ground clutter data presented in this paper were collected by an operational radar at THALES Air Defence[1], placed 13 meters above ground and illuminating the ground at low grazing angle. Ground clutter complex echoes were collected in $N = 868$ range bins for 70 different azimuth angles and for $m = 8$ recurrences, which means that vectors size is $m = 8$. Near the radar, echoes characterize non-Gaussian heterogeneous ground clutter whereas beyond the radioelectric horizon of the radar (around 15 kms) only Gaussian thermal noise (the dark part of the map) is presented (Figure 1). To emphasize the areas of impulsive clutter, Figure 2 represents in 3 dimensions, the same range bin-azimuth map as on Figure 1: the third dimension (vertical) shows the power of the clutter.

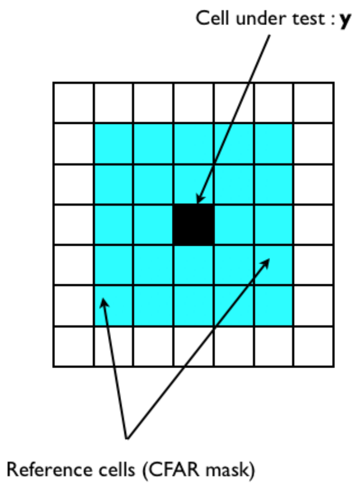

Figure 3: CFAR mask

The analysis of these radar data allows to adjust the detection threshold λ for a given PFA. A common procedure is to set this threshold, which is a system design parameter, based on the designer perception of tradeoffs between false alarms and missed detection. Traditionally, the experimental detection threshold adjustment is determined by counting, by moving a rectangular CFAR-mask of size 5×5. For all central cells of the mask (i.e. the cell under test), the dark cell on Figure 3, corresponding to the studied observation $\mathbf{y}$ (8-vector), a value of $\hat{\Lambda}(\widehat{\mathbf{M}})$ is calculated. The covariance matrix $\widehat{\mathbf{M}}$ has been estimated with the set of $N = 24$ 8-vectors, considered as the secondary data, $\mathbf{y}_1, \ldots, \mathbf{y}_{24}$, and situated around the tested cell. These reference cells are the light blue cells on Figure 3.

3.1 Validation of Eqn.(20) on real data

One purpose of this paper is to validate the theoretical relationship (20) between the detection threshold and the PFA thanks to counting of the real data when the covariance matrix is estimated by the FP estimate.

Moreover, when it is assumed that the covariance matrix $\mathbf{M}$ is known, one has $\lambda = 1 - P_{fa}^{-\frac{1}{1-m}}$ (see for example [15]). Notice that this equation has just a theoretical interest because in practice, $\mathbf{M}$ is always unknown.

Remark 3.1
Note that the counting system on the real data makes sense only thanks to the M-CFAR property of the adaptive detector. Indeed, there is no valid reason why all the sets of 24 data have the same covariance matrix.

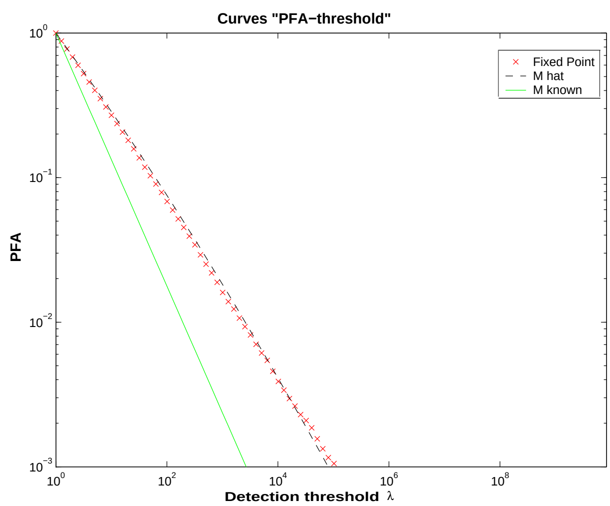

Figure 4: Validation of PFA-threshold relationship

On Figure 4, the solid curve corresponds to the theoretical relationship "PFA-threshold" if $\mathbf{M}$ is known while the dotted curve represents the theoretical relationship "PFA-threshold" when $\mathbf{M}$ and is assumed unknown and estimated by $\widehat{\mathbf{M}}_{FP}$.

The curve made of crosses ($\times$) represents the experimental (made with CFAR masks by counting) relationship "PFA-threshold" when $\mathbf{M}$ is estimated by $\widehat{\mathbf{M}}_{FP}$. It perfectly matches the theoretical relationship. Obtaining this result has

[1] Authors are grateful to Thales Air Defence for the analysis of their data

been possible only because the detector $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ satisfies the M-CFAR property, essential in an heterogeneous clutter.

Thus, this application validates Eqn. (20) and an essential consequence of this result is that thanks to Eqn. (20), the clutter training is not essential any more for the adjustment of the detection threshold.

3.2 Robustness to the clutter transitions

Figure 5 presents, for all the points of the range bin-azimuth map, the GLRT calculated for the FP estimate $\widehat{\mathbf{M}}_{FP}$. This map was made from the 8 available maps of real data.

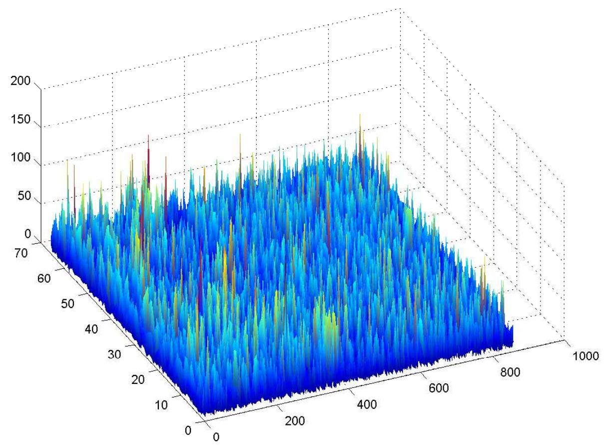

Figure 5: Detector map

We can conclude from Figure 5 that in spite of the clutter heterogeneity, on the right hand side of Figure 1, the use of the FP estimate in the GLRT allows to obtain a completely uniform likelihood ratio map. This experimental result ensures a constant false alarm regulation, even in the transitions areas. Moreover, it is in a good agreement with the theory and is directly provided by the SIRV-CFAR property of $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$.

4. CONCLUSION

In this paper, the M-CFAR property of the adaptive detector GLRT $\hat{\Lambda}(\widehat{\mathbf{M}}_{FP})$ built with the FP estimate of the covariance matrix $\mathbf{M}$ has been established. This result has been used in a radar application on real non-Gaussian data. This property stated the independence between the GLRT distribution and the real covariance matrix $\mathbf{M}$ of the data.
Moreover, a goal of this paper was the analysis of non-Gaussian experimental ground clutter signals. For that purpose, we first validated a theoretical relationship between the detection threshold and the PFA established in [12], thanks to non-Gaussian data. Then, we highlighted the GLRT robustness to the clutter transitions thanks to it SIRV-CFAR property.

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