OFF-GRID RADAR TARGET DETECTION WITH THE NORMALIZED MATCHED FILTER: A MONOPULSE-BASED DETECTION SCHEME

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ABSTRACT

In this paper, we tackle the problem of off-grid radar target detection based on the Normalized Matched Filter. State-of-the-art solutions to this problem include the use of the Generalized Likelihood Ratio test (GLRT), whose implementation can be costly, subspace detectors, or oversampling. In this paper, we introduce a new solution for improving detection of an off-grid target. It is inspired by monopulse angle estimation methods. Using simulations, we show that our solution is comparable to the GLRT under Gaussian noise hypothesis with a known covariance matrix in terms of detection probability. As such, its detection performance appears to be often better than those of detectors in the same computational cost range, and significantly better in some cases.

***Index Terms*—** Radar, Detection, Off-grid, Monopulse, Normalized Matched Filter

1. INTRODUCTION

The goal of radar systems is to detect the presence of targets with unknown parameters in unknown environment. Classically, in presence of unknown parameters, the commonly used detection strategy is the Generalized Likelihood Ratio test (GLRT) that replaces the unknown parameters by their Maximum Likelihood estimators (MLE) in the detection test. Unfortunately, analytical MLE solutions are not available for some target parameters of interest (Doppler shift, distance, and direction for example).

Therefore, for ease of implementation, most detection strategies assume that target parameters lie over a discrete set, called the grid. However, target parameters have no reason to fall exactly on the grid, since they are distributed over a continuous range. This mismatch between the tested parameters and the real target parameters deteriorates the response of most state-of-the-art tests. For example, in homogeneous Gaussian environment, it is well known that the Matched Filter (MF) test can approximately suffer from a maximum of 3dB loss for common grid sampling strategies (e.g. at the resolution cell). This tends to deteriorate the detection probability. The off-grid impact has also been shown to be particularly dramatic for other detection schemes like the Normalized Matched Filter (NMF) test. In some cases, the detection probability may vanish to 0 even for high Signal to Noise ratio (SNR) [1], especially for common low probability of false alarm (P_{FA}).

To overcome this problem, the most obvious solution is to approximate the GLRT on a refined parameter grid, but this can be costly in terms of computations. Other works have addressed this issue in adaptive context with a steering vector mismatch (bounded angle mismatch) [2], but they are not yet suited for low P_{FA} radar contexts [3]. The authors in [4] proposed a two-stage detector to reduce the loss in performance. Those solutions do not cure the asymptotic SNR loss under grid mismatch [5]. Some authors have proposed to model the mismatch through relevant subspaces [6] using subspace detection framework.

In this paper, we propose a new method that solves the off-grid problem. It enables to achieve a detection probability of 1 at high SNR, even for arbitrary low P_{FA}. Moreover, this is achieved at a low computational cost.

Section 2 presents the signal model and the off-grid detection problem. We present our method in Section 3. Finally, its performance is evaluated in Section 4 through simulations.

Notations: Matrices are in bold and capital, vectors in bold. For any matrix $\mathbf{A}$ or vector, $\mathbf{A}^T$ is the transpose of $\mathbf{A}$ and $\mathbf{A}^H$ is the Hermitian transpose of $\mathbf{A}$. $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Gamma})$ is the circular complex Normal distribution of mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Gamma}$.

2. THE OFF-GRID RADAR DETECTION PROBLEM

In radar detection, the main problem consists in detecting a complex signal $\mathbf{s} \in \mathbb{C}^N$ corrupted by an additive noise $\mathbf{n}$ (clutter, thermal noise, etc.). This problem can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{r} = \mathbf{n}\,, \\ H_1 : \mathbf{r} = \alpha\, \mathbf{s}(\boldsymbol{\theta}) + \mathbf{n}\,, \end{cases} \tag{1}$$

where $\mathbf{r}$ is the complex N-vector of the received signal, α is an unknown complex target amplitude and $\mathbf{s}(\boldsymbol{\theta})$ stands for

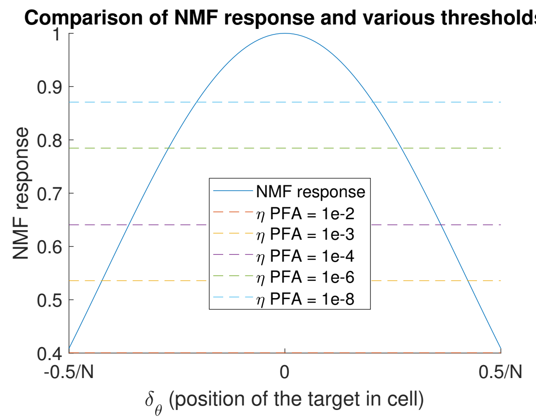

Fig. 1. Comparison of the NMF response with various thresholds $\eta = 1 - P_{FA}^{1/(N-1)}$ when $\mathbf{\Gamma} = \mathbf{I}, N = 10$.

a generally known *steering vector* characterized by unknown target parameters $\boldsymbol{\theta}$ (time-delay, Doppler or angle). In the sequel, we will assume $\mathbf{n}$ is a zero-mean complex circular Gaussian noise vector with known covariance matrix $\mathbf{\Gamma}$ but with unknown variance σ^2 i.e. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{\Gamma})$. This context is known as a partially homogeneous Gaussian environment. Without loss of generality, we will here assume $\boldsymbol{\theta}$ to be the scalar Doppler shift of the target and we here investigate only the normalized Doppler steering vector $\mathbf{s}(\theta)$:

$$\mathbf{s}(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{2i\pi\theta}, \dots, e^{2i\pi(N-1)\theta}\right]^T. \quad (2)$$

This model of steering vector is often encountered in radar Range-Doppler detection schemes where the problem consists in estimating a complex sinusoid embedded in noise after range Matched Filter processing.

For unknown parameters $\{\boldsymbol{\lambda}\}_{i\in[0,1]}$ depending on each hypothesis $\{H_i\}_{i\in[0,1]}$ (either parameters of interest and/or nuisance parameters), the usual procedure to derive the best strategy relies on the Generalized Likelihood Ratio (GLR) statistic, namely the ratio $\Lambda(\mathbf{r})$ between the Probability Density Function (PDF) $f_{H_1}(.)$ of the data under H_1 and the PDF $f_{H_0}(.)$ under H_0 where the unknown parameters are replaced by their MLE estimate:

$$\Lambda(\mathbf{r}) = \frac{\max_{\boldsymbol{\lambda}_1} f_{H_1}(\mathbf{r})}{\max_{\boldsymbol{\lambda}_0} f_{H_0}(\mathbf{r})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (3)$$

When $\boldsymbol{\lambda}_1 = \{\alpha, \sigma\}$ and $\boldsymbol{\lambda}_0 = \{\sigma\}$ with $\theta = \theta_0$ known, the corresponding GLRT is known as the NMF (Normalized Matched Filter) [7, 8]:

$$t_{\mathbf{\Gamma}}(\mathbf{r}, \theta_0) = \frac{\left|\mathbf{s}(\theta_0)^H \, \mathbf{\Gamma}^{-1} \, \mathbf{r}\right|^2}{\left(\mathbf{s}(\theta_0)^H \, \mathbf{\Gamma}^{-1} \, \mathbf{s}(\theta_0)\right) \, \left(\mathbf{r}^H \, \mathbf{\Gamma}^{-1} \, \mathbf{r}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (4)$$

This test is also widely used for adaptive radar in non-Gaussian contexts [9, 10] for example when the noise is distributed according to a Complex Elliptically Symmetric (CES) distribution [11].

When θ is unknown, what is done in practice is to run a variety of tests. The set of the parameters Θ used for testing is called the grid. For a parameter θ_0 on the grid, we will define the cell of width Δ associated with this parameter as

$$\nu_{\theta_0} = [\theta_0 - \Delta/2, \theta_0 + \Delta/2]\,. \quad (5)$$

The grid step size Δ is classically taken such that $\mathbf{s}(\theta_0)$ and $\mathbf{s}(\theta_0 + \Delta)$ are orthogonal. In our case, $\Delta = \dfrac{1}{N}$ i.e. the grid samples are separated by the Doppler resolution.

Each test is run with the assumption that the target parameters are equal to those under test. If a target is detected for any of those tests, it will be considered as detected. However, target parameters θ are never exactly equal to the parameters under test, as target parameters are continuous.

If the point θ_0 where the NMF is tested is different from the parameter θ of the target, the target is said to be off-grid. This induces a mismatch ($\theta \neq \theta_0$) between the real target steering vector $\mathbf{s}(\theta)$ and the steering vector $\mathbf{s}(\theta_0)$ under test. Unfortunately, it was shown in [5] that the NMF detector is very sensitive to steering vector mismatch, potentially leading to a dramatic deterioration of the detection performance: in particular for a mismatch larger than the detection threshold, it was shown that the asymptotic detection probability tends to 0 at high SNR: this phenomenon occurs [1] for P_{FA} as high as 10^{-3}. Indeed, the NMF value can be interpreted as a squared cosine between $\mathbf{\Gamma}^{-1/2}\mathbf{s}(\theta_0)$ and $\mathbf{\Gamma}^{-1/2}\mathbf{r}$ [5]. As soon as the mismatch causes this angle to lie outside the detection cone defined by the detection threshold η in (4), no detection occurs even without any noise. This can be seen in Figure 1, which displays the NMF response without any noise as a function of the mismatch: this response falls below the detection thresholds at the edge of the cell. This pathological behavior motivates the search for robust detection schemes.

The most natural solution is to consider the GLRT with unknown $\theta \in \nu_{\theta_0}$:

$$GLRT(\mathbf{r}, \theta_0) = \max_{\theta \in \nu_{\theta_0}} t_{\mathbf{\Gamma}}(\mathbf{r}, \theta) \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (6)$$

Unfortunately, a closed-form expression of this test is unknown, it has to be approximated using possibly computationally-heavy methods.

3. PROPOSED MONOPULSE-INSPIRED SCHEME

In this section, we introduce a new method for coping with the mismatch. We took inspiration from monopulse methods [12], and consider the following function:

$$h_{\mathbf{\Gamma},\theta_0}(\mathbf{r}) = \frac{t_{\mathbf{\Gamma}}\left(\mathbf{r}, \theta_0 - \dfrac{\Delta}{2}\right) - t_{\mathbf{\Gamma}}\left(\mathbf{r}, \theta_0 + \dfrac{\Delta}{2}\right)}{t_{\mathbf{\Gamma}}\left(\mathbf{r}, \theta_0 - \frac{\Delta}{2}\right) + t_{\mathbf{\Gamma}}\left(\mathbf{r}, \theta_0 + \dfrac{\Delta}{2}\right)}. \quad (7)$$

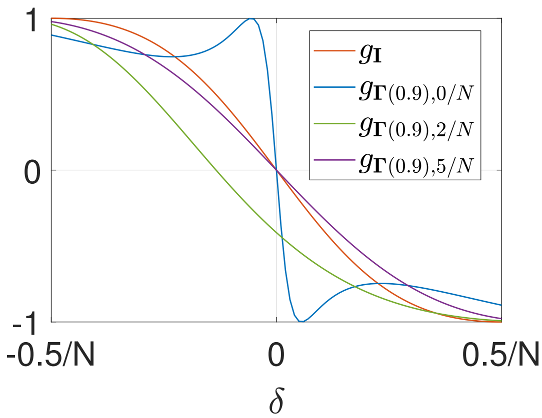

Fig. 2. Candidate $g(.)$ functions for N=10. $\mathbf{\Gamma}(\rho)$ refers to the covariance matrix model in (10)

The principle is the following: without noise, knowing two test points of the NMF, namely $\theta_0 + \frac{\Delta}{2}$ and $\theta_0 - \frac{\Delta}{2}$, in the main lobe (here, the resolution cell) is enough to provide an estimate of θ, as the distribution of energy between those points is directly linked to the mismatch:

$$\delta = \theta - \theta_0. \tag{8}$$

The quantity (7) known as the monopulse ratio can be seen as an approximation of the MLE [12]. The link between δ and $h_{\mathbf{\Gamma},\theta_0}(\mathbf{r})$ is made thanks to the noiseless $g_{\mathbf{\Gamma},\theta_0}$ function defined as follows:

$$g_{\mathbf{\Gamma},\theta_0}(\delta) = h_{\mathbf{\Gamma},\theta_0}(\mathbf{s}(\theta_0 + \delta)) \tag{9}$$

Our goal is to find an estimate $\hat{\theta} = \hat{\delta} + \theta_0$ of θ, to plug it in the NMF in order to approximate the GLRT in (6). For that purpose, $\hat{\delta}$ is obtained by computing $\hat{\delta} = g^{-1}_{\mathbf{\Gamma},\theta_0}(h_{\mathbf{I},\theta_0}(\mathbf{r}))$. Of course, this only makes sense if $g_{\mathbf{\Gamma},\theta_0}$ is invertible. As can be seen in Figure 2, this is not always the case, depending on $\mathbf{\Gamma}$ and θ_0. For the particular case $\mathbf{\Gamma} = \mathbf{I}$, the functions $g_{\mathbf{\Gamma},\theta_0}$ are invertible. This is why we will use these functions, even in colored noise. Moreover, they do not depend on θ_0 and will be simply denoted by:

$$\begin{aligned} g(\delta) &= g_{\mathbf{I},\theta_0}(\delta) \\ &= \frac{\left(\frac{\sin\left(\pi N(\delta+\Delta)\right)}{\sin\left(\pi(\delta+\Delta)\right)}\right)^2 - \left(\frac{\sin(\pi N(\delta-\Delta))}{\sin(\pi(\delta-\Delta))}\right)^2}{\left(\frac{\sin(\pi N(\delta+\Delta))}{\sin(\pi(\delta+\Delta))}\right)^2 + \left(\frac{\sin(\pi N(\delta-\Delta))}{\sin(\pi(\delta-\Delta))}\right)^2}. \end{aligned}$$

The function g takes its values between -1 (at the right edge of the cell) and 1 (at the left edge of the cell).

To summarize, our proposed method is as follows: for every θ_0 on the grid:

1. compute $t_{\mathbf{I}}\left(\mathbf{r}, \theta_0 - \frac{\Delta}{2}\right)$ and $t_{\mathbf{I}}\left(\mathbf{r}, \theta_0 + \frac{\Delta}{2}\right)$;
2. compute $\hat{\delta} = g^{-1}\left(h_{\mathbf{I},\theta_0}(\mathbf{r})\right)$;
3. run the final tests $t_{\mathbf{\Gamma}}\left(\mathbf{r}, \hat{\delta} + \theta_0\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta_g$.

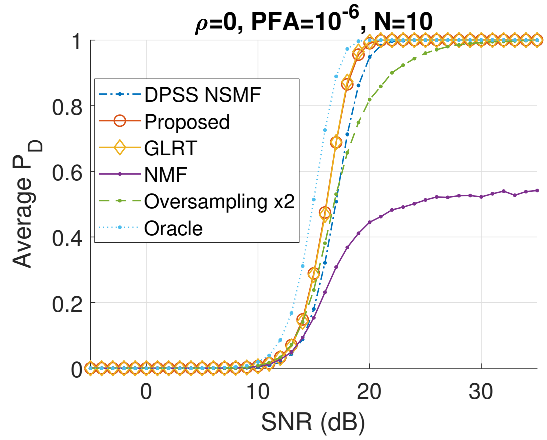

Fig. 3. P_D of the detectors in white Gaussian noise, for a P_{FA} of 10^{-6}, $N = 10$.

Finding η_g analytically is much more involved than for the NMF. This is why η_g was computed resorting to Monte Carlo methods.

Let us describe some properties of this approach. Firstly, when the SNR tends to infinity, $\mathbf{r}$ is equivalent to $\alpha\mathbf{s}(\theta)$ and our procedure yields the true parameter value i.e. $\hat{\theta} = \theta$. This ensures that the probability of detection tends to 1. Secondly, when $\mathbf{\Gamma} = \mathbf{I}$, $\hat{\theta}$ is an approximate MLE [12] and our test is an approximate GLRT. Thirdly, simulations show that our procedure is still close to the GLRT even when $\mathbf{\Gamma} \neq \mathbf{I}$.

Finally, let us evaluate the computational load of this scheme. In practice, the method has to be applied in each of the N resolution cells. In total, $t_{\mathbf{\Gamma}}$ needs to be computed $2N$ times. Consequently, the overall computational load is equivalent to that of approximating GLRT using 2 points per cell, neglecting the cost of inverting g, which is minimal since g can be tabulated offline. It turns out that our approach outperforms existing methods with equivalent computational cost.

4. NUMERICAL RESULTS

In this section, the performance of our detection scheme is illustrated using Monte Carlo simulations. In each simulation, target parameter θ is drawn at random, uniformly in a given cell. For a given cell under test, we compared the performance of different detectors as a function of SNR. The detectors we evaluated are the DPSS NSMF with subspaces of dimension 2 introduced in [1], our scheme under the name ”Proposed” in the figures, an approximation of the GLRT done computing 50 tests per cell, the classical and mismatched NMF under

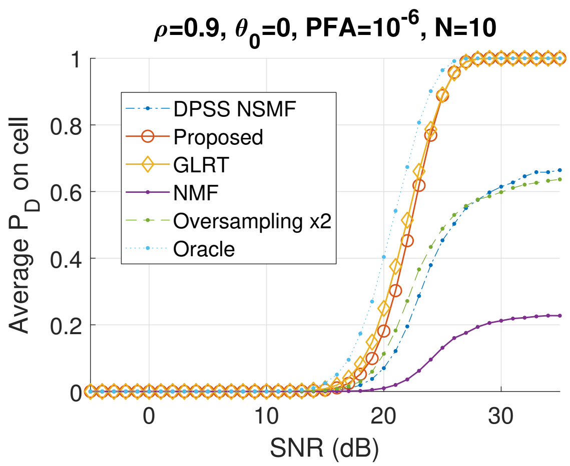

Fig. 4. P_D versus SNR with $\rho = 0.9$ at cell $\theta_0 = 0$ for a P_{FA} of 10^{-6}, $N = 10$.

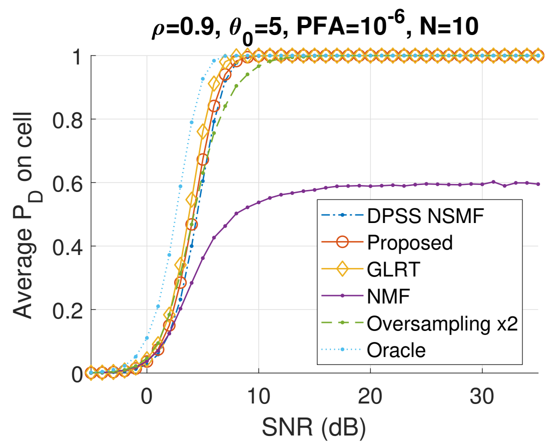

Fig. 5. P_D versus SNR with $\rho = 0.9$ at cell $\theta_0 = 5$ for a P_{FA} of 10^{-6}, $N = 10$.

the name "NMF" and a crude approximation of the GLRT exploiting the maximum of two tests per cell under the name "Oversampling x2". Performance for known θ is also provided as "Oracle". Oracle and the GLRT are given as benchmarks. Other detectors ("DPSS NSMF","Oversampling x2") are in the same computational range as our proposed method. Thresholds are computed for each detector, using $20\, P_{FA}^{-1}$ Monte Carlo simulations. For each SNR, 20000 Monte Carlo simulations are run, considering steering vectors of length $N = 10$ and setting $P_{FA} = 10^{-6}$.

Figure 3 first compares the performance of our method with the other detection schemes under additive white noise ($\mathbf{\Gamma} = \mathbf{I}$). Note that in this case, all the cells are equivalent in terms of detection probability. It can be seen that our detector is very close to the GLRT even though its computational cost is lower. As such, it does better than the detectors it is compared to, at a similar cost.

Our proposed detection scheme was also tested against colored Gaussian noise. Its covariance matrix was generated according to the following well-known model:

$$\mathbf{\Gamma}(\rho) = \mathcal{T}\left(\begin{bmatrix}1 & \rho & \dots \rho^{N-1}\end{bmatrix}\right), \tag{10}$$

where $\mathcal{T}(.)$ is the Toeplitz operator. We here tested all the schemes against colored noise of covariance matrice $\mathbf{\Gamma}(0.9)$. It should be noted that when $\mathbf{\Gamma} \neq \mathbf{I}$, tests behave differently for each resolution cell. In Figure 4, the target is located in the cell centered around $\theta_0 = 0/N$, and in Figure 5, $\theta_0 = 5/N$. The relationship between $|\alpha|^2$ and the SNR in dB is defined as:

$$\mathrm{SNR} = 10 \log_{10}\left(\frac{|\alpha|^2}{\sigma^2}\right). \tag{11}$$

Note that for the oracle detector, the detection probability depends only on the non-centrality parameter $\frac{|\alpha|^2}{\sigma^2}\, \mathbf{s}(\theta)^H \mathbf{\Gamma}^{-1} \mathbf{s}(\theta)$.

This non-centrality parameter characterizes the ratio between the target power and the noise power in the cell under test. It partly explains why the performance depends on the choice of the cell under test.

In cell ν_0, the shape of the whitened steering vectors variety makes it so that both DPSS NSMF and oversampling by a factor of two do not approach a detection rate of 1 as SNR becomes very high. However our scheme performs only a bit worse than GLRT, and as expected its detection rate increases towards 1 when the SNR is high. In cell $\nu_{5/N}$, all detectors perform much better than in cell $\nu_{0/N}$ and than in white noise, which was to be expected because of the non-centrality parameter. In this case, the shape of the whitened steering vectors variety also makes it possible for the detectors to detect targets asymptotically. As a result, the probability of detection of DPSS NSMF and oversampling by a factor of two tends toward 1 as SNR increases. In this context, our scheme does approximately as well as the DPSS NSMF detector asymptotically, which makes it slightly worse than GLRT, as it was before.

5. CONCLUSION

This paper is motivated by detector performance degradation in the case of off-grid targets. The proposed solution is based on monopulse technique and can be considered as an approximate GLRT for off-grid targets in Gaussian noise. It also overcomes the problem of the off-grid NMF not reaching a probability of detection equal to 1 when SNR tends to infinity. It outperforms existing detectors of the same computational cost order and provides detection performance close to the true GLRT, whatever the noise covariance matrix.

Future works will investigate the derivation of the P_{FA}-threshold relationships, and the adaptive case as well as non-Gaussian additive noise environments.

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